Journal of Economics and Business Research,
ISSN: 2068 - 3537, E – ISSN (online) 2069 – 9476, ISSN – L = 2068 – 3537
Year XXI, No. 1, 2015, pp. 98-107

A Literature Review Fuzzy Pay-Off-Method – A Modern Approach in Valuation

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Abstract
This article proposes to present a modern approach in the analysis of updated cash flows. The approach is based on the Fuzzy Pay-Off-Method (FPOM) for Real Option Valuation (ROV). This article describes a few types of models for the valuation of real options currently in use. In support for the chosen FPOM method, we included the mathematical model that stands at the basis of this method and a case study.
Keywords: Cash-Flow, Pay-off-Method, ROV – Real Option Valuation, Fuzzy, NPV – Net Present Value**.**

Introduction

The analysis of updated cash flows is to be found in the future revenue actualization method. This method can be applied within the income approach, alongside three other techniques: the updated value technique; the revenue and expenditure estimation and direct capitalization.

The update is a technique used to convert a series of future income in their present value using an appropriate discount rate. In order to apply the method, the following steps must be followed:

- Selecting a period of ownership of the investment
- Predicting future cash flows
- Selecting the discount rate
- Converting future cash flows in their updated value

In order to use the updating method, we must be used with the cash flow analysis. The general formula used for updated cash flows is the following (ANEVAR, IROVAL 2008):

$$Updated\ value = \frac{CF_1}{(1+k)^1} + \frac{CF_2}{(1+k)^2} + \frac{CF_3}{(1+k)^3} + \cdots + \frac{CF_n}{(1+k)^n} = \sum_{i=1}^{n} \frac{CF_i}{(1+k)^i}$$

where:

CF_i = cash flows related to a time-span/year

K = discount rate

i = the number of periods/years of investment ownership

Real options are investment possibilities and possibilities within investment projects. Real option valuation is valuing these possibilities as “real” options. Valuation started by using the same methods that have been used for financial option valuation.

Next we treat the real options as a modeling problem. The three major components of modeling the value of an (real) option are:

- to model of the future value distribution;
- to calculate the expected value of the future value distribution while mapping negative values of the distribution zero;
- to model the calculation of the PV - present value of the expected value.

In present there are four types of models for the valuation of real options currently in use. These are the following:

1. *differential equation solutions*
2. *discrete event and decision models*
3. *simulation based models*
4. *fuzzy logic based methods*

Types of models representative for the four aforementioned valuation models are the following:

1. *The Black & Scholes Model*
2. *The Binomial Option Valuation Model*
3. *Datar-Mathews Model for ROV*
4. *FPOM (Fuzzy Pay-off Method) for ROV.*

Literature review

Black & Scholes Model – B&S M(1973)

This model is a stochastic (GBM) process used to yield a continuous log-normal distribution of future asset value. It consists in calculation of expected value as a probability weighted average of the positive side of the future value distribution, also discounting the expected value to present value with the risk-free rate of return, continuous compounding. Same discount rate will be used for revenue and cost. The input parameters are fed into a formula and it out pops the answer (Collan, Carlsson and Majlender, 2003).

B&S M is a closed form solution, based on the replication argument: Any two assets that yield same cash-flows under the same circumstances must be worth the same price. From a Mathematical point of view it is very elegant and is based on a strict set of assumptions about markets (complete/efficient). The result is presented as a single number (Buckley and Esfandiar, 2008).

Binomial Model -BM (1979)

The BM is a binomial tree process used to create a future value distribution. It uses the backwards iteration to find the option value. Iteration includes discounting with a compounding risk-free rate of return for cost and revenues. With a large number of time steps the result converges with the result from the Black & Scholes model. BM is based on strict assumptions about the markets and needs the input info for standard deviation and up/down probability. The result is presented as a single number, as in the previous model (Boyle, 1986).

Datar –Mathews Model – DMM (2004)

The DMM creates future value distribution, using manager-created cash-flow scenarios as input for a Monte Carlo simulation project that creates a probability distribution (Datar, Mathews and

Johnson, 2007). The expected value is calculated as the probability, weighted average of the positive side of the future value distribution. Thus, the discount rate could be separated from the costs and CF revenues, based on selecting a range of periods. They are no strict assumptions about the markets/reality. The model is designed for spread-sheet software and generates the same answer as Black & Scholes, reachable with same assumptions. The result is presented as a single number, as in the previous models (Mathews and Datar, 2007).

