Lotfi A. ZADEH

Professor in the Graduate School
Director, Berkeley Initiative in Soft Computing (BISC)
Address: 729 Soda Hall #1776
Computer Science Division
Department of Electrical Engineering and Computer Sciences
University of California
Berkeley, CA 94720-1776
E-mail: *zadeh@eecs.berkeley.edu*

COMPUTING WITH WORDS AND PERCEPTIONS (CWP) – A SHIFT IN DIRECTION IN COMPUTING AND DECISION ANALYSIS

Note: *The paper was presented during the ceremony of conferring honorary degree to Prof. Lotfi A. Zadeh at „Aurel Vlaicu" University of Arad, Romania, Arad, 03.07.2003*

-

Abstract

In computing with words and perceptions, or CWP for short, the objects of computation are words, propositions and perceptions described in a natural language. In science, there is a deep-seated tradition of striving for progression from perceptions to measurements, and from the use of words to the use of numbers.

Reflecting the bounded ability of sensory organs and, ultimately, the brain, to resolve detail, perceptions are intrinsically imprecise. Perceptions are f-granular in the sense that (a) the perceived values of attributes are fuzzy; and (b) the perceived values of attributes are granular, with a granule being a clump of values drawn together by indistinguishability, similarity, proximity or functionality.

F-granularity of perceptions is the reason why in the enormous literature on perceptions one cannot find a theory in which perceptions are objects of computation, as they are in CWP.

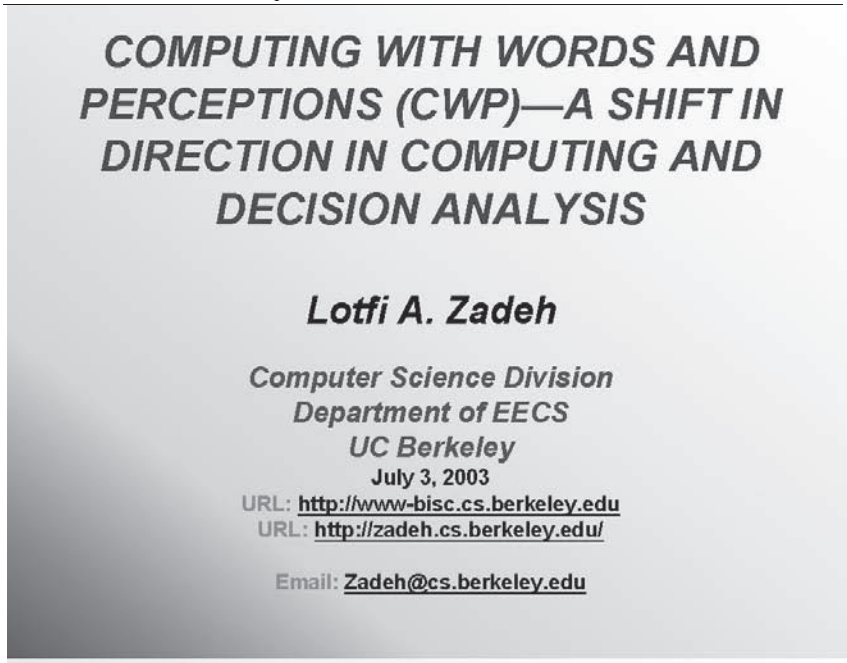
PNL (precisiated natural language) associates with a natural language, NL, a precisiation language, GCL (Generalized Constraint Language), which consists of generalized constraints and their combinations and qualifications.

The principal function of PNL is to serve as a system for computation and reasoning with perceptions. The need for redefinition arises because standard bivalent – classic-based definitions may lead to counterintuitive conclusions.

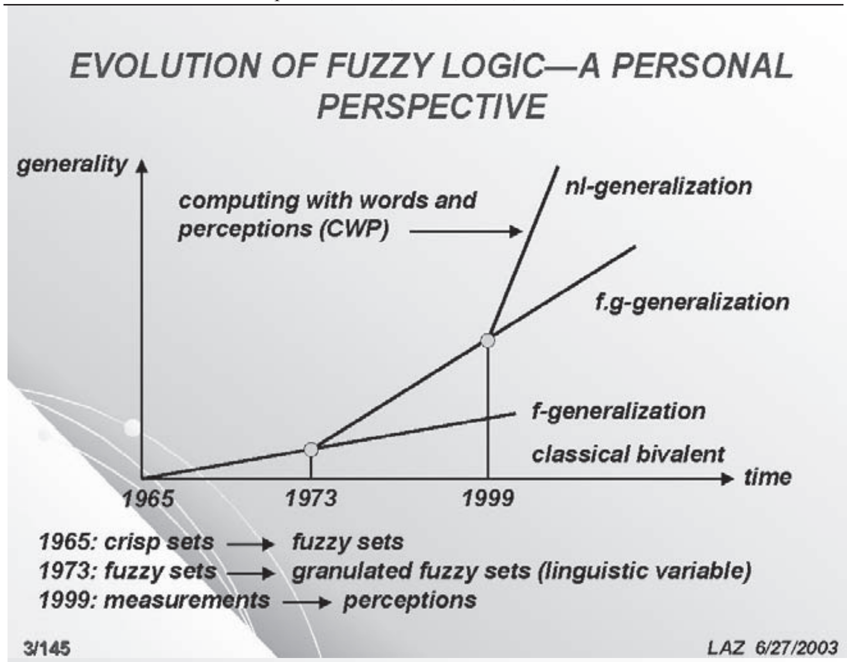
Computing with words and perceptions provides a basis for an important generalization of probability theory, namely, perception-based probability theory (PTp).

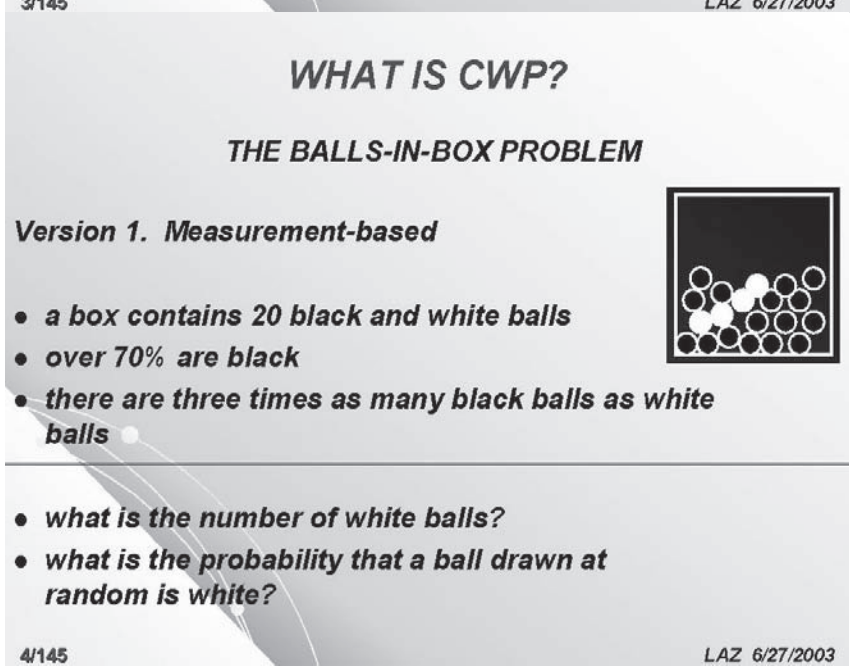
The importance of CWP derives from the fact that it opens the door to adding to any measurement-based theory.

