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INTERACTIVE PROCESSING OF MEASURING AND MEASUREMENTS IN THE YOUNGER GRADES IN PRIMARY SCHOOLS

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Abstract: *Measuring and measurements is the name of an exceptionally important topic, which has its place in the program curriculum of Mathematics, in each of the four younger grades of primary school. Basis for methodical transformation is the wide integration of the topic, especially with the teaching subjects of World around us and Primary science teaching. Methodical frames for processing the topic are made is such a way that they satisfy all the important characteristics of interactive teaching/learning. Having that in mind, authors of the paper first describe the concept and the characteristics of interactive math teaching. In the main part of the paper there are functionally described methodical frames for interactive processing of measuring and measurements in younger grades of primary school. For illustration of the suggested methodical frames, a model of interactive processing of measuring and measurements of surfaces has been created. In the conclusion, there are stated the elements of evaluation of the described methodical frames of processing the topic and the constructed interactive processing model of measuring and measurements of surfaces.*

Key words: *Methodical transformation and frames, processing model, measuring and measurements, interactive and integrated teaching.*

Introduction

The concept of measuring and measurements, and the way of calculating measures of some sizes represents a necessary condition for general education in many areas, therefore many subjects. Measurements are expressed with measuring numbers and measuring units, and measuring certain sizes comes down to determination or calculation of the right measuring numbers. The previously stated is the main reason why teaching contents about measures and measurements are included in teaching mathematics to all younger grades of primary school.

Measuring of values is one component of the activities of almost every working man in the modern world. If the work or research subject is made of immeasurable sizes, we are forced to determine and measure other relevant sizes. For example, quality of teaching and learning are both immeasurable sizes, and the main reason for that is the non-existent measuring unit. Quality of teaching is most frequently determined by measuring the quantity of adopted knowledge and the know-how of a pupil. Determination of the domain of measuring numbers and measuring units represent a very complex activity. Therefore it is necessary for their “measurement”, to have a lot of theoretical knowledge and experimental work. Before we engage in such a complicated activity, it is necessary to previously theoretically found and “assess” the quality of pre-prepared methodical frames and models of teaching.

Modern methods of teaching mathematics, except the educational and scientific foundation, are classified as priority condition for quality teaching and learning mathematics, interactivity and individualization. In assignment for research, authors of this paper had in mind that the theories of interactive learning recommend greater representation of integrated teaching of different subjects and areas, especially in younger grades of primary school. From the point of operative objectives and tasks of processing measuring and measurements in teaching mathematics have the highest degree of correlation in teaching subjects World around us or Primary science teaching. In books and in teaching there are many aberrations, both in domain of methodical transformation and in methodical frames of processing measuring and measurements.

Having in mind the previously stated, it can be concluded that the addressed topic is one of the prioritized researches of methodical transformation and modern teaching, especially interactive. Theoretically based and empirically evaluated models, that is, concrete methods of interactive processing of measuring and measurements in younger grades of primary school, cannot be found in any bibliographical data or on the internet. Main objective of the paper is to, in accordance with theoretical researches and scientifically based methodical transformations, compose original methodical frames for interactive processing of measuring and measurements in younger grades of primary school.

The idea and the characteristics of interactive teaching of mathematics

In wider meaning, the concept of interaction is defined as an activity which is interchangeable between at least two subjects. Interactive teaching/learning, as one of the most common strategies of modern education, is the short-term of interaction. It includes exchange of activities of the following subjects: teacher, pupils and the educational media. This implies all combinations of the stated subjects, which contain at least one pupil.

So far, didactic-methodical researches and knowledge about interactive teaching/learning of mathematics put in the foreground the activity of the pupil. The role of the teacher is to guide, encourage and to teach the pupil how to learn mathematics. At the same time, behind every student activity there is a necessary feedback from the teacher (tutor), because it is a proven psychological need of every individual. For analysis of every interaction in teaching Suzic N., (2005) suggests four aspects: “cognitive, emotional, target and active”.

Instead of memorizing facts and formalisms in interactive teaching of mathematics, it is necessary to use such methods which would enrich the teaching with pupil's reflective activities. The outcomes of interactive learning are characterized by more efficient influence of the development of cognitive and conative abilities of the pupil, they encourage criticism, creativity, etc. "With interactive learning, thanks to social interaction, we conduct changes in our thinking, in our emotions and the behaviour of people:" (Milijevic, S., 2003). That is why it is important that interaction, from the early childhood, is permanently and correctly conducted.

The statements refer to, firstly, mastering of the teaching contents in teaching mathematics because so far the contents were not respected enough. With the help of interactive learning, the learned is better used in new situations encountered during further learning of mathematics, the learning transfer is greater, and the learned is longer contained and remembered.