Pay-Off Method for Real Option Valuation (2009)

In POM manager created cash-flow scenarios are used to create (simplified) future value distributions that are treated as fuzzy numbers (often triangular / trapezoidal). The ”expected value” is calculated from the fuzzy numbers as the possible mean of the positive side of the fuzzy number. Discounting is done by using separate discount rates for revenues and costs. The POM for ROV compounds interval at the discretion of the user, is no strict assumptions about the markets/reality, is spread-sheet compatible and is designed for the practitioner –”as easy as possible”.

The main observations of the Fuzzy Pay-Off Model – FPOM are the following:

- The fuzzy NPV of a project is (equal to) the pay-off distribution of a project value that is calculated with fuzzy numbers.
- The mean value of the positive values of the fuzzy NPV is the possible mean value of the positive fuzzy NPV values.
- ROV calculated from the fuzzy NPV is the possible mean value of the positive fuzzy NPV values multiplied with the positive area of the fuzzy NPV over the total area of the fuzzy NPV.

The ROV value can be derived from the fuzzy NPV. These are the blocks that together make the FPOM for ROV.

Mathematical Model for FPOM

In POM for ROV the real option value is calculate from the fuzzy Net present Value – NPV, with the formula (Collan, Fullér and Mézei, 2009):

$$ROV = \frac{\int_0^{\infty} A(x)dx}{\int_{-\infty}^{\infty} A(x)dx} \times E(A_+)$$

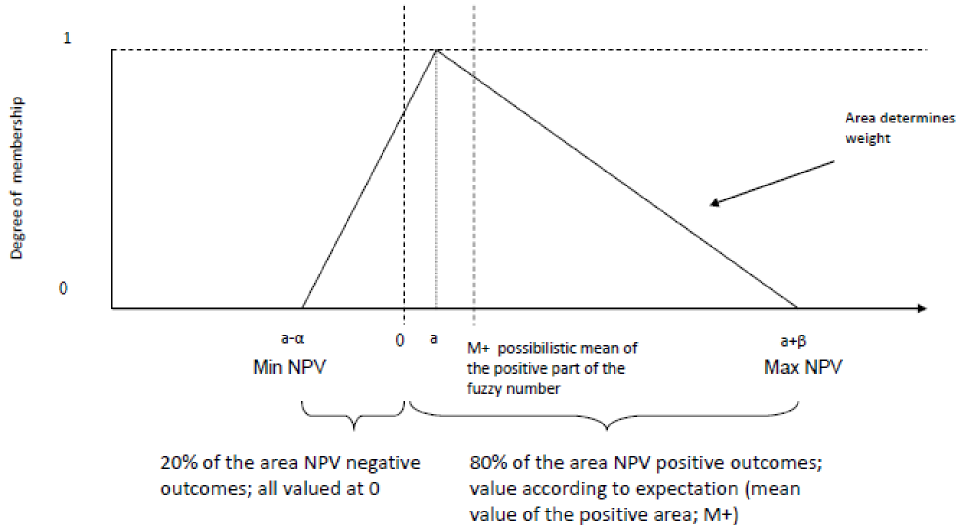
where (see Figure 1):
A - stands for the fuzzy NPV
E(A+) - denotes the fuzzy mean value of the positive side of the NPV
$\int_{-\infty}^{\infty} A(x)dx$- computes the area below the whole fuzzy number A
$\int_0^{\infty} A(x)dx$- computes the area below the positive part of A

A positive ROV indicates that it is profitable to invest.

The most used fuzzy numbers are trapezoidal and triangular fuzzy numbers, because they make many operations possible and are intuitively understandable and interpretable.

In this article we use the triangular fuzzy numbers.

