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SWITCHING CONTROL BY PHASE TRAJECTORY

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ABSTRACT:

The paper is discussing the instability that can occur in the case of switching controllers. The qualitative analyze of the switching phenomenon is performed by means of the phase trajectory of the difference between the controllers outputs. The significant regions of the phase plane are investigated. The second and the fourth quadrants are recommended for switching, while the first and the third quadrants must be avoided. An analogy with the sliding mode method is proposed

KEYWORDS:

Switching controllers, phase trajectory, qualitative control, fuzzy interpolative controller, sliding mode.

1. INTRODUCTION

The switching controllers problem

The limits of the frequency analyze methods are showing in control problems for which the initial values are playing important roles. Such a case appears in the switching controllers' applications. Two relevant conclusions, proved by specific benchmark studies and an extended overview of the existing literature are presented in [1]:

- a switching system can be potentially destabilized by an appropriate choice of the switching signal, even if the switching is between a number of Hurwitz-stable close loops systems; this possibility exists even in the case of identical switched systems;
- the implementation particularities of the control system can produce the same effects.

There are applications where the switching controllers instability can produce fatal consequences. For example, in particular circumstances, the instability produced when switching between automate pilot and manual pilot may cause airplane crashes. In fact these kind of effects may appear in each technological process, when automate and manual controls are switching.

This paper is continuing a previous work [2] with new arguments that were partially discussed in [3].

2. POSSIBLE EXPLANATIONS FOR THE SWITCHING CONTROLLERS INSTABILITY

Detailed and precise studies of the switching linked phenomena were reported mostly for the linear systems [4]. For the nonlinear systems it is more difficult to draw precise conclusions because of the huge diversification of the problem and of the lack of a unified theory.

The frequency analysis (using the transfer functions) has few chances to produce positive results in this case, taking in consideration the fact that this theory is essentially founded on the hypothesis of the null initial conditions. That is why Hurwitz-stable systems can be destabilized by perturbations.

There are some possible explanations for the quasi unpredictable instabilities that appear occasionally in the switching controllers systems:

- a commutation represents a discontinuity by itself, transitory effects are inherent;
- control algorithms need a perfect state initialization for the moment of the switching;
- the digital control systems are fundamentally affected by the digitization operation (sampling and encoding); that is why most of the time the digital control systems are actually working in open loop and odd unexpected dynamic effects are always possible.

In [2] and [3] we analyzed the switching controllers' problem with the help of the phase trajectories. The phase trajectory can fully support the design of switching controllers for linear systems [4]. Our aim is to move this technique towards nonlinear, replacing the precise control of the phase trajectory that can be achieved in linear systems by the Clocksin-Morgan qualitative analysis [5]. The method was already used for self adaptive control [6] and can be easily implemented with fuzzy-interpolative expert systems [7].

3. A BENCHMARK STUDY: A PI TO PD SWITCHING CONTROL FOR A DC DRIVE

Let us consider the case of a d.c. electric drive (P = 12kW, U_{nom} = 220 V, n_{nom} = 685 rpm) whose speed is controlled either by a PI controller (proportional gain = 25, integral = 10 gain) or by a PD one (proportional = 25 gain, derivative = 0.1 gain) [1]. The main window of the Matlab-Simulink model is presented in fig. 1. This configuration is taking advantage of the precision of the PI controller in steady regimes and the robustness of the PD controller in transient regimes.

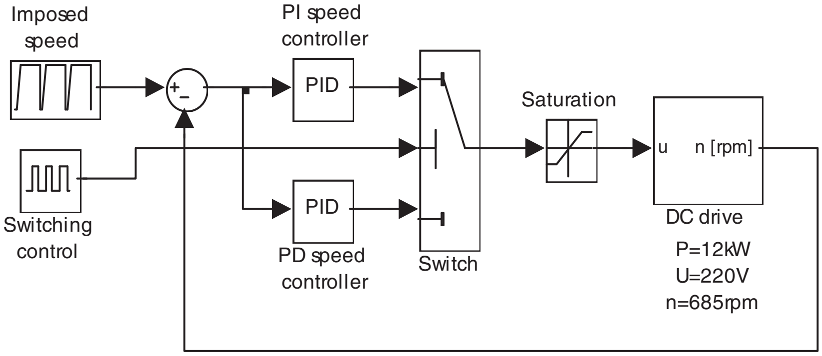

Fig. 1. The d.c. driver and the two switched PI and PD controllers

The scenario of the simulations is the following: we will impose a 600 rpm speed step that should be accomplished in one second and we will introduce a loading torque at t = 5s. The basic idea is to switch from the PD controller to the PI one after 5s, for instance at t = 7s. The resulting performance for the previous scenario is the expected one.