Interactive learning means pupil's independent work, with the usage of modern didactic means or the guidance of a teacher. Teaching mathematics has to be conducted as an interactive process in which the pupil and the teacher have a collaborative (cooperative) relationship, which gradually increases the pupil's activity.

In interactive processing of the right teaching units (classes) of mathematics in younger grades of primary school, the method we suggest approximately means the application of these phases:

1. Interactive repetition of previously adopted relevant knowledge of the pupil;
2. Setting and defining problem situations or examples, which provoke adoption of new knowledge;
3. Interactive processing of the provided teaching content;
4. Completing, uniting and generalizing of the processed content;
5. Verification summary.

The basis for preparation and realization of classes are teaching sets, which are suitable enough for interactive teaching and learning mathematics. **Connecting** the modern system, method, shape and means in interactive teaching is very significant and depends on the age group of pupils, objectives and teaching tasks, as well as program contents of every topic, that is, teaching units. In this paper we will concisely describe only those systems and methods which affect the most the level and quality of interactivity in teaching, that is, in learning mathematics.

In interactive teaching there is a need for integrating scientific and teaching areas, as well as within one teaching subject as within various subjects. The need for integration of teaching does not depend only on the degree of correlation of teaching subjects, from the point of operative objectives and tasks, but also on many other factors, above all psychological and physical characteristics of the children.

The fact that children in primary school age, and especially in younger grades, have strongly emphasized need for simultaneous continuation of different activities has long been known and proven. That on its own is sufficient reason to integrate different areas into modern teaching. The reason for wider implementation of a well organized and accomplished **integrated teaching** is also in its ability to fulfil complex objectives and tasks. That means that for nearly equal amount of time spent, better teaching results are achieved, in sense of their rationality and suitability.

Application of **exemplary teaching** dominantly affects the level and quality of interactivity in learning or forming concepts, as well as in learning mathematical rules. The adjective exemplary is derived from the noun example, which means instance or pattern. Consequently, with nominal definition, exemplary teaching can be determined as model.

Some authors consider that exemplary teaching should be used to point out characteristic teaching contents, which address the methodically whole and model way. At the same time, pupil should process as large part of the content as possible by themselves, in accordance with the model or with minimal help from the teacher. In that way, the activity of the teacher would be significantly rationalized, and the activity of the pupil increased. Despite the advantages stated, exemplary teaching, understood in that way, cannot be sufficiently successfully applied.

For interactive teaching of mathematics, especially in younger grades of primary school, wisely selected and processed examples are most frequently used in a frame of one teaching unit, and rarely in terms of teaching subject. The most suitable example is the one whose analysis can intrigue the pupils and be processed interactively. In that case the entire interactive processing of the provided teaching contents can be relieved of excess examples, and the inductive conclusion can enrich with brain activity. In the first place it activates analogical thinking of the pupil. It is known that analogy is based on observation and comparison, but it also demands separation of the important from the irrelevant. It means that with the previously stated application of exemplary teaching, students are being trained also for the application of more complex brain activities.

In differentiating teaching of mathematics, the starting point should be the fact that we must provide a mutual knowledge fund, necessary to every pupil. The objective is to maximize the usage and development of brain activities, inclinations, interests, and so on, with every student individually. "In terms of equal program demands, the problem of differentiation of teaching mathematics comes down to optimal usage of *obviousness and concretization, motivation, degree of difficulty of the tasks and the level of help to the pupils"* (Petrovic, N., Mrdja, M., 2005). If it is about interactive teaching, it imposes **differentiated minimum support** from the teacher in all cases**.**

The most complex task for the teacher, in preparation of this differentiated processing of teaching units, is formulating and structuring of differentiated support to pupils. Modern methods of teaching mathematics, approximately sets the hierarchy of different types of support (instructions): "motivational support, feedback support, strategic support directed to the content and contextual support" (Zech, F., 1999).

Motivational support more or less encourages the pupil and directs him to activity. Feedback support notifies the pupils whether they are on the right track to solve the problem. Every previously given support also demands a feedback. Based on the feedback, the pupil is introduced to the course and result of the stage of his work, for which he had received help. Strategic support implies general instructions for tackling the problem, based on Polya's scheme: "comprehension or understanding, creating a plan, implementation of the plan and accuracy check, discussion and interpretation of the solution" (Polya, G., 1980).

For every class of differentiated processing, teacher has to meticulously and carefully prepare strategic support guided towards the content and the contextual support. Strategic support guided towards the content directs the pupil to the methods of solving specific tasks and gives more specific directions, related to the beginning of solving the task. Contextual support implies those means, which give more specific directions for the concepts and rules given, for specific relations between them, for exactly specified support values and the result.