Keywords: *fuzzy, CWP (computing with words), PNL (precisiated natural language), probability theory.*


COMPUTING WITH WORDS AND PERCEPTIONS (CWP)—A SHIFT IN DIRECTION IN COMPUTING AND DECISION ANALYSIS
Lotfi A. Zadeh
Computer Science Division
Department of EECS
UC Berkeley
July 3, 2003
URL: http://www-bisc.cs.berkeley.edu
URL: http://zadeh.cs.berkeley.edu/
Email: Zadeh@cs.berkeley.edu


BACKDROP
2/145
LAZ 6/27/2003


EVOLUTION OF FUZZY LOGIC—A PERSONAL PERSPECTIVE
generality
computing with words and perceptions (CWP)
nl-generalization
f.g-generalization
f-generalization
classical bivalent
time
1965
1973
1999
1965: crisp sets → fuzzy sets
1973: fuzzy sets → granulated fuzzy sets (linguistic variable)
1999: measurements → perceptions
3/145
LAZ 6/27/2003


WHAT IS CWP?
THE BALLS-IN-BOX PROBLEM
Version 1. Measurement-based
• a box contains 20 black and white balls
• over 70% are black
• there are three times as many black balls as white balls
• what is the number of white balls?
• what is the probability that a ball drawn at random is white?
4/145
LAZ 6/27/2003

CONTINUED

Version 2. Perception-based

- *a box contains about 20 black and white balls*
- *most are black*
- *there are several times as many black balls as white balls*

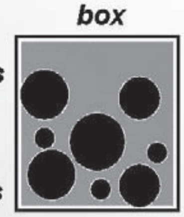
- *what is the number of white balls?*
- *what is the probability that a ball drawn at random is white?*

5/145 LAZ 6/27/2003

CONTINUED

Version 3. Perception-based

box



- *a box contains about 20 black balls of various sizes*
- *most are large*
- *there are several times as many large balls as small balls*

- *what is the number of small balls?*
- *what is the probability that a ball drawn at random is small?*

6/145 LAZ 6/27/2003

MEASUREMENT-BASED

- ***a box contains 20 black and white balls***
- ***over seventy percent are black***
- ***there are three times as many black balls as white balls***
- ***what is the number of white balls?***
- ***what is the probability that a ball picked at random is white?***

PERCEPTION-BASED *(version 1)*

- ***a box contains about 20 black and white balls***
- ***most are black***
- ***there are several times as many black balls as white balls***
- ***what is the number of white balls***
- ***what is the probability that a ball drawn at random is white?***

7/145 LAZ 6/27/2003

COMPUTATION (version 1)

