

CHILDREN'S WAYS OF THINKING WHEN COPING WITH EVERYDAY MATHEMATICAL SITUATIONS

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Abstract: *Already at a very young age, children come across mathematical situations as part of their everyday life in preschool and at home. Engagement in mathematical situations helps children in developing their mathematical thinking. Listening to children while they are coping with such situations, help us learn more about their ways of thinking. This study describes the ways of thinking of four children aged 5.2–6.4 when dealing with nine everyday mathematical situations, involving addition, subtraction and division into equal groups. The children have given correct answers to almost all situations and used a variety of solution strategies. They have used mental calculations in some of the situations, while in others they have used their fingers and/or manipulatives. The children have paid attention to the numbers given in the situations and one child has surprised us by showing number sense, when using the compensation method for solving $7+5$ and $8+4$.*

Keywords: *mathematics; everyday situations; preschool children.*

1. Introduction

In recent years, there has been an increasing comprehension that more resources should be invested in early childhood education. This opinion is being held not only by educators and early childhood researchers. It is also embraced by economists who claim that investment in early childhood education pays off financially, being the key to a prosperous society. Investment in young children comes to the fore in future generations, as the children of today will be the adults of tomorrow (Barnett & Masse, 2007; National Scientific Council on the Developing Child, 2007; OECD, 2011; The World Bank, 2008).

Dealing with mathematics in early childhood is important for two main reasons. The first reason has to do with the fact that mathematics is a part of children's everyday life and is a necessary everyday activity. For example, when children are counting how many cards are there in the box; when they are checking if the number of pencils is sufficient for the number of children sitting in a group; when they compare whether they have received the same number of cucumber slices for dinner like their brothers. The second reason is grounded in the fact that mathematical activities at this age lay the foundations for the development of children's mathematical thinking. Good practices in early childhood will promote children's mathematical thinking, becoming the basis for further development of mathematical thinking at school.

Studies show that very young children can already build mathematical ideas, sometimes even abstract ones (Baroody, Lai & Mix, 2000). Many times they surprise us with their ability to cope with mathematical tasks as well as with their inquisitiveness and willingness to engage in activities related to mathematics. Greenes, Ginsburg, and Balfanz (2004) describe children's competences when field testing the program Big Math for Little Kids: "...we observed children doing mathematical work at a higher level than we expected. Indeed, we were surprised at what the children managed to accomplish" (p. 164). Hachey (2013) adds that, "We now know that prior to elementary school, young children engage in surprisingly complex intuitive mathematical thinking with regard to numbers, geometry, measurement, algebraic thinking, and data analysis" (p. 419).

However, many preschool teachers tend to make young children engage more in language than in mathematics. This might be because they either believe that language is more important than mathematics at this age or they themselves have no interest in mathematics or are afraid of it (Copley, 2000; Lee & Ginsburg, 2007; Ginsburg, Lee & Boyd., 2008). Studies show that some preschool, pre-service and in-service teachers, do not like mathematics and even hate it (Gresham, 2007; Markovits, 2012; Markovits & Patkin, 2018; Zacharos, Koliopoulus, Dokimaki & Kossoumi, 2007), and probably avoid opportunities to engage in mathematics with the children. Involving children in mathematical situations can provide a means for understanding young children's ways of coping with mathematics (Charlesworth & Leali, 2012). Exposure of preschool teachers to children's ways of thinking when dealing with mathematical situations, is important not only for understanding how children think and for paying attention to the mathematical abilities they already possess, but also for learning how to advance the development of children's mathematical thinking, Jung, Kloosterman, and McMullen (2007) conclude that young children have natural intuitions for solving problems and that teachers should listen to the children while solving problems in order to better understand their ways of thinking.

This paper describes the ways four young children coped with everyday mathematical situations, involving addition, subtraction and partition into equal groups.

2. The study

2.1. Research aims

This study aimed to learn about children's ways of thinking when coping with everyday mathematical situations. Furthermore, it aimed to observe whether the children use the same strategies and manipulatives when dealing with different situations or change strategies and manipulatives for different situations.