In teaching practice you ought to be prepared to give support which could not have been foreseen. It is important to have in mind that the differentiated help in interactive teaching can be conducted with all teaching methods, forms and means, even with verbal method in frontal form. For achieving good results it is necessary that the pupils get to know and adopt the principal of **minimum support.** It means that the pupils do not pay attention to all the support they have been given, but only to the one that is necessary for them. According to the rule support is given indirectly, often in the form of suggestive question, and it has to be followed by a suitable feedback.

The described definition in teaching mathematics is most suitable for elevating the level of its interactivity. Unlike the so called differentiation on a higher level of work, in this case it is about one and only level, **flexible differentiation.** The term flexibility is used having in mind that the pupil chooses the support which he finds necessary. Flexible differentiation finds its full implementation in interactive teaching of mathematics, only if it is permeated with the elements of **problem teaching.**

Cooperative teaching represents interactive work of a group of participants on the accomplishment of specific educational objectives and tasks. The fact that, in teaching, teacher's presence is obligatory does not mean that he interactively participates in every cooperative group. A contrary, his presence and activity should only be where there is a need for it and while it lasts. Work in small groups of pupils is the most suitable form for accomplishing cooperative teaching because it provides a high level of interactivity in a comfortable and pleasant atmosphere.

In acquiring knowledge and skills, which have been foreseen as mandatory for all pupils, forming of heterogeneous groups is desirable. At the same time, it needs to be kept in mind that the process of acquiring knowledge and skills also takes place outside the class. It means that the most suitably formed groups are the ones that pupils join voluntarily and implement cooperative activities without greater obstacles. "The main prerequisite for conducting pupil-directed teaching is the cooperative atmosphere among the pupils" (Roeders, P., 2003)

In accordance to the rule, reasonably heterogeneous groups in interactive teaching of mathematics have the greatest success in accomplishing objectives and tasks. "If in some area there can be an established hierarchy, then it is best to group together those pupils who differ according to the hierarchy of learning in one or two steps the most" (Bennett & Cass, 1988).

Groups of talented pupils can tackle the complicated and problem tasks with the help of cooperative interactive work more successfully and creatively, apposed to individual work or working in heterogeneous groups. Homogeneous cooperative groups of pupils, who adopt of mathematical knowledge and skills with more difficulties, can interactively work only with differentiated support. However, successful work in a group of pupils is possible only with maximum activity of every individual.

Methodical frames for interactive processing of measuring and measurements in younger grades of primary school

Processing of measuring and measurements of some variable sizes is usually preceded by processing of comparison of their values. Due to that fact, we begin this part of the paper with concise mathematical basis for **comparison** of the size values. Values of variable sizes can be compared if in the set they belong to, that is, the domain, there is a strictly defines relations order. (R, AS, T).

The set is completely arranged, if for every two elements x and y from that set stands: Either is x in relation to y, or is y in relation with x, or is x equal to y (notice: "or' has the meaning of exclusive disjunction). In other words, all the values of those sizes are mutually **comparable.** For example, set N is completely arranged, and all natural numbers are mutually comparable. Values of certain sizes are only partially comparable. It means that in a certain domain there are at least two elements X and Y to which the previously stated does not apply.

Measurement of some size is expressed with measuring number and measuring unit. For the selected measuring unit, each value of **measurable** size is equal to exactly one measuring number. Determination of the measuring number is called measuring, and considering the previously stated it can be referred to as mapping. Therefore, values of measuring units are in the domain of mapping, and the co-domain is real numbers.

Measuring number of the size's value, is determined, by measuring, as a number which indicates how many measuring units are contained in the measuring value. It is also important to notice that the selection of value's measuring unit of variable mathematical sizes is not conditioned by anything. On the other hand, for measuring certain variable sizes, significant for everyday life and science, measuring units are, according to the rule, determined by general convention (deal) as constant values, therefore, sizes.

In younger grades of primary school examples of measuring sizes have to be elected so that the measuring numbers are in the set N. That means that the examples should be given inversely, that is, to first choose the measuring number from the N set and the measuring unit, and to ask the pupil to measure the so "given" value, that is, to determine its measure. For example, if the pupils are supposed to measure a straight line, therefore to determine its length so that it is 3cm long, "precisely that straight line" has to be given. All other measurable sizes are dealt with by analogy.

For accomplishing complex objectives and tasks of measuring and measurements in younger grades of primary school, we especially use integrated teaching of mathematics with the subjects World around us and Primary science teaching. At the same time, we keep in mind, first and foremost, the integrating of development of logical, mathematical and naturalistic abilities.