- ***measurement-based***

  X = number of black balls

  Y_2 number of white balls

  $X \geq 0.7 \cdot 20 = 14$

  $X + Y = 20$

  $X = 3Y$

  $X = 15$; $Y = 5$

  $p = 5/20 = .25$

- ***perception-based***

  X = number of black balls

  Y = number of white balls

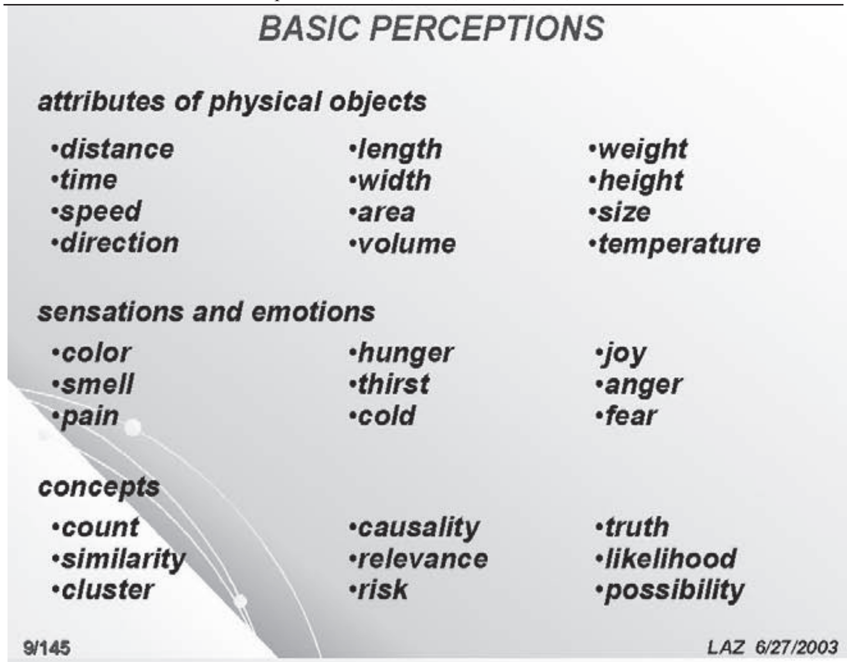
  $X = most \times 20^*$

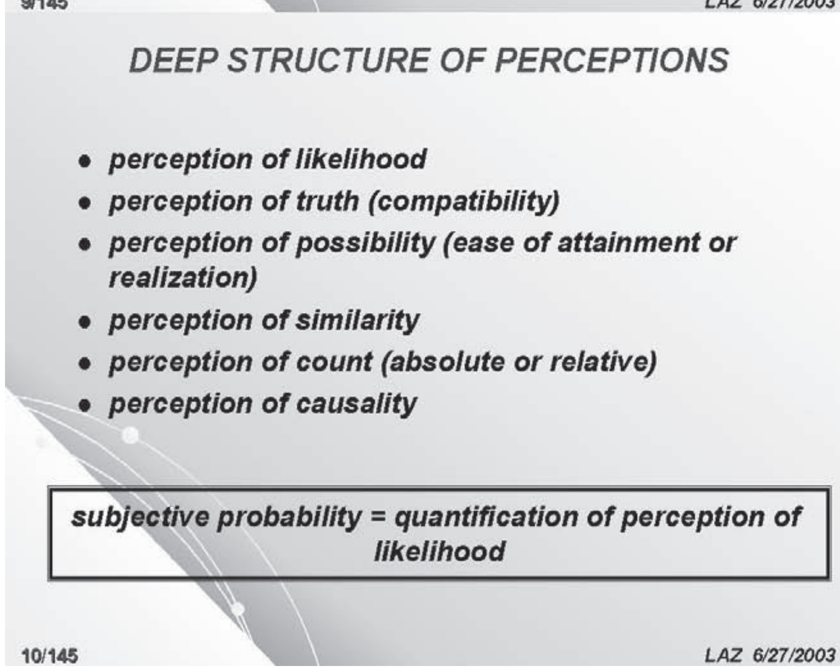
  $X = several\ {}^*Y$

  $X + Y = 20^*$

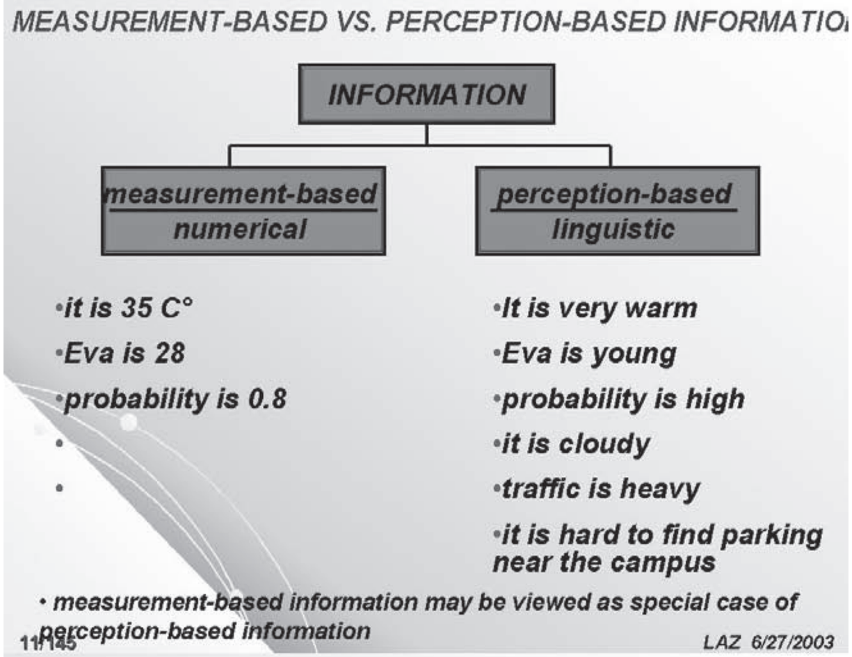
  $P = Y/N$

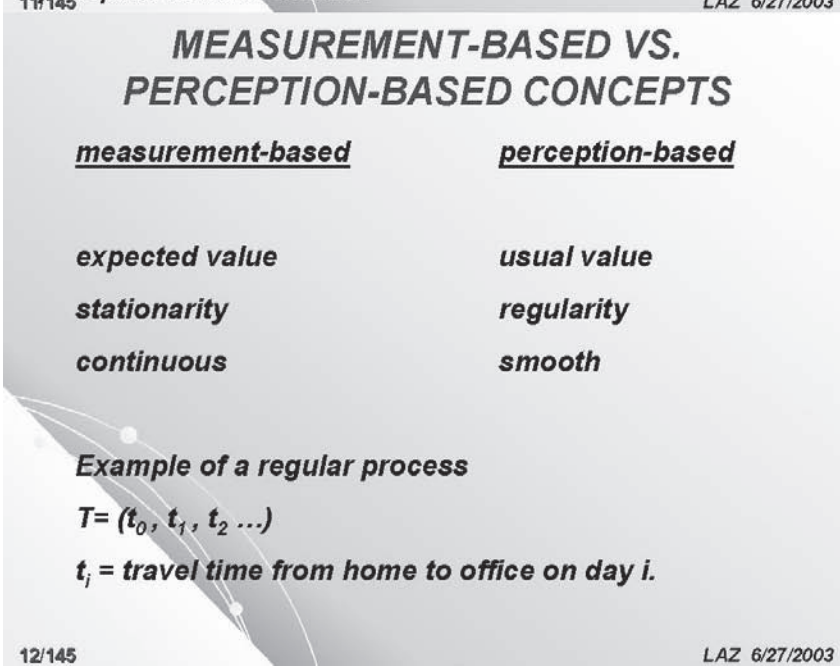
8/145 LAZ 6/27/2003

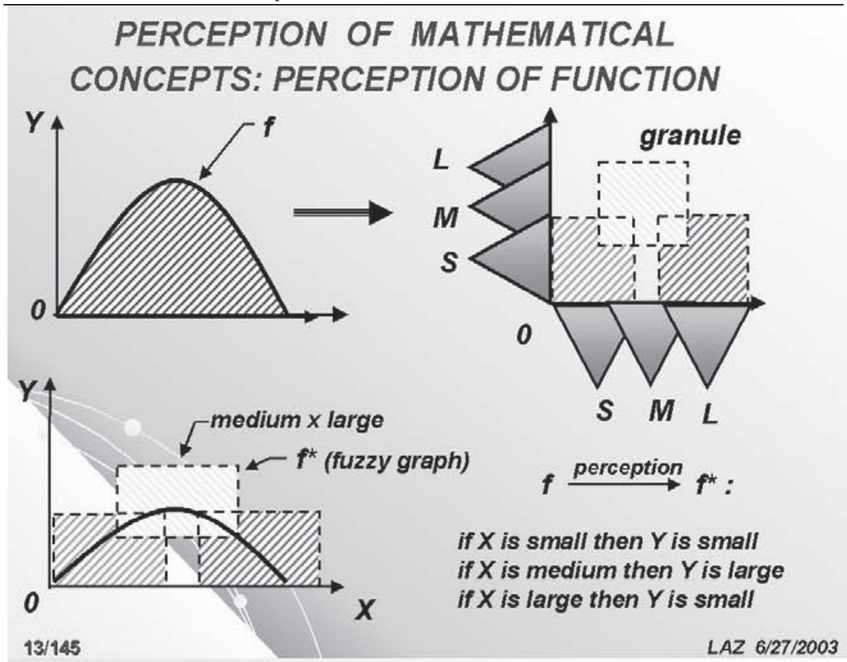

BASIC PERCEPTIONS
attributes of physical objects
•distance
•time
•speed
•direction
•length
•width
•area
•volume
•weight
•height
•size
•temperature
sensations and emotions
•color
•smell
•pain
•hunger
•thirst
•cold
•joy
•anger
•fear
concepts
•count
•similarity
•cluster
•causality
•relevance
•risk
•truth
•likelihood
•possibility
9/145
LAZ 6/27/2003

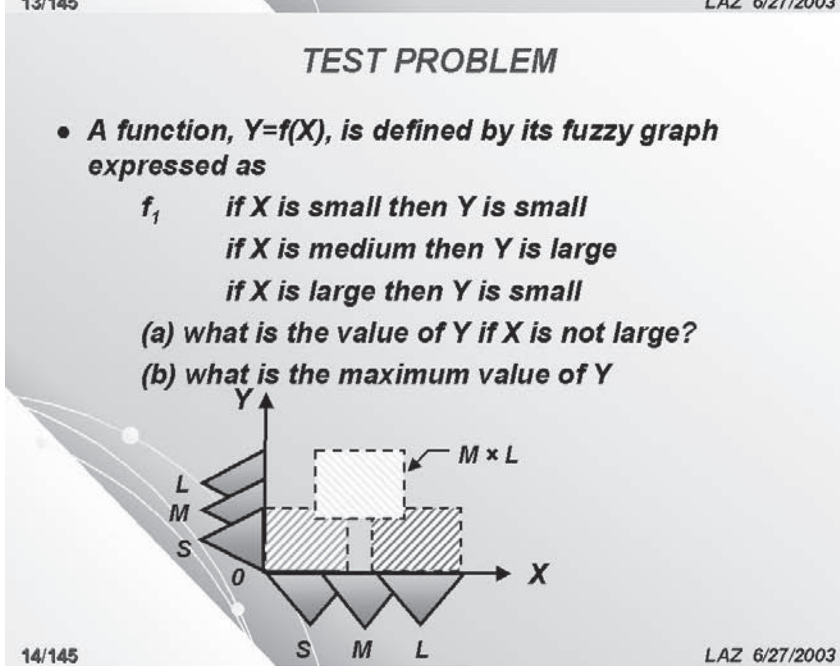

DEEP STRUCTURE OF PERCEPTIONS
• perception of likelihood
• perception of truth (compatibility)
• perception of possibility (ease of attainment or realization)
• perception of similarity
• perception of count (absolute or relative)
• perception of causality
subjective probability = quantification of perception of likelihood
10/145
LAZ 6/27/2003

Scientific and Technical Bulletin Series: Electrotechnics, Electronics, Automatic Control and Computer Science, Vol. 2, No. 1, 2005**,** ISSN 1584-9198

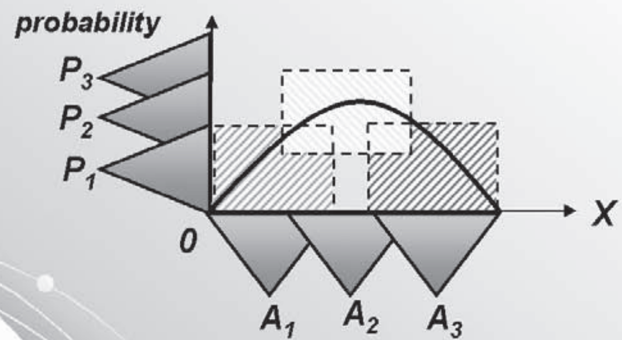





PERCEPTION OF MATHEMATICAL CONCEPTS: PERCEPTION OF FUNCTION
Y
f
0
granule
L
M
S
0
S M L
Y
medium x large
f* (fuzzy graph)
0
X
f perception f* :
if X is small then Y is small
if X is medium then Y is large
if X is large then Y is small
13/145
LAZ 6/27/2003


TEST PROBLEM
• A function, Y=f(X), is defined by its fuzzy graph expressed as
f1 if X is small then Y is small
if X is medium then Y is large
if X is large then Y is small
(a) what is the value of Y if X is not large?
(b) what is the maximum value of Y
Y
M × L
L
M
S
0
X
S M L
14/145
LAZ 6/27/2003

BIMODAL DISTRIBUTION (PERCEPTION-BASED PROBABILITY DISTRIBUTION)



$P(X) = P_{i(1)} \backslash A_1 + P_{i(2)} \backslash A_2 + P_{i(3)} \backslash A_3$

Prob $\{X \text{ is } A_i\}$ is $P_{j(i)}$

P(X)= low\single+high\medium+low\large

15/145 LAZ 6/27/2003

COMPUTING WITH WORDS AND PERCEPTIONS—A SHIFT IN DIRECTION IN COMPUTING AND DECISION ANALYSIS

- Computing with words and perceptions, or CWP for short, is a mode of computing in which the objects of computation are words, propositions and perceptions described in a natural language.

16/145 LAZ 6/27/2003

CONTINUED

- ***Perceptions play a key role in human cognition. Humans—but not machines—have a remarkable capability to perform a wide variety of physical and mental tasks without any measurements and any computations. Everyday examples of such tasks are driving a car in city traffic, playing tennis and summarizing a book.***