Now let us change the PI to PD switching and the moment of the switching before the loading of the drive, for instance at t = 3s. The system becomes unstable as one can see in **fig. 2**.

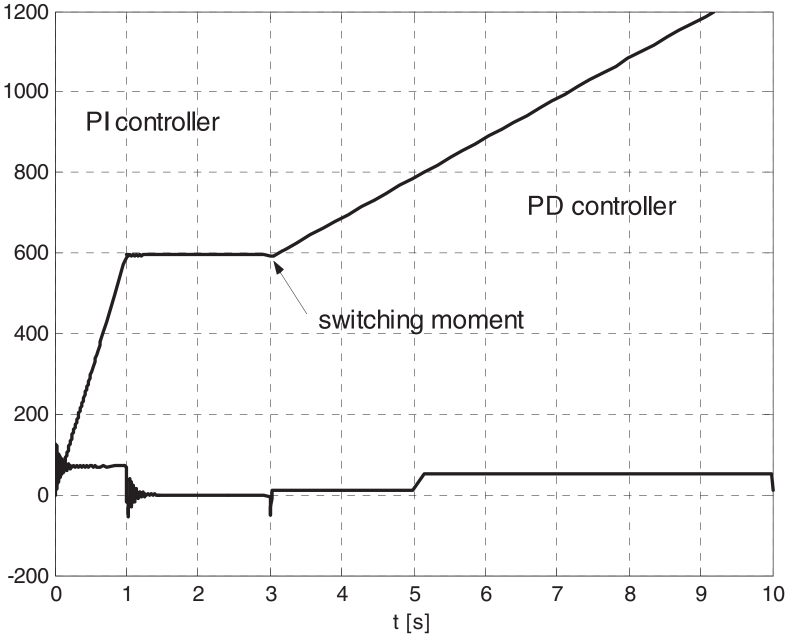

Fig. 2. Instability induced by a PI to PD commutation at t = 3s

Analyzing the state variables one can easily notice that the outputs of the two controllers are very different in the moment of the commutation (see **Fig. 3**). It is natural to assume that the origin of the instability is connected to this difference.

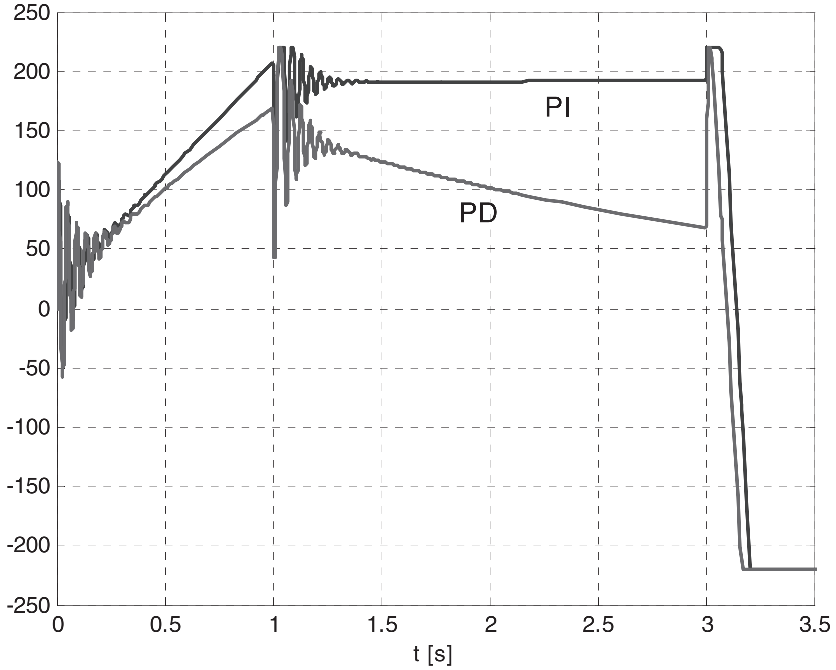

Fig. 3. The outputs of the two controllers

Other empirical observations drawn from simulations [2]:

- the instability may evolve in both senses: positive as in fig. 3 but also negative;
- the instability appears as well in the case of same type controllers;
- the instability is finally producing the saturation of the controller, but its causes are not necessarily linked to the saturation;
- changing the parameters of the integration used for the simulation is producing significant changes and even the disappearing of the instability; in a correct perspective we can conclude that the instability is appearing only in digital systems and it is linked to the sampling operation and the integration method.