For this purpose we list two quotes. Gardner, H. (1984), lists four basic components of logical and mathematical abilities. “The ability to manage sequences of the thinking process, acknowledging the relations between elements, ability of abstraction, critical attitude”. Roeders, P. (2003) as basic naturalistic abilities he lists: “identification and classification of patterns in nature, sensitivity to changes in time patterns and comprehension of different features in the natural surroundings.”

Measuring value with the help of money does not fit in the definition of the term measurement. Claims like “every possession has its value” (referring to the value expressed through money), are incorrect. It is known that a lot of things, but not all, can be measured with money. It means that the general domain of sizes measurable with money cannot be precisely determined.

If we limit ourselves to domain of variable size, which is sold for money and is called merchandise, even then for a price of an item (domain element) we cannot say it is a measurement. For a price to become a measurement, it would have to be unique for every single item, and it is known that it varies, first and foremost depending on the place and the time of sale. So we can talk about the price as a measure of value of the goods only in certain store at a certain time. If you keep in mind the inflation, devaluation, floating rate, then currencies cannot be considered as measuring units either, because they are not constant.

In accordance with the previously stated, in processing of the topic we should make the term measurement and measuring relative. We should emphasize the concept of money, coins and paper bills, their role and meaning. In processing of teaching units, relying on limited experience of second grade pupils, interactive dialogue should be dominantly used. For example, the following could be used as help for students with feedback.

What do you have to give to the salesperson in order to buy something from him? How do you call the quantity of money for which you buy something? Salesperson sells, who buys? What is the word for everything that can be bought for a certain price in one store? Do we get money only by selling goods? How do we call the amount of money a worker receives for his work? Beside the pay check, can money be received for a special achievement, like in sports, for example? How do we call the first prize someone gets in games of chance? Can a premium be received in cash?

In this way, the role and significance of money would be interactively processed as well as proper use of the names of following terms. Introductory course of the term currency, that is, coins and paper bills, should be conducted in demonstrative method.

Time is considered to be continual variable size, whose domain has cardinal value in the continuum. According to current theories, the creation of the Universe was started with the Big Bang. It is only natural that that moment represents time values equal to measuring number zero. Under the condition that there is a measuring unit, time is a variable size, moreover it is permutation of time and not negative real numbers. Concept of time as continual variable size makes sense only in the world of changes. In unstable universe, that is, the world of chaotic changes, it is not possible to determine a measuring unit. Humans as intelligent beings, appear in relatively stable universe, that is, in a world of legitimate changes. Then it is possible to determine measuring units, and also time measuring.

Time measuring based on the day and year has been gradually accepted as primary measuring units for measuring time. **Day** is time needed for Earth to rotate around its axis. **Year** is the time needed for Earth to orbit around the Sun following an elliptical orbit. Note that the given time units, as continual variable sizes are in fact time intervals.

From the previously stated we can conclude that by measuring time we determine "exact time" or the value of time and "duration" or a time interval. Time of the new era can be presented with the help of a numeric ray, so that the interval from zero to one represents the first year, one to two the second year and so on. By that we mean an astronomical year, given by the previous definition, not the so called calendar year. In that case the dots which mark the days could not be determined with natural numbers, which is not practical for general use.

Time interval for every year, is divided into months according to the calendar, and all of them except February have a constant number of days. Calendar also depends on the daylight saving time. Bigger units than the year are decade, century or millennium, and the smaller are hour, minute and second. Often the time intervals are expressed with months as measuring units. However, time interval is precisely determined only if it is expressed with months stated by name for an entire year.

Anyone who helps or directs children in the process of developing and forming of the concept of time and its measuring should have in mind the stated theoretical foundations for it. It especially goes for pre-school teachers and teachers, when they prepare and conduct certain educational activities. Development of the concept of time, as a variable size, in pre-school age dominantly takes place spontaneously and empirically. However, it is desirable that the children have previous guidance from the pre-school teacher. When they start school, majority of children know the days of the week. Concept of a day, as a time interval, is mostly related to some activities in certain parts of it, especially night sleep.

One of the biggest tasks for a teacher is to explain the concept of the day to children with the help of interactive dialog. That cannot be done with only the astronomical definition. With that, it is necessary to introduce ideas such as its two major parts: **daytime** and **night-time.** That way all the confusion that the use of the word day causes with children is avoided. Among the significant tasks of a teacher is introducing the children to the fact that the Earth is shaped like a ball and revolves around its axis once every day. After that, with interactive dialogue it is desirable to use the feedback support. Here we suggest specific contextual support which we find to be the most important.