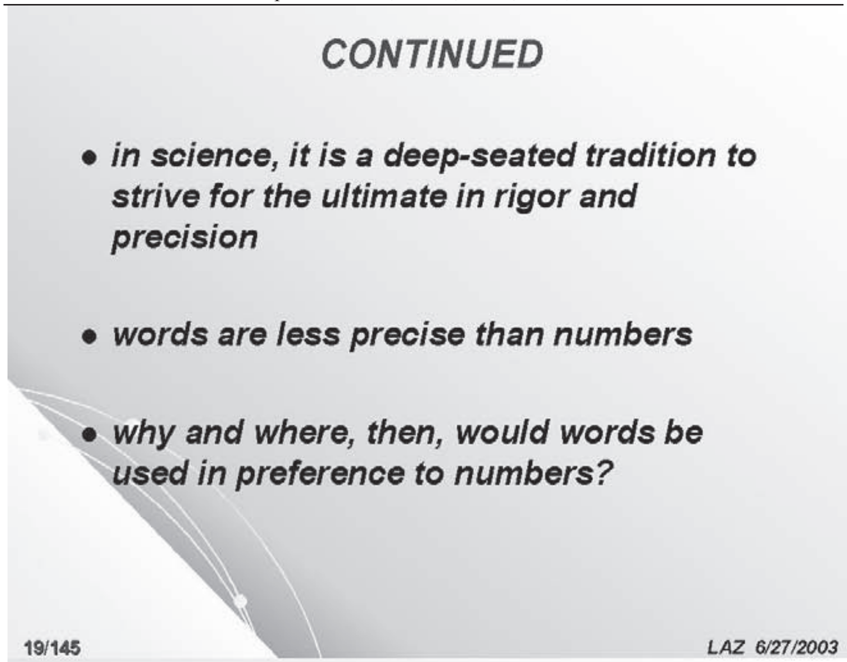
17/145 LAZ 6/27/2003

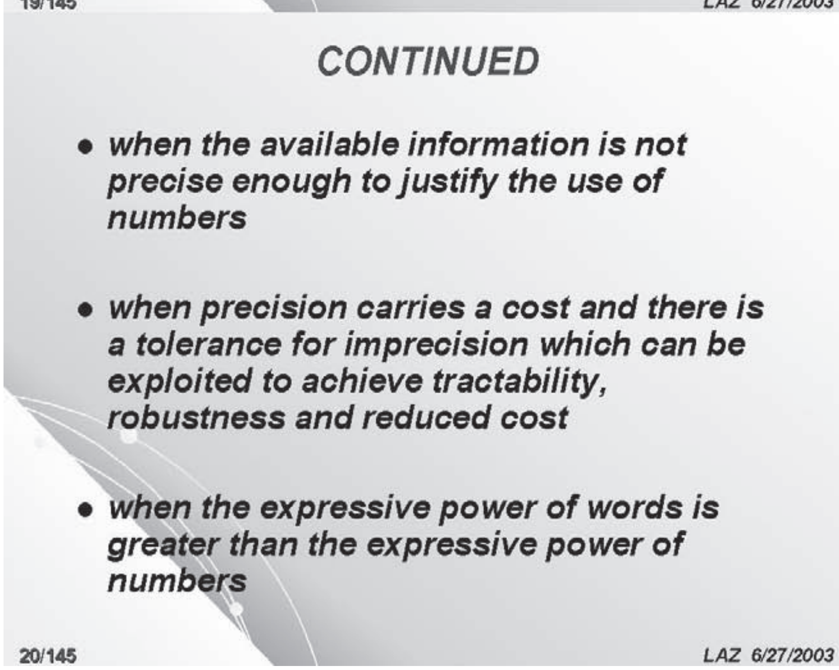
COMPUTING WITH WORDS AND PERCEPTIONS (CWP)

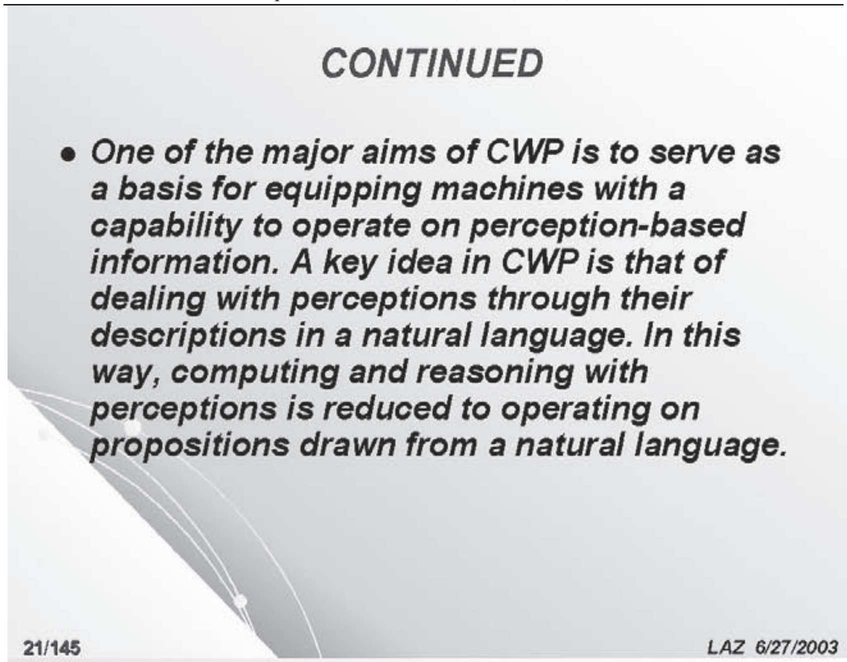
Key points

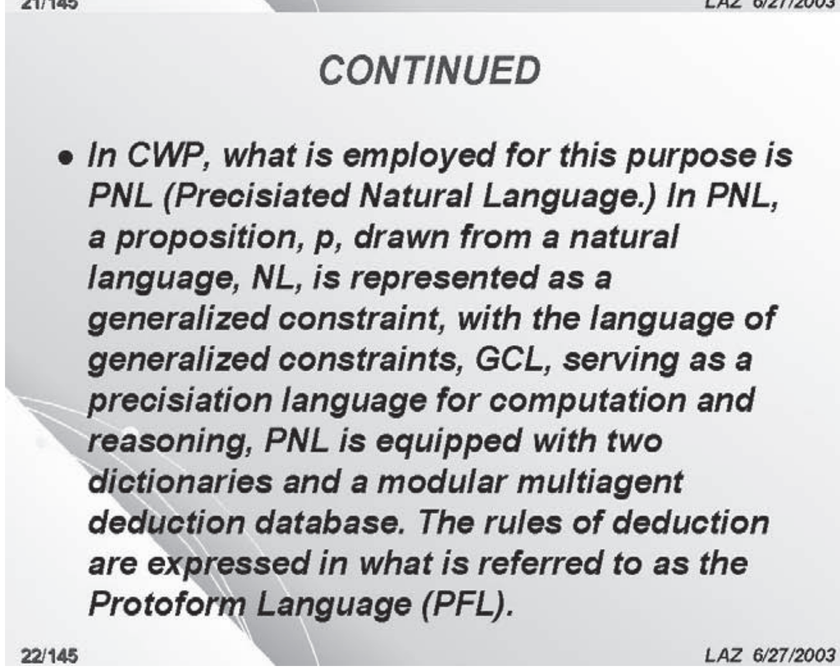
- ***In computing with words and perceptions, the objects of computation are words, propositions, and perceptions described in a natural language***
- ***A natural language is a system for describing perceptions***
- ***In CWP, a perception is equated to its description in a natural language***