Based on these observations one can conclude that the switching controllers' instability is linked to the initial conditions, as well as to the very essence of the numeric calculus.

4. THE QUALITATIVE ANALYSIS OF THE PHASE TRAJECTORY OF THE ERROR

Since the Laplace operational calculus is useless as a tool in this case, we will replace it with the *phase trajectory of the error* (PTE). Originally PTE refers to the control error in close loop control systems. In this case the error *er* will be defined as the difference between the outputs of the switching controllers PI(t) and PD(t) [2]:

$$er(t) = \mathrm{PI}(t) - \mathrm{PD}(t) \qquad (1)$$

er and its derivate *cer* corresponding to the fig. 6 simulation are shown in fig. 4.

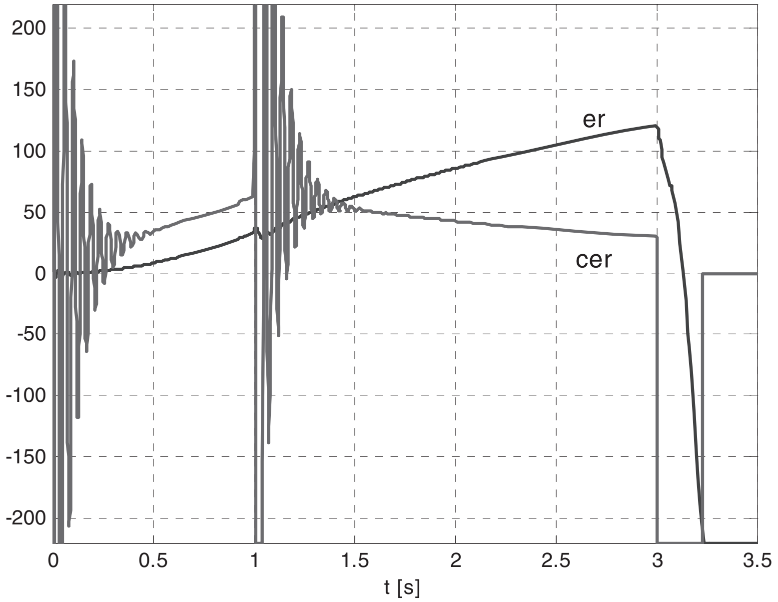

Fig. 4. The error and its derivate, for the case of the switching controllers

The PTE that is resulting after filtering the high frequency components (see fig. 5) can be used for analyzing the causes of the oscillatory or unstable commutations, as well as for the choice of a low risk switching moment. The technique is identical for the case of nonlinear controllers and/or processes. The option for this linear example is made rather for methodological reasons: the time responses are very simple and easy to interpret.

A first observation is that the commutations executed while the PTEs are located into the first or the third quadrants presents high risks for instability, while the second and the forth quadrants are recommendable for commutations. The risk of producing instability decreases massively according to our experience if we have in mind this primary criterion when choosing the switching moment. Our observations are pointing to a particular high risk zone in the first (third) quadrant: the one limited by max(*cer*) and max(*er*) [2].