Is the place of our residence turned towards the Sun the entire day so that the sunshine can reach us? What is the name of that time of the day when sunshine can reach us? What is the name of the daytime when the Sun is "right above us"? Is noon the middle of the day, or the middle of daytime? What is the name of that time of the day when the sunshine cannot reach us? What is the name of the middle of the night? Do night-time and daytime last the same every day of the year? In which season does daytime last the longest and night-time the shortest? Do all days, daytime and nighttime, last the same?

Let us assume that, during the first two grades, the adopted stated concepts are described to all the students in the same way. Then at the end of the second grade, before introducing the concepts of the day, hour, minute, week and month, the level adoption of previously explained concepts should be briefly strengthened with an interactive dialog. On the contrary, there should be enough time provided for the adoption previously stated concepts with the help of integrated teaching of World around. In processing of the hour and minute illustrative-demonstrative method and teaching set is commonly used, besides the previously stated methods suitable for interactive teaching. The most suitable is that, during the six classes provided all program contents, without any strict division to teaching units, are being interactively processed.

In the third grade among the first teaching units there are two classes provided for adopting the concept of the year, decade and century. Under the previously stated presumptions, it is enough to repeat the already adopted knowledge of the student about the year. In interactive processing of the concepts of decade and century, after introducing the pupil with the suitable definitions, there should be an emphasis on their use. Integrative teaching of Primary sciences teaching should be used in those purposes. For example, with interactive dialog the next feedback support could be provided to the pupils.

What is the smallest number of years that a man in his seventh decade of life has lived? Can a dog enter the seventh decade of life? Can a man enter a second century of his life? Name some plants and animals, for which you know that they can live for over a century. In which century of the new era did Serbs raise rebellions and manage free themselves of slavery under the Turks? How many centuries did the slavery approximately last?

Mass is a variable size which expresses characteristics of solid bodies, fluids and gases, determined by the quantity of matter which they contain. Not to go deeper into the substance of mass, we can claim that it is measured by physical size, called force of gravity. The measurement of that force is called **weight,** and her basic measuring units, pound and kilopond are tentatively equal to the measuring units of the mass, gram and kilogram.

Mass of one kilogram at a sea level, that is, the ocean has the weight of one kilopond. It should be noticed that mass at a greater elevation weighs less. It also needs to be mentioned that in a larger part of the universe there are so called conditions of weightlessness. For example, astronauts have to be trained to manage in the conditions of weightlessness. People on the moon do not change their mass, but their weight becomes significantly smaller. If in the future people get to Mars, they will face the problem of significantly bigger body weight.

Processing of measuring mass is planned for third grade teaching with two classes. At the beginning of the first processing class, in interactive dialog, the pupils need to be explained the difference between mass and weight, in accordance to the theoretical basis and previously acquired knowledge. After that, using demonstrative and experimental methods, the pupils should verify the independence of the body mass from size of the space fill that is, their volume.

For example, two matching balls or cubes (glass and metal marble or plastic and metal cube), would be put on pupils palms and they would assess which one feels heavier. Based on that they conclude which one has a bigger mass. In that way they understand, based on the introduction, the concept of mass as a body feature determined exclusively by the amount of matter. The teacher can use the demonstrative and experimental method by using a scale to compare mass of equal dishes filled with different matter, for example flour and water.

In the second part of the class pupils are getting to know the devices for measuring mass, names and tags of measuring units through an interactive application of dialog and illustrative and demonstrative method. For remembering relations of the units: gram, kilogram and tone it is enough to emphasize the fact that every next of them is **thousand times** bigger than the previous. In the second class, in solving the tasks the priority should be given to demands that the students identify the reasons for usage of every one of the stated measuring units.

Word **length** is a homonym, commonly used. It is nominally tied to the concept of measuring straight lines (segments) or the shortest distance between two points, in geometry as well as in real space. Length is also used to express measurement of all limited lines, which is a superior or wider concept than the previous one. However, measuring units in both spaces are segments therefore it is not possible to directly determine measuring number for curved lines.

In practice we are usually satisfied to get an approximate result when calculating length of curved lines. We usually do that in a way that on a curved line, between the end points, we take a big enough number of “closer” points which make an appropriate inscribed broken line with their tendons. If we use tangent lines in all points, their intersections and end point of the curved line, the right inscribed broken line has been determined. Constructed in such a way that lines can be measured and the arithmetic mean of their segments can be taken as approximate length of the curved line. Therefore, it is possible to conduct the described procedure so that it “satisfies” the previously required measuring accuracy. Approximate length of the path crossed, no matter the shape of the path, is measured with special instruments which all the modern vehicles have.