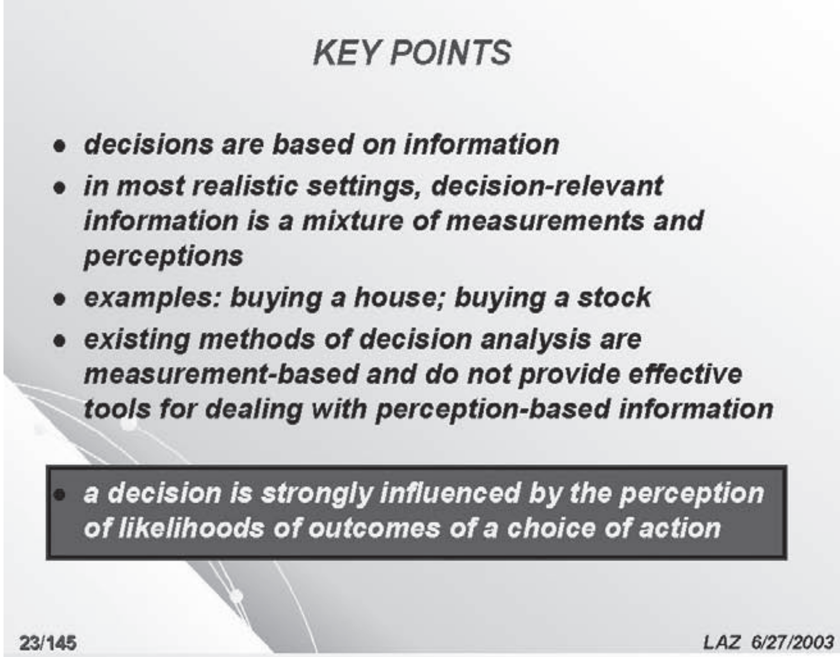
18/145 LAZ 6/27/2003

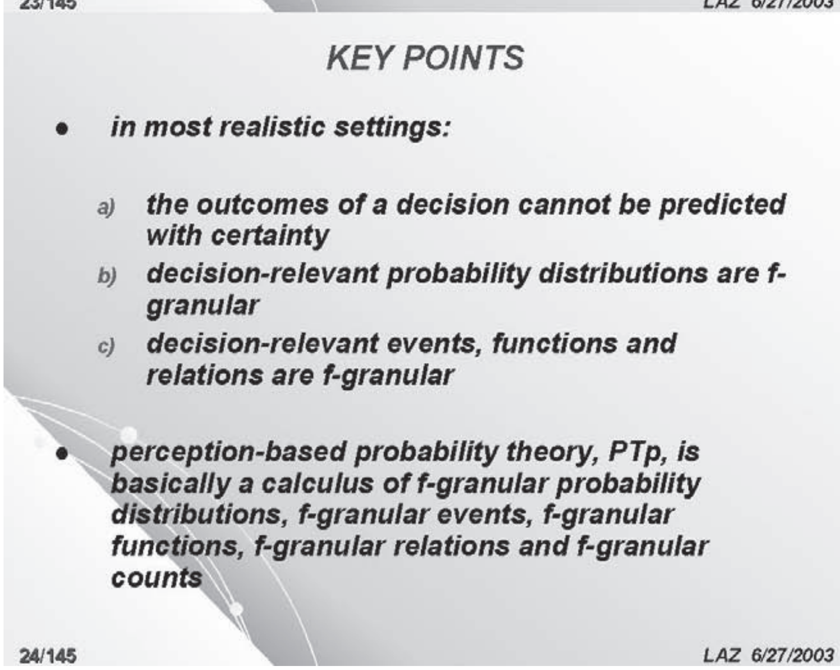

CONTINUED
• in science, it is a deep-seated tradition to strive for the ultimate in rigor and precision
• words are less precise than numbers
• why and where, then, would words be used in preference to numbers?
19/145
LAZ 6/27/2003


CONTINUED
• when the available information is not precise enough to justify the use of numbers
• when precision carries a cost and there is a tolerance for imprecision which can be exploited to achieve tractability, robustness and reduced cost
• when the expressive power of words is greater than the expressive power of numbers
20/145
LAZ 6/27/2003


CONTINUED
• One of the major aims of CWP is to serve as a basis for equipping machines with a capability to operate on perception-based information. A key idea in CWP is that of dealing with perceptions through their descriptions in a natural language. In this way, computing and reasoning with perceptions is reduced to operating on propositions drawn from a natural language.
21/145
LAZ 6/27/2003


CONTINUED
• In CWP, what is employed for this purpose is PNL (Precisiated Natural Language.) In PNL, a proposition, p, drawn from a natural language, NL, is represented as a generalized constraint, with the language of generalized constraints, GCL, serving as a precisiation language for computation and reasoning, PNL is equipped with two dictionaries and a modular multiagent deduction database. The rules of deduction are expressed in what is referred to as the Protoform Language (PFL).
22/145
LAZ 6/27/2003


KEY POINTS
• decisions are based on information
• in most realistic settings, decision-relevant information is a mixture of measurements and perceptions
• examples: buying a house; buying a stock
• existing methods of decision analysis are measurement-based and do not provide effective tools for dealing with perception-based information
• a decision is strongly influenced by the perception of likelihoods of outcomes of a choice of action
23/145
LAZ 6/27/2003


KEY POINTS
• in most realistic settings:
a) the outcomes of a decision cannot be predicted with certainty
b) decision-relevant probability distributions are f-granular
c) decision-relevant events, functions and relations are f-granular
• perception-based probability theory, PTp, is basically a calculus of f-granular probability distributions, f-granular events, f-granular functions, f-granular relations and f-granular counts
24/145
LAZ 6/27/2003

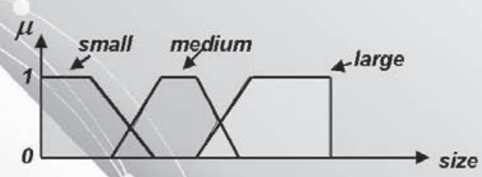
OBSERVATION

- ***machines are driven by measurements***
- ***humans are driven by perceptions***
- ***to enable a machine to mimic the remarkable human capability to perform a wide variety of physical and mental tasks using perception-based information, it is necessary to have a means of converting measurements into perceptions***

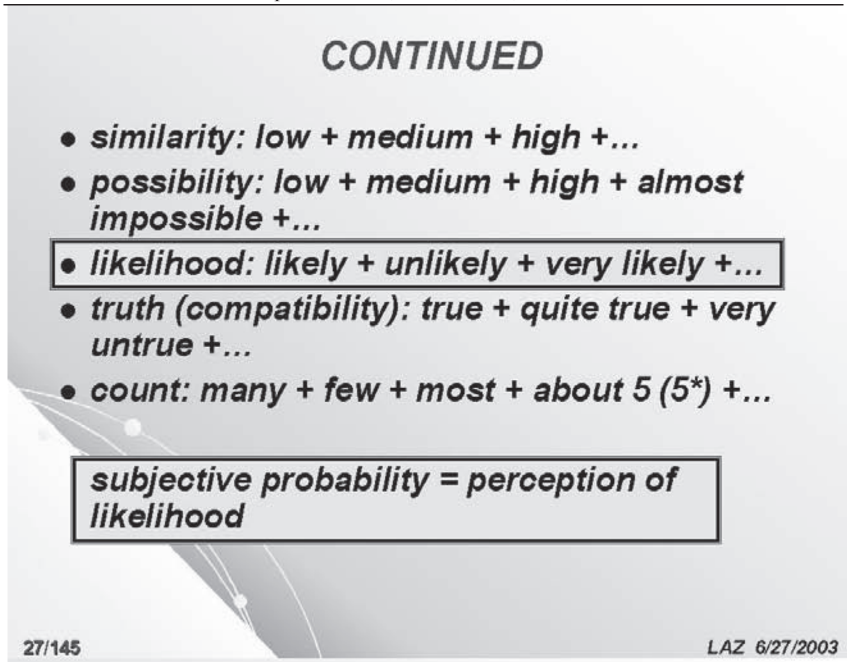
25/145 LAZ 6/27/2003

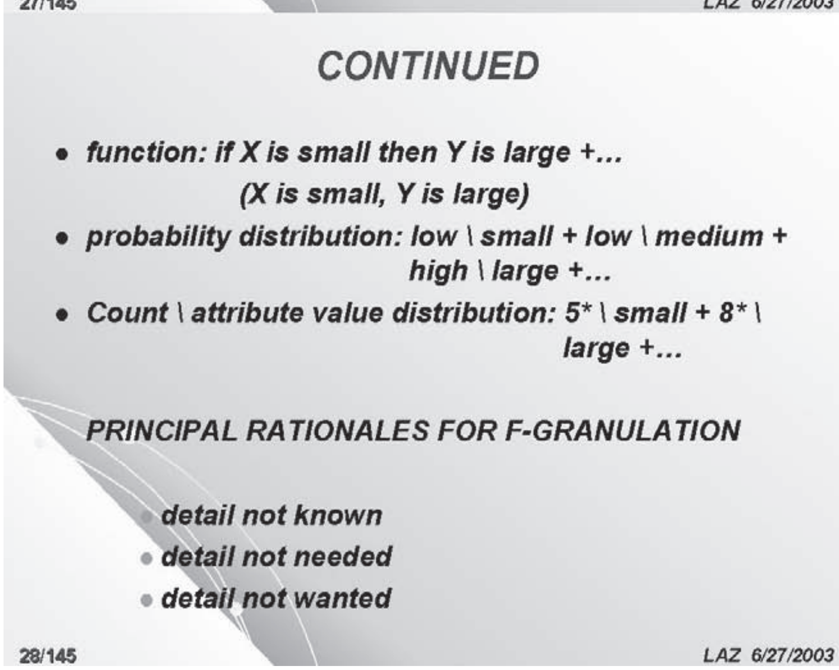
BASIC PERCEPTIONS / F-GRANULARITY

- ***temperature: warm+cold+very warm+much warmer+…***
- ***time: soon + about one hour + not much later +…***
- ***distance: near + far + much farther +…***
- ***speed: fast + slow +much faster +…***
- ***length: long + short + very long +…***