2.2. Research participants

Four children aged 5.2 – 6.4 took part in the study. Emily (6.3) and Sara (6.4) were in the last year of preschool and would attend school after the summer vacation. Daniel (5.2) and Noah (5.5) will complete one more year at preschool before attending the 1st grade.

2.3. The interviews

Each child was individually interviewed. Nine situations were presented by the interviewer, one at a time, and the children were asked to explain their way of thinking when coping with each situation. Manipulatives, such as cubes, beads and corks were on the table and the children were told they could use them.

2.4. The everyday mathematical situations

Mathematical everyday situations are situations involving numbers that children encounter in their everyday activities in preschool and at home. This study refers to everyday situations which are actually word problems children will face at school. The list of the everyday mathematical situations is presented:

1. Abigale had four spoons. She received two more spoons. How many spoons does she have now?
2. Daniel had four cars. He received two more cars. How many cars does he have now?
3. Amir had five bananas. He ate two bananas. How many bananas are left?
4. Jonathan had seven oranges. He ate three oranges. How many oranges are left?
5. Rona had seven colors on the table. She put three colors in her pencil-case. How many colors were left on the table?
6. Mother had eight stickers. She divided the stickers equally between her

- two daughters. How many stickers each daughter received?
7. Dana had six colors. She divided them equally between two pencil-cases
How many colors did she put in each pencil-case?
 8. Emma had seven jellybeans. She received five more. How many
jellybeans does she have now?
 9. Michael had eight dolls. He received four more. How many dolls does he
have now?

Three types of situations were presented: Addition situations categorized by Carpenter, Fenemma, Frank, Levi, and Empson (1999, cited in Jung et al., 2007) as *join with unknown result* (situations 1,2,8,9); subtraction situations categorized as *separate with unknownresult* (situations 3-5) and division to equal groups, categorized as *partitive division* (8,9).

The numbers given in the first two situations were small numbers and similar in both (the numbers four and two) in order to check whether the children would pay attention to the repetition of the numbers and decide not to solve again the second situation. The numbers in the other two addition situations were bigger, so when the children added them, the sum was more than ten. The same numbers were also repeated in two of the subtraction situations (the numbers seven and three).

3. Results

All four children were very excited to solve the situations presented to them. The following is a description of the way each child coped with the situations.

Daniel (5.2) gave correct answers to all nine situations. For situations 1 and 2, he gave the answer six and said: "I just know" for the first situation and "It is the same as before" for the second situation. Regarding the subtraction situations (3,4), he counted backwards. He explained about situation 4: "You count backwards: seven, six, five, four". For situation 5, he said that the answer was seven and explained that he counted backwards and specified: "This is similar to your last question". In situation 6, he used his fingers and said: "Four and four is eight". He did not use his fingers in situation 7 and just said: "Three. I know that three and three is six". Daniel used his fingers in situations 8 and 9. In situation 8 he lifted seven fingers to represent the seven jellybeans. Then he counted three more fingers up to ten and counted (without using fingers or manipulatives) eleven and twelve, saying that the answer was twelve. He continued using his fingers in situation 9. He lifted four fingers on each hand to represent the eight dolls, lifted one more finger on each hand and said this was already ten. Then he said that he needed to add two more dolls and this was twelve.

Noah (5.5) gave correct answers to all nine situations. In the first situation he said: "Six. I practice math with my brother. Who does not know that four and two is six?". In the second situation he asked: "Same question again? I already told you that two and four is six". In all subtraction situations (3, 4 and 5), he calculated in his head and said, for example in situation 5: "Also four. I thought in my head that seven minus three was four". He explained that he was also thinking in his head about situations 6 and 7: "If it is equal than it is the same so four and four is eight" (situation 7). In situation 8 he said immediately "twelve" and explained: "I join my brother in his math assignments. I remember he told me that seven and five was twelve". In situation 9 he said: "Twelve. I have already told you many times and you keep asking. My brother told me that seven and five was twelve. Here it is the same. You give one to the four and you take one from the eight. It is twelve."