It is common that in real, three-dimensional space relatively determined names are used for dimensions: length, width and height. Height is always determined according to direction (cross-axis) normal to the plane of the ground, around the place where the man stands. A man determines the width in the direction of his wide open arms, parallel to the ground. Length of the body is considered to be the longest segment, whose end points belong to the surface of the body. Whether they are called length, width or height, all three sizes are determined by length.

At the end of the first grade, there are two lessons provided for preliminary formation of the concepts of segment and meter, as basic measuring units. Basis for the interactive realization of the teaching objective and task is the teaching set and the application of demonstrative and experimental methods. In interactive dialog with pupils teacher emphasizes the necessity for measuring of all lengths there should be unified and constant measuring unit. That measuring unit is called meter and is the official measuring unit for length worldwide. It is desirable to point out to the pupils that there is a long kept special stick, exactly one meter long whose length hadn't changed over time. People who needed precise length measuring could, based on that stick, compare and coordinate their "meters".

For understanding, measuring and assessing the length in meters, two classes of mathematics are not enough. That is why integrated teaching with the subject World around us should be used and in that way additional interactive processing could be conducted. Having that in mind, special attention should be devoted to segments of curved lines. For example, the teacher can set differentiated support with feedback. Can the length of a string be measured if it is in a ball? Can the length of the entire ball of string be measured? What if the ball of string is rolled out and the string "stretched", can the length be measured in that case? At the end an experiment with 3m long ball of string is conducted.

In the second grade, there are two lessons planned for drawing and comparing segments as an introduction for widening the knowledge about the length. After that, the teacher should prepare and conduct four lessons for introducing measuring units (meter, decimetre and centimetre) and the lengths expressed in those units to the pupils. At the same time to point out the need for introduction of measuring units smaller than a meter. For that purpose, try measuring with a "carpentry meter" model of a segment, whose measuring number is in meters and does not belong to the set N, for example, the edges of a classroom floor. After introducing the concepts of decimetre and centimetre, we measure the given segment with "role meter", and express its length in centimetres. Foundation for further interactive processing is the teaching set and the application of demonstrative and experimental method.

In the third grade there is one lesson planned for revision and broadening the knowledge and skills of the pupils about length, adopted in the second grade. That lesson should be conducted interactively, by using similar methods, shapes and means which have been used in the previous processing. After that, there are two classes planned for introducing millimetre and kilometre and their use in the determination of lengths of certain sizes.

Models of sizes whose length is expressed in millimetres are in the teaching set or are given in the notebook. At the same time, pupils use triangle of ruler and tackle the tasks individually. Models of sizes, whose length is in kilometres, can be found only outside the classroom. Experiments, which would be used to immediately measure, are practically impossible. However, it is necessary that the teacher with his pupils walks the distance of one kilometre in the school surroundings (the starting and end point can be closely determined with the help of a vehicle) and determine the time for walking distance.

In that way, pupils can closely determine the length of their movements, whose length is shorter or longer than one kilometre. That kind of experiment is supposed to be recommended to them with the objective of acquiring the ability of assessing the length of certain sizes. Assessing the segment by observing makes sense, if the observer is in the right place and the measured size is not too big. In order to realize the objectives and tasks described, it is necessary to integrate the teaching with the primary sciences teaching.

Part of the space which is consumed by the body is called **volume.** Under that concept we consider the surface that limits the body and the inner area of the body. Word volume is also used for expressing measurements of parts of a space limited in those ways. In the geometry of Euclid the body consists of points which are the primary concept. Straight line and plane are also primary concepts; however, the body cannot contain only their limited parts.

Bodies in real space can be made of different materials, and consist of the so called physical points and points of vacuum. For measuring unit of volume, the most practical solution is to use a body with the shape of a cube with edges of 1dm. Part of the space which that body occupies is called a cubic decimetre and is marked with 1dm3. If its interior happens to hold a liquid substance, then we say that its volume is one litre, a volume determined like that is considered to be a measuring unit for the volume of liquid and is marked with ***l.***

Processing of measuring volume of liquid is planned in the third grade with two lessons. Foundation for interactive processing besides the teaching set is also the implementation of illustrative and demonstrative methods. With introducing the concepts of measuring of units smaller and bigger than a litre it is desirable to remind the pupils of previously used units for length. For example, centilitre is relatable to centimetre, and hectolitre is relatable to hectometre. Even though the previously mentioned eases the performance of certain tasks, it is necessary to use integrated teaching with the subject Primary science teaching.