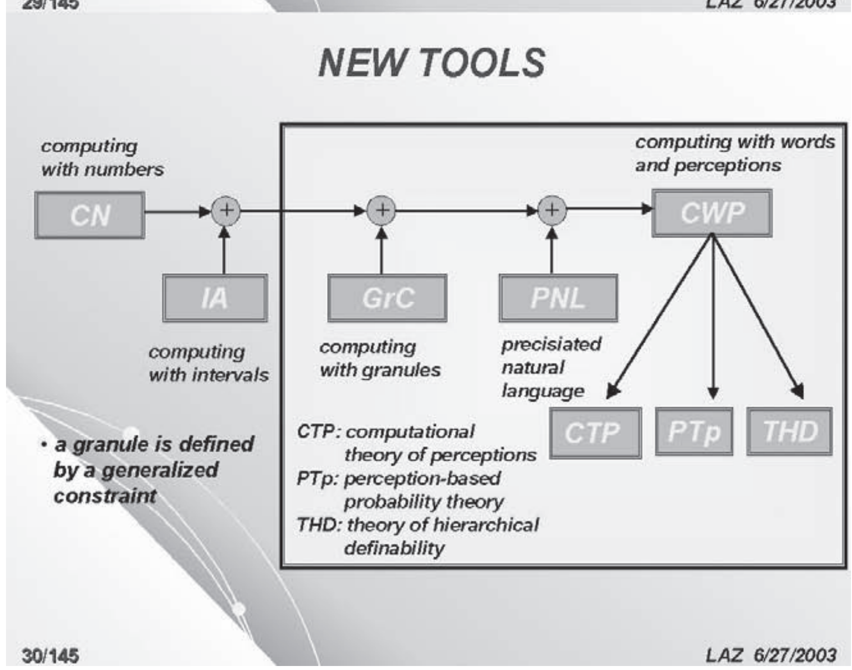


26/145 LAZ 6/27/2003


CONTINUED
• similarity: low + medium + high +...
• possibility: low + medium + high + almost impossible +...
• likelihood: likely + unlikely + very likely +...
• truth (compatibility): true + quite true + very untrue +...
• count: many + few + most + about 5 (5*) +...
subjective probability = perception of likelihood
27/145
LAZ 6/27/2003


CONTINUED
• function: if X is small then Y is large +...
(X is small, Y is large)
• probability distribution: low \ small + low \ medium + high \ large +...
• Count \ attribute value distribution: 5* \ small + 8* \ large +...
PRINCIPAL RATIONALES FOR F-GRANULATION
• detail not known
• detail not needed
• detail not wanted
28/145
LAZ 6/27/2003


New Tools
29/145
LAZ 6/27/2003


NEW TOOLS
computing with numbers
CN
IA
computing with intervals
GrC
computing with granules
PNL
precisiated natural language
computing with words and perceptions
CWP
CTP
PTp
THD
• a granule is defined by a generalized constraint
CTP: computational theory of perceptions
PTp: perception-based probability theory
THD: theory of hierarchical definability
30/145
LAZ 6/27/2003

GRANULAR COMPUTING GENERALIZED VALUATION

valuation = assignment of a value to a variable

X = 5	0 ≤ X ≤ 5	X is small	X isr R
point	interval	fuzzy interval	generalized

singular value

measurement-based

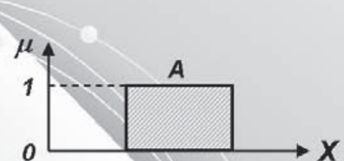
granular values

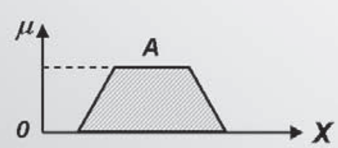
perception-based

31/145 **LAZ 6/27/2003**

F-GENERALIZATION

- ***f-generalization of a theory, T, involves an introduction into T of the concept of a fuzzy set***
- ***f-generalization of PT, PT$^+$, adds to PT the capability to deal with fuzzy probabilities, fuzzy probability distributions, fuzzy events, fuzzy functions and fuzzy relations***

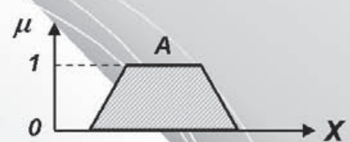




32/145 **LAZ 6/27/2003**

F.G-GENERALIZATION

- ***f.g-generalization of T, T^{++}, involves an introduction into T of the concept of a granulated fuzzy set***
- ***f.g-generalization of PT, PT^{++} , adds to PT^{+} the capability to deal with f-granular probabilities, f-granular probability distributions, f-granular events, f-granular functions and f-granular relations***



33/145 **LAZ 6/27/2003**

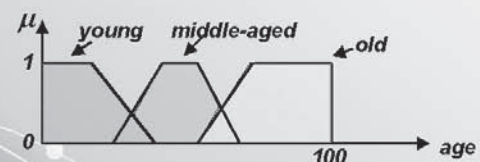
EXAMPLES OF F-GRANULATION (LINGUISTIC VARIABLES)

color: red, blue, green, yellow, …

age: young, middle-aged, old, very old

size: small, big, very big, …

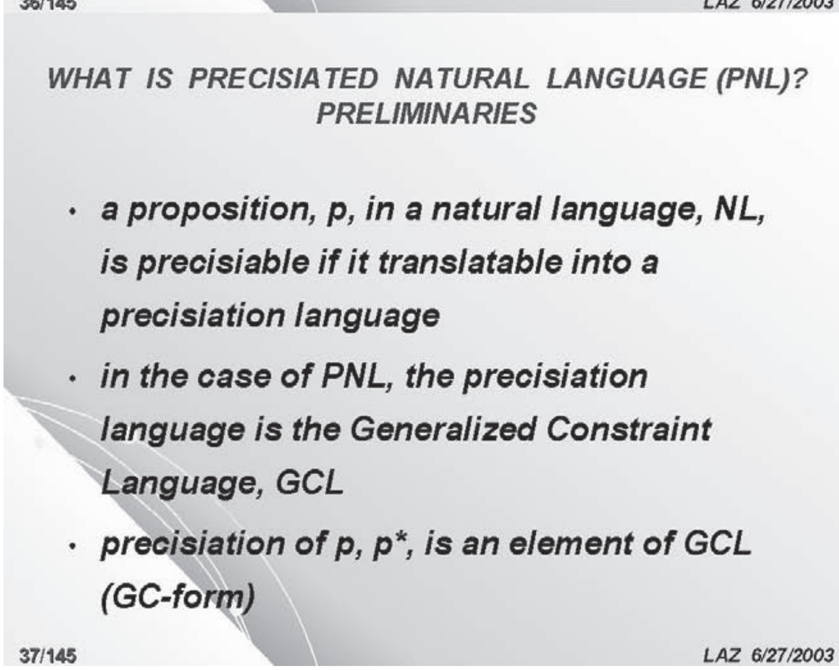
distance: near, far, very, not very far, …



• humans have a remarkable capability to perform a wide variety of physical and mental tasks, e.g., driving a car in city traffic, without any measurements and any computations
• one of the principal aims of CTP is to develop a better understanding of how this capability can be added to machines

34/145 **LAZ 6/27/2003**


PRECISIATED
NATURAL LANGUAGE
PNL
36/145
LAZ 6/27/2003


WHAT IS PRECISIATED NATURAL LANGUAGE (PNL)?
PRELIMINARIES
• a proposition, p, in a natural language, NL, is precisiable if it translatable into a precisiation language
• in the case of PNL, the precisiation language is the Generalized Constraint Language, GCL
• precisiation of p, p*, is an element of GCL (GC-form)
37/145
LAZ 6/27/2003

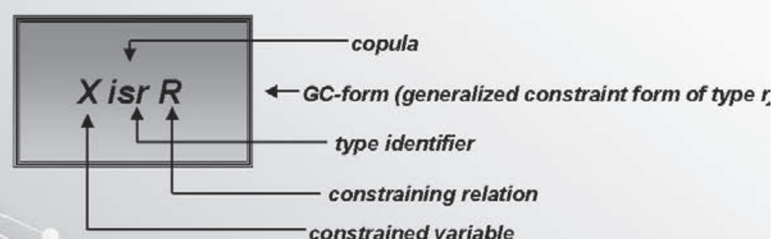
WHAT IS PNL?