Fig. no. 1. A triangular fuzzy number A, defined by three points [a,α,β] describing the NPV of a prospective project (percentages are for illustration purpose only) (Collan, Fullér and Mézei, 2009)



The membership function of the right-hand side of a triangular fuzzy number truncated at point a-α-z, where 0≤z≤α.

$$(A|z)(t) = \begin{cases} 0 \; if \; t \leq a - \alpha + z, \\ A(t) \; otherwise \end{cases}$$

So, we get the expected value of the triangular fuzzy number (Collan, Fullér and Mézei, 2009):

$$E(A_+) = \begin{cases} (a-\alpha) > 0 \; then \; E(A_+) = a + \dfrac{\beta - \alpha}{6} \\ a > 0 > (a-\alpha) \; then \; E(A_+) = a + \dfrac{\beta - \alpha}{6} + \dfrac{\alpha - a^3}{6a^2} \\ \alpha + \beta > 0 > a \; then \; E(A_+) = \dfrac{\alpha + \beta^3}{6\beta^2} \\ \alpha + \beta < 0 \; then \; E(A_+) = 0 \end{cases}$$

Case Study

Using FPOM for ROV in Analyzing the Cash Flow

In our article we present the problem of valuating the uncertain cash-flows from APPLE INC Company. The financial data refers to a three-year time period 2011-2013 and have as source the Yahoo! FINANCE Web site.

The FPOM approach is based on three steps (Muzzioli and Torricelli, 2000):

Step 1: Build a number of cash-flow scenarios and perform present value & NPV calculations;

Step 2: Create the pay-off distribution;

Pay-off distribution	Fuzzy number
Maximum scenario NPV	a+β
Best guess scenario NPV	a
Minimum scenario NPV	a-α
Distance between best guess scenario NPV and maximum scenario NPV	β
Distance between best guess scenario NPV and minimum scenario NPV	α

Step 3: Calculate descriptive for additional decision-support.

The input information available is in the form of three future cash-flow scenarios: Best estimate – most likely, Minimum possible – pessimistic, Maximum possible – optimistic.

Table no. 1 and the Figure no. 2 reflect data from the Cash Flow.

Fig. no. 2. Cash-Flow Diagram- APPLE INC Company
Data refers to a three-year time period 2011-2013

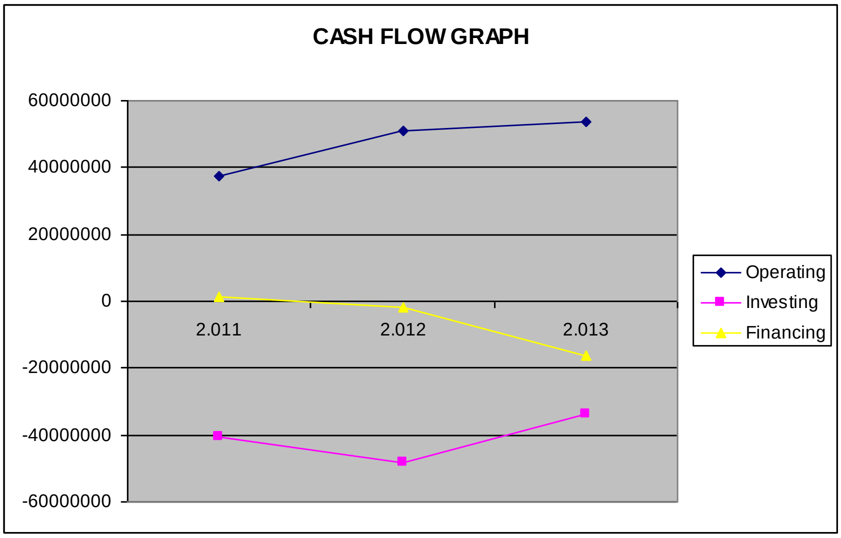


Table no. 1. Cash-Flow from Financial Statement of APPLE INC Company - data refers to a three-year time period 2011-2013

Structures	***2011***	***2012***	***2013***
Net Income	**25.922.000**	**41.733.000**	**37.037.000**
Operating Activities, Cash Flows Provided By or Used In			
Depreciation	1.814.000	3.277.000	6.757.000
Adjustments To Net Income	4.036.000	6.145.000	3.394.000
Changes In Accounts Receivables	-1.791.000	-6.965.000	-1.949.000
Changes In Liabilities	8.664.000	9.843.000	8.320.000
Changes In Inventories	275.000	-15.000	-973.000
Changes In Other Operating Activities	-1.391.000	-3.162.000	1.080.000
Total Cash Flow From	***37.529.000***	***50.856.000***	***53.666.000***