Model of interactive processing of measuring and measurements of surfaces

For processing and revising the material, teaching subjects of **Measuring and measurement of surfaces** is planned for eight lessons of mathematics in the fourth grade of primary school. Foundations for production of model of interactive processing of topics are theoretical determination of interactive teaching of mathematics and the teaching set Mathematic for the fourth grade of primary schools with practice tasks, part 1, whose publisher is Kreativni Centar, Belgrade, and the authors are Dr. Mirko Dejic, Dr. Jasmina Milinkovic and M.A. Olivera Djokic. Commitment to the stated teaching set is conditioned by the fact that it is very suitable for interactive teaching and learning.

First two lessons of interactive processing of the topic are meant for **comparing surfaces.** For the preparative stage of the first lesson (stage 1 described in this paper, in frames of five phases of interactive processing of teaching units), the foundation is page 53 of the teaching set. At the same time it is enough for pupils to tackle part 1 and 2 of the task, with differentiated support from the teacher.

In operative stage of the lesson (stage 2, 3 and 4) as an example we use tasks 1 and 2 on page 54 of the teaching set. Except this example, in this stage of the lesson, pupils notice and compare with the naked eye the surfaces of flat figures, which limit the bodies to the inside of a classroom. At the same time, teacher notes that only surfaces of flat figures are being compared, and the pupils are comparing the noticed surfaces based on observation from at least two different places. After that, they put notebooks, books, ruler and triangle on the plain surface of their desks.

They put those items one on top of the other, in pairs, that is, they overlap the chosen co-respondent surfaces of the items. First they determine those matching items whose overlapped surfaces can be compared with certainty. For other matches, they use overlap as an addition to the assessment of comparison of the surfaces with the naked eye.

In verification stage of the lesson (stage 5) besides the summary, a pupil is required to bring a blank paper sheet from the block for art education and paper scissors for the next class.

For the second lesson, the foundation is task 3 from page 54 of the teaching set. During the interactive development of the task, considering that the pupils are using a big enough paper sheet, the previously mentioned dimensions are being doubled. Pupils first draw the figures and then they cut them out. Before comparing the figures, based on the task pupils are supposed to compare them with the naked eye on the drawing and also after the "cut-out".

In this way, the pupils are able to make sure of the advantages of comparing surfaces by overlapping them opposed to comparing them with the naked eye.

In verification summary, the pupil may be required to, with the differentiated support of the teacher, define the positions of the observer of the surface, from which he would be the most sure to compare them with the naked eye. In order for the lesson to have all the characteristics of interactive teaching, the role of the teacher is supposed to be only to direct and correct.

Lessons 3 and 4 of interactive processing of the topic are devoted to **surface of the figure.** For the third class we have prepared a more detailed scenario in accordance to the previously described characteristic of interactive teaching.

In the preparative stage of the lesson (approx. 10 min.) we revised the processed measurements and measuring from the previous grades, with the help of interactive dialog. At the same time, we use the first part of the foil which we state in the whole and which contains differentiated feedback support.

For which sizes have you measured values up till now?

Money, segment line, time, mass and volume of liquid

A pupil ran a hundred meters in twenty five seconds.

a) Which expression do we use to state the measurement of time of his run and is there a special name for it?

It is stated with the expression 100 m, and it is called length.

b) Which expression is used to determine measurement of the time of his run and is there a special name for it?

It is determined with the expression 25 s, and there is no special name for it.

c) How do we call numbers 100 and 25 in the stated expressions?

We call them measuring numbers.

d) How are meter and second universally called?

They are called measuring units.

e)Is the claim correct: measurement of size values are expressed with measuring number and measuring unit?

It is correct.

In one barrel there is 2 hl of wine and in the other 200 l.

a) In what relation are the measuring numbers of volume of liquid in those two barrels?

Measuring numbers are different.

b) In what relation are the volume measures of liquid in those two barrels?

Measures are equal because 2 hl=2 x 100 l =200 l

c) What makes it possible for measuring numbers to be different if the measurements are equal?

It is possible because the measuring units are different.

How do we call the measurement for the surface of a figure?

Measurement for the surface of a figure is called surface.

Since surface is a measurement, in which way do we express it?

Surface is expressed with measuring number and measuring unit.

Which are the measuring numbers of the surfaces of the rectangles A and B, if the measuring unit is square K?

Measuring numbers for surfaces of the rectangles A and B are the smallest numbers which determine how many squares K (measuring unit) are needed for the rectangles to be covered.

In operative stage of the lesson, (approx. 20 min) as an example and interactive processing we use a task on page 55 of the teaching set, with the part of the foil that relates to the surface.