- *PNL is a sublanguage of precisiable propositions in NL which is equipped with two dictionaries: (1) NL to GCL; (2) GCL to PFL (Protoform Language); and (3) a modular multiagent database of rules of deduction (rules of generalized constrained propagation) expressed in PFL.*

38/145 LAZ 6/27/2003

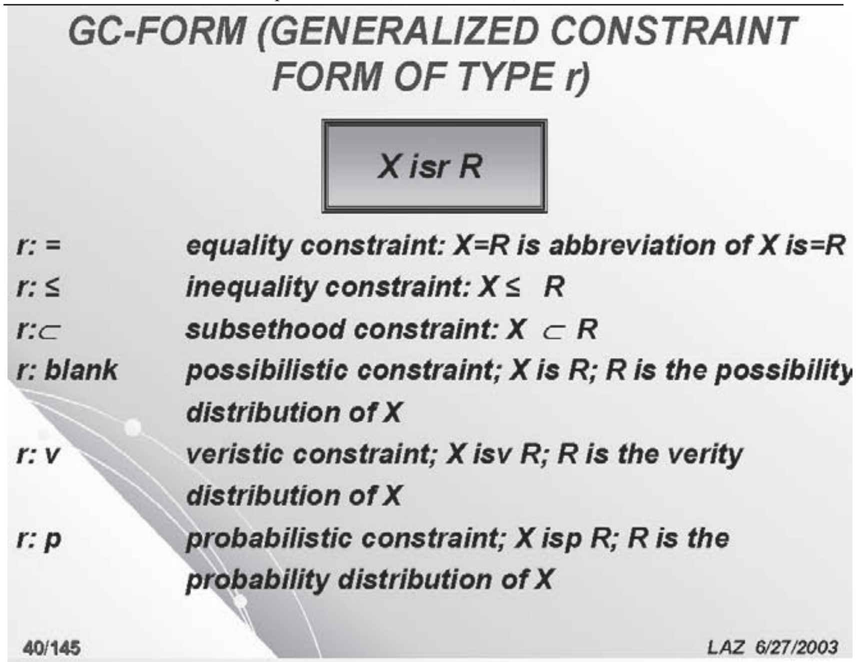
GENERALIZED CONSTRAINT

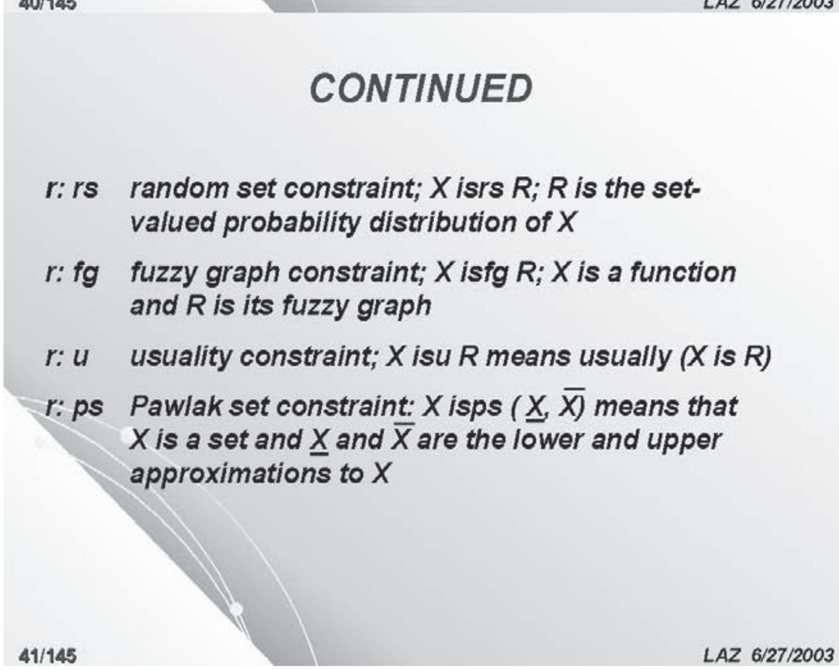
- *standard constraint:* $X \in C$
- *generalized constraint:* *X isr R*

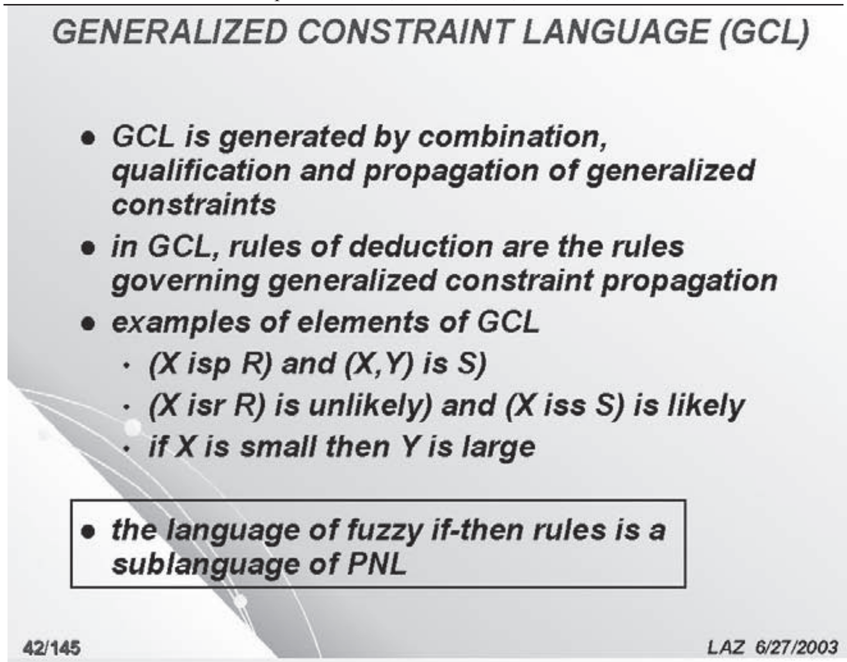


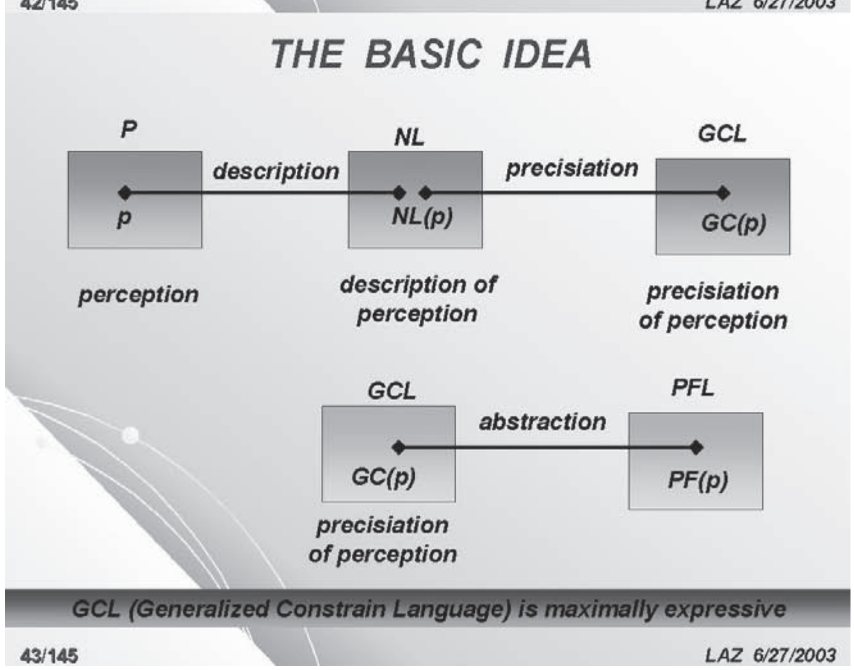
- $X = (X_1, \ldots, X_n)$
- *X may have a structure: X=Location (Residence(Carol))*
- *X may be a function of another variable: X=f(Y)*
- *X may be conditioned: (X/Y)*
- $r := / \leq / \ldots / \subset / \supset /$ blank / v / p / u / rs / fg / ps / ...