Operating Activities			
Investing Activities, Cash Flows Provided By or Used In			
Capital Expenditures	-4.260.000	-8.295.000	-8.165.000
Investments	-32.464.000	-38.427.000	-24.042.000
Other Cash flows from Investing Activities	-3.695.000	-1.505.000	-1.567.000
Total Cash Flows From Investing Activities	***-40.419.000***	***-48.227.000***	***-33.774.000***
Financing Activities, Cash Flows Provided By or Used In			
Dividends Paid	0	-2.488.000	-10.564.000
Sale Purchase of Stock	831.000	665.000	-22.330.000
Net Borrowings	0	0	16.896.000
Other Cash Flows from Financing Activities	-520.000	-1.226.000	-1.082.000
Total Cash Flows From Financing Activities	***1.444.000***	***-1.698.000***	***-16.379.000***
Effect Of Exchange Rate Changes	0	0	0
Change In Cash and Cash Equivalents	***-1.446.000***	***931.000***	***3.513.000***

Table no. 2 present the ROV obtained for Operating, Investing, Financing Cash Flow, respectively Change in Cash and Cash Equivalents.

We observe that the obtained values are positive, negative and zero. In case of Operating and Financing Cash Flow ROV is positive, that means it is profitable to invest.

The obtained data is in concordance with the Cash Flow trends presented in Figure no. 2.

Conclusions

The FPOM method is simpler to apply than other, more complex methods, mentioned in section 2.

Using triangular fuzzy numbers make very easy implementations possible with the most commonly used spreadsheet software; this opens avenues for ROV to find its way to more practitioners.

Table no. 2. ROV obtained with FPOM

Operating Cash Flow		2.011	2.012	2.013	NPV		a>α	
maximum	a+β	37.529	50.856	53.666	128.229	β= 12.529	$E(A_+)$=	25.838
best estimated	a	25.000	38.000	42.000	94.558		A_{total}=	10.015
minimum	a-α	17.500	26.000	31.800	67.719	α= 7.500	A_+=	6.265
							ROV=	16.163

Investing Cash Flow		2.011	2.012	2.013	NPV		a<0	
maximum	a+β	15.000	12.000	17.000	39.855	β= 55.419	$E(A_+)$=	-47.769
best estimated	a	-40.419	-48.227	-33.774	-111.413		A_{total}=	40.000
minimum	a-α	-65.000	-72.000	-51.000	-171.267	α= 24.581	A_+=	27.710
							ROV=	-33.091

Financing Cash Flow		2.011	2.012	2.013	NPV		a>α	
maximum	a+β	3.200	200	-7.800	-3.509	β= 700	$E(A_+)$=	2.441
best estimated	a	2.500	-275	-1.500	836		A_{total}=	878
minimum	a-α	1.444	-1.698	-16.379	-14.314	α= 1.056	A_+=	350
							ROV=	973

Change In Cash and Cash Equivalents		2.011	2.012	2.013	NPV		α+β<0	
maximum	a+β	-2650	1340	4750	2.795	β= -1.204	$E(A_+)$=	0
best estimated	a	-1446	931	3513	2.502		A_{total}=	1725
minimum	a-α	800	500	2800	3.634	α= -2.246	A_+=	602
							ROV=	0

The method is flexible as it can be used when the fuzzy NPV is generated from scenarios or as fuzzy numbers from the beginning of the analysis.

As information changes and uncertainty is reduced, this should be reflected in the fuzzy NPV, the more there is uncertainty the wider the distribution should be, and when uncertainty is reduced the width of the distribution should decrease. Only under full certainty should the distribution be represented by a single number, as the method uses fuzzy NPV there is a possibility to have the size of the distribution decrease with a lesser degree of uncertainty, this is an advantage over probability based methods.

In case when all the values of the fuzzy NPV are greater than zero, the single number NPV equals ROV, which indicates immediate investments.

The FPOM for ROV opens possibilities for making ROV evaluation tools that will help managers to construct real option analyses - ROA for systems of RO that are present currently in many types of investments.

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