In verification stage of the lesson (approx. 15 min), beside the summary, pupils commence their work on the practice task, on pages 56-58 of the teaching set. With differentiated support of the teacher those tasks are finished at home, as homework.

Foundations for realization of the fourth lesson are tasks on pages 56-58 of the teaching set. In the first part of the lesson the teacher gives feedback on the results of the given tasks and also the homework. In continuation of the class pupils are performing tasks individually, that is, individually with the differentiated support of a teacher.

Four teaching classes are devoted to the interactive processing of **surface units.**

For the realization of the fifth and sixth class, that is, interactive processing of the teaching unit foundations are pages 59 and 60 of the teaching set.

Having in mind that it is the last processing of the teaching unit from the section of measurements and measuring additional material is necessary to relate interactively the section with appropriate content from Primary science teaching. It is of extreme importance that the surface units are related to appropriate measuring units for length. In accordance with the stated, the processing should be interactively supplemented with the following additional explanations.

1. In preparative stage of the fifth lesson the following text should be used.

If people used different kind of measuring units for values of sizes, for their use in the practice they would have to get a measuring unit with the measuring number which has been used in that occasion. That is why in modern world, in accordance to a general agreement, unified measuring units are used: meter (m), second (s), gram (g) etc. In order to have as precise measuring of the size values as possible, nowadays devices for extremely precise determination of measuring units are used. However, for many years, special institutes were in charge of "keeping" exact measuring units of certain sizes, and the rest were used according to them.

With regard to those specified measuring units, other, bigger or smaller units are determined. It is done because of various needs for measuring "smaller" or "bigger" size values. In accordance with the previously stated, other measuring devices are used for measuring, for example ruler for a notebook, ruler for a blackboard, measuring device for a path crossed in a vehicle or aircraft, a clock, a scale, and so on.

2. With the help of interactive processing, in both classes, the relation of every unit for surface with a suitable unit for length should be established. As an example for the stated, the following ought to be used as graphic display for 1 square meter on page 59 of the teaching set.

3. The way converting 1 square meter into surface, expressed with smaller units of measurement dm2, cm2 and mm2 (1m2=(10 x 10)dm2=100dm2; 1m2=(100 x 100) cm2 = 10 000 cm2 and 1m2= (1000 x 1000)mm2= 1 000 000 mm2), with the help of analogy due to be transferred to reaching the formula which connects all other units for surface.

4. To get to the framed formulas, pupils should use the previously mentioned way, rather than just formally memorize them.

In the verification stage of the sixth lesson, beside the summary, pupils commence their work on the practice tasks on pages 61 and 62 of the teaching set. With differentiated support from the teacher, those tasks are to be finished for homework.

Foundations for realization of the seventh lesson are the tasks on pages 61 and 62 of the teaching set. In the first part of the lesson the teacher provides feedback on the solutions of the tasks, and then moves on to homework. In continuation of the lesson pupils tackle the tasks individually, unlike individualized but with the differentiated support of the teacher.

In the so called mixed inscriptions of surfaces, that is, with more measuring units it needs to be kept in mind that it is a unique surface which is expressed with several appropriate surfaces. That is why pupils need to express the inscription with verbally also, for example, inscription 2 *ha* 5 *a* 37 *m2,* would be: surface inscription like this consists of the surfaces of two hectares, five acres and thirty seven square meters.

Foundations for realization of the eighth lesson are tasks on pages 63 and 64 of the teaching set. In the first part of the class the teacher provides feedback on the solutions of the tasks, followed by the homework. In continuation of the class the pupils tackle the tasks individually, that is, individualized with differentiated support of the teacher.

The so called conversions of measuring numbers for units different from the square meter are to be performed in the way described in the example. The objective of that part of definition is also that different “conversion and calculations” of surfaces with different measuring units are practiced to the point of gaining **skill** and in that way also the formulas in the teaching set to become permanently adopted.

Conclusion

In this part of the paper we specifically want to point out the used structure of interactive processing of teaching units, which contains five stages. For the overall course of interactive processing of teaching units the second stage is of extreme importance, that is, the setting and defining the problem situation or example. Example, therefore exemplary teaching is used for learning concepts and simpler rules. Problem situation, that is, problem teaching is used for learning more complex rules and solving problems.

Other than flexible differentiation described in this paper, we also point to the connection of all factors which influence the quality of interactive teaching. Particularly significant role in elevating the quality of interactive processing of measuring and measurements is the one of the integrated teaching of mathematics with teaching subjects World around us or Primary science teaching.

It is our opinion that processing of measuring and measurements according to methodical frames and model of processing described in this paper is characterized by significant representation of interactivity, which can be experimentally verified and the assessment of the pupil’s progress can be performed.

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