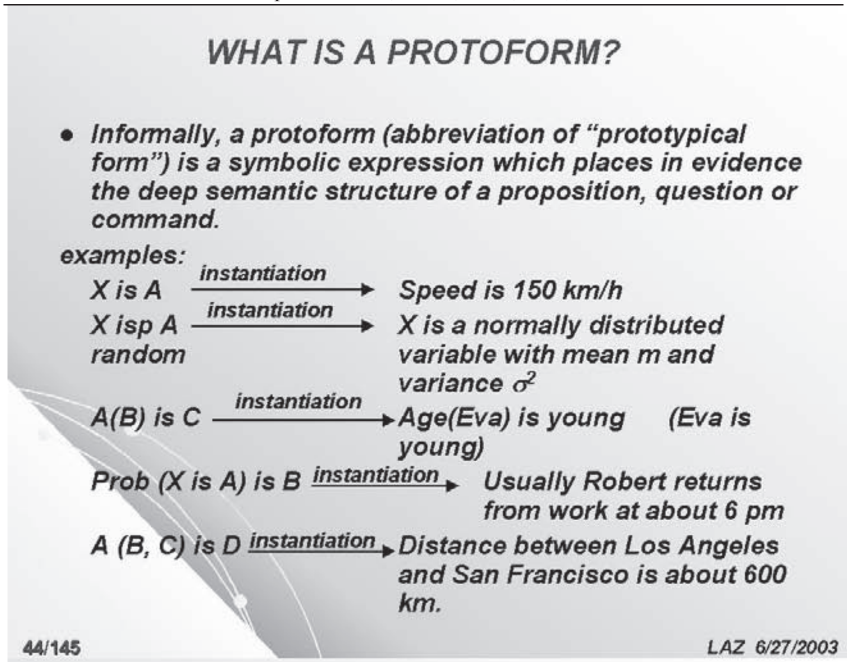
39/145 LAZ 6/27/2003

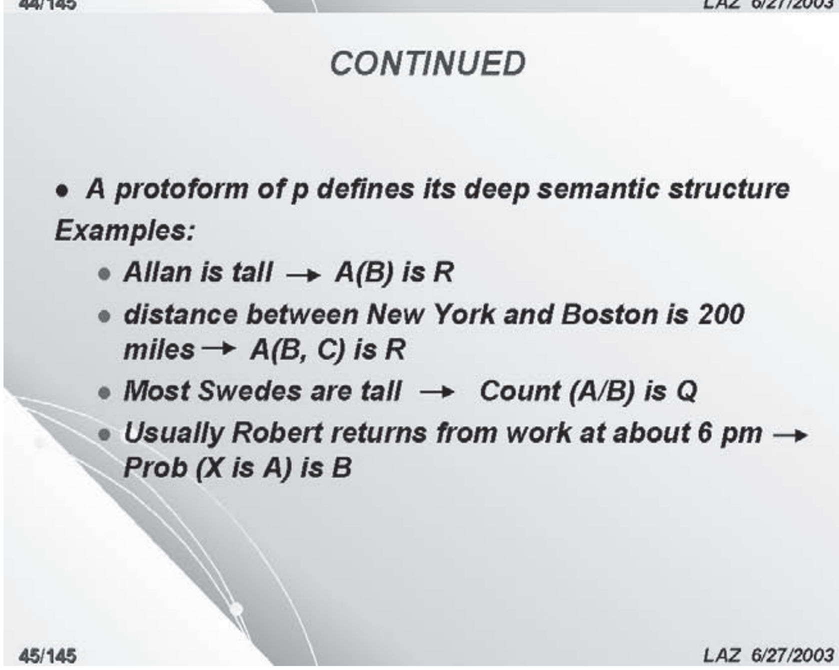

GC-FORM (GENERALIZED CONSTRAINT FORM OF TYPE r)
X isr R
r: = equality constraint: X=R is abbreviation of X is=R
r: ≤ inequality constraint: X ≤ R
r:⊂ subsethood constraint: X ⊂ R
r: blank possibilistic constraint; X is R; R is the possibility distribution of X
r: v veristic constraint; X isv R; R is the verity distribution of X
r: p probabilistic constraint; X isp R; R is the probability distribution of X
40/145
LAZ 6/27/2003


CONTINUED
r: rs random set constraint; X isrs R; R is the set-valued probability distribution of X
r: fg fuzzy graph constraint; X isfg R; X is a function and R is its fuzzy graph
r: u usuality constraint; X isu R means usually (X is R)
r: ps Pawlak set constraint: X isps ($\underline{X}$, $\overline{X}$) means that X is a set and $\underline{X}$ and $\overline{X}$ are the lower and upper approximations to X
41/145
LAZ 6/27/2003


GENERALIZED CONSTRAINT LANGUAGE (GCL)
• GCL is generated by combination, qualification and propagation of generalized constraints
• in GCL, rules of deduction are the rules governing generalized constraint propagation
• examples of elements of GCL
· (X isp R) and (X,Y) is S)
· (X isr R) is unlikely) and (X iss S) is likely
· if X is small then Y is large
• the language of fuzzy if-then rules is a sublanguage of PNL
42/145
LAZ 6/27/2003


THE BASIC IDEA
P
NL
GCL
description
precisiation
p
NL(p)
GC(p)
perception
description of perception
precisiation of perception
GCL
PFL
abstraction
GC(p)
PF(p)
precisiation of perception
GCL (Generalized Constrain Language) is maximally expressive
43/145
LAZ 6/27/2003


WHAT IS A PROTOFORM?
Informally, a protoform (abbreviation of "prototypical form") is a symbolic expression which places in evidence the deep semantic structure of a proposition, question or command.
examples:
X is A —instantiation→ Speed is 150 km/h
X isp A random —instantiation→ X is a normally distributed variable with mean m and variance σ²
A(B) is C —instantiation→ Age(Eva) is young (Eva is young)
Prob (X is A) is B —instantiation→ Usually Robert returns from work at about 6 pm
A (B, C) is D —instantiation→ Distance between Los Angeles and San Francisco is about 600 km.
44/145
LAZ 6/27/2003


CONTINUED
A protoform of p defines its deep semantic structure
Examples:
Allan is tall → A(B) is R
distance between New York and Boston is 200 miles → A(B, C) is R
Most Swedes are tall → Count (A/B) is Q
Usually Robert returns from work at about 6 pm → Prob (X is A) is B
45/145
LAZ 6/27/2003

DICTIONARIES

1:

proposition in NL	*precisiation*
p	*p* (GC-form)*
most Swedes are tall	Σ *Count (tall.Swedes/Swedes) is most*

2:

precisiation	*protoform*
p (GC-form)*	*PF(p*)*
Σ *Count (tall.Swedes/Swedes) is most*	*Q A's are B's*

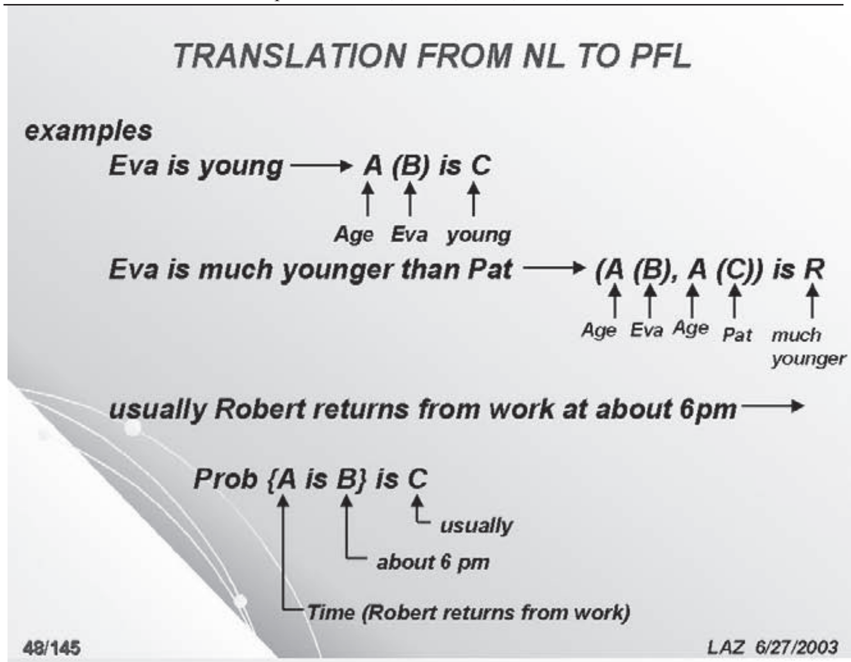
46/145 LAZ 6/27/2003

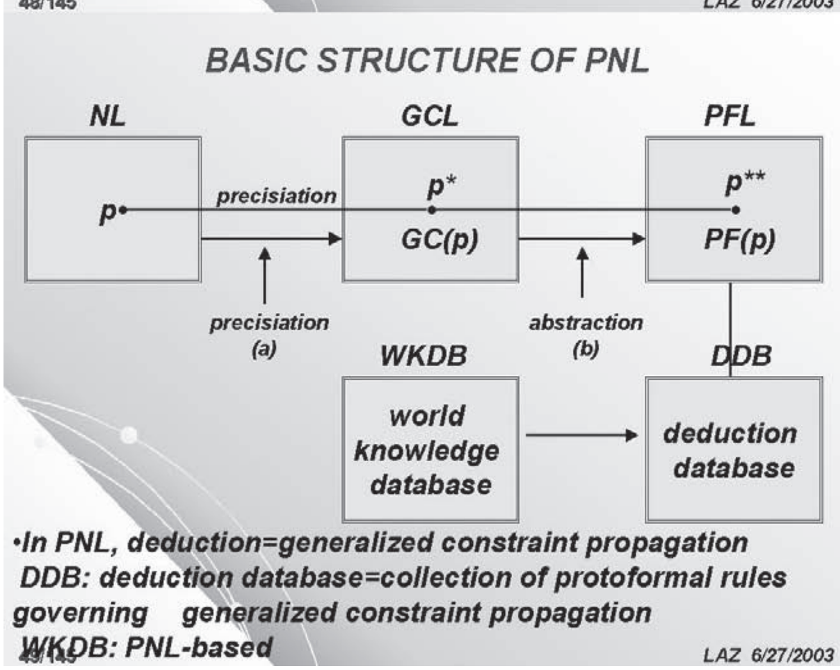
EXAMPLE OF TRANSLATION