Emily (6.3) gave correct answers to four out of five addition and subtraction situations (1-5). In the first situation she said that the answer was six and explained: "I thought about it very well. In my head I put one row of four and one row of two. Together it is six". She

again relied on the rows in her head but this time she did a manipulation: "I thought again and again. In my head I took one from the first row and added it to the second row and it is three and three. Together it is six". She used her fingers for situation 3 and said: "I did it with my fingers, five and he ate two, so four are left". She used her fingers also for situations 4 and 5, reaching the correct answer. Emily encountered difficulties in the division situations. About situation 6, she first said "I don't know". Then she used her fingers and said that it would be three. About situation 7, *six colors equally divided between two pencil-cases*, she said that "there will be two colors in each pencil-case because there are two pencil-cases". When situation 8 was presented to her, she said: "Oh... this is already hard. I think the answer is eight. Mother does not give me such difficult questions". Regarding situation 9, she said that the answer was also eight. She lifted 4 fingers on each hand and said: "Four on each hand, eight together".

Sara 6.4 gave correct answers to all nine situations. About the first situations she said: "Six spoons. I did it in my head. Mother exercises with me at home". In the second situation she said that the answer was six once again because the numbers were the same. She said that she took five bananas and took off two, so three bananas were left in situation 3. In situation 4 she used corks. She arranges seven corks in a row, moved three of them aside and said: "Four". When situation 5 was presented, she immediately said "Four colors. It is the same exercise as before". She used the corks in the division situations (6 and 7) but demonstrated different strategies. For situation 6, she took eight corks and divided them equally into two groups. She moved one cork to the left and one to the right until she moved all the corks, saying that the answer was four. For situation 7, she arranged the six corks in two rows, three in each row, and said the answer was three. For the addition situations 8 and 9, Sara used her fingers. In both situations she represented the big number by lifting her fingers. Then she lifted the rest of her fingers up to ten. For the remainder of the second number she used corks. Thus, in both situations, she lifted ten fingers and added two corks, saying that the answer was twelve.

5. Discussion

The children used a variety of strategies when they dealt with everyday mathematical situations presented to them. They explained what they were doing and, thus, enabled us to listen to their ways of thinking.

Three of the children, Daniel (5.2), Noah (5.5) and Sara (6.4) gave correct answers to all nine situations. Emily had difficulties with the division situations and the addition situations which involved bigger numbers. This is interesting because she actually solved situation 2 by dividing the six cars in her head into two equal groups. Maybe she was less exposed to the phrasing of division situations and met addition situations that mostly involved small numbers.

The children were paying attention to the numbers involved in the situations. Thus, Daniel (5.2), Noah (5.5) and Sara (6.4) noticed the similar numbers in the first addition situations (1 and 2). They did not solve the second situation but said that the answer was the same as in the previous question. All three of them also noticed the similar numbers in the subtraction situations (4 and 5).

The children used a variety of strategies in coping with the situations: counting forwards, counting backwards, division into two equal groups by putting one cork in each of the groups or by arranging the group in two rows. They sometimes used mental computation and calculated in their heads, especially when small numbers were involved. Moreover, they used their fingers and applied manipulatives. When the fingers were not enough (answer bigger than ten), they combined fingers and manipulatives. It seems that the children used different ways of solution and adjusted the way of solution to the situation presented.

Noah (5.5) surprised us all along the interview and showed a high level of mathematical understanding. It seems that he not only knows how to add, subtract and divide into two equal groups, but he has already developed numbers sense and used a given exercise together with the *compensation method* in solving $8 + 4$. He solved the previous exercise, $7 + 5$, and said the answer was 12. For $8 + 4$, he took 1 from the 8 and added it to the 4 to get an exercise similar to the one he had solved previously, saying: "Here it is the same. You give one to the four and you take one from the eight. It is twelve."

Preschool teachers can undoubtedly learn a lot from listening to children while coping with everyday mathematical situations. They can realize that there are children, even at preschool, who think already by using numbers sense.

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