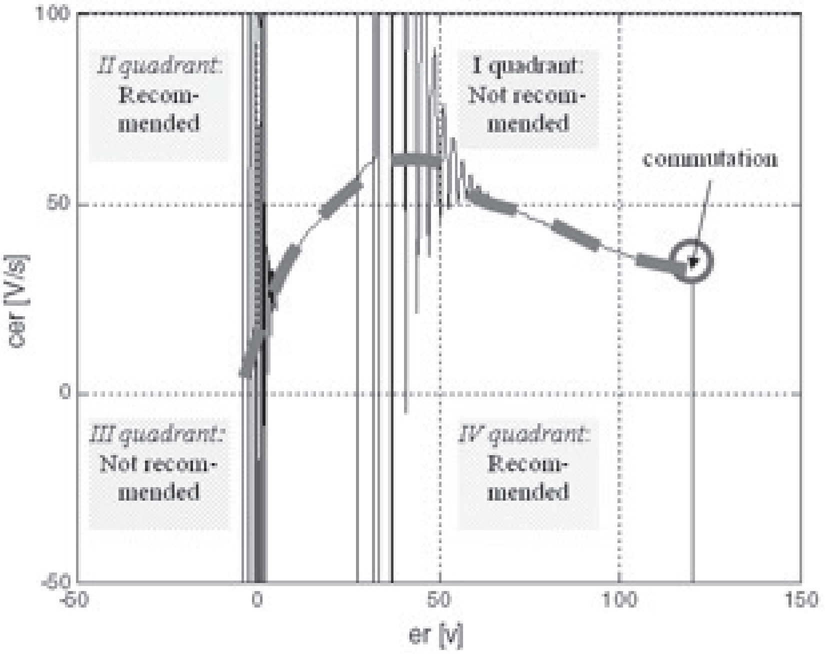

Fig. 5. The PTE and the not recommendable switching in the first quadrant

5. THE CONTROL OF THE SWITCHING

A further step after pointing the high risk zones would be to recommend the best possible commutation methods. The commutation methods can be *passive* or *active*.

a) **The passive commutation** is choosing the best possible commutation moments. In this matter we are proposing an analogy with the sliding mode method. Although the meanings of *er* and *cer* are totally different (in the case of the sliding mode *er* is a control error while in the case of the switching controllers *er* is the difference between the outputs of the switching controllers) the idea of imposing a switching law in the II and IV quadrants of the phase plane is offering good results in the case of the switching controllers. Instead of becoming a sliding contour the switching law has only the purpose to trigger the switching action.

It is to remark that if we are accepting a precise switching criterion and if we leave the unconnected controller in open loop, we will loose the possibility to control the precise moment of the switching, since there are no guarantees that the unconnected controller, that is very often saturated, will behave by itself such way that the phase trajectory can reach the sliding contour.

b) **The active commutation** is forcing the unconnected controller to track the output of the active controller. This way the commutation can be performed in any moment. If the unconnected controller’s output is to slow, one can still find a satisfactory solution by imposing to it an initial value equal to the other controller’s output in the moment of the commutation. In the case of very pretentious plants (nuclear reactors for instance) the tracking control law should be sliding mode. Adaptive tracking controllers able to control the commutations with no risks may be realized using the Takagi-Sugeno controllers [8] and also by the fuzzy-interpolative methodology, because the phase trajectory analyze is similar to the one that is performed in FSAICs (Fuzzy Self-Adaptive Interpolative Controllers) [6] by the adaptive corrector.

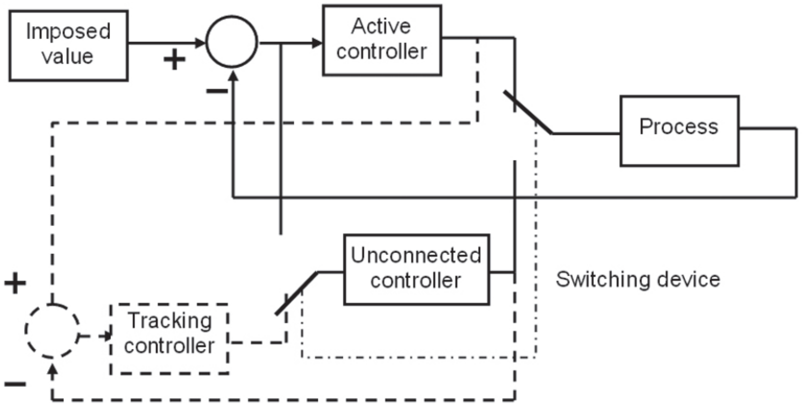

Fig. 6. The following controllers active commutation

6. CONCLUSIONS

The switching controllers' instability is caused by particular initial conditions and by the numeric calculus that is characterizing digital systems. The phenomenon is extremely dangerous and barely predictable. For this problem no rigorous theoretical apparatus is disposable for the moment, as far as we know. A common sense recommendation for decreasing the risk is to smooth as much as possible the commutations.

Among other existing methods for smoothing the commutations, a very feasible one is relying on the on-line analyze of the phase trajectory of the difference between the outputs of the switching controllers, which can be considered as a switching error. Such way one can choose the best moments for switching the controllers, in quadrants II and IV of the phase plane of the switching error, and one can avoid the risky quadrants I and III. A sliding mode law can be imposed to the unconnected controller's output in order to follow the output of the active controller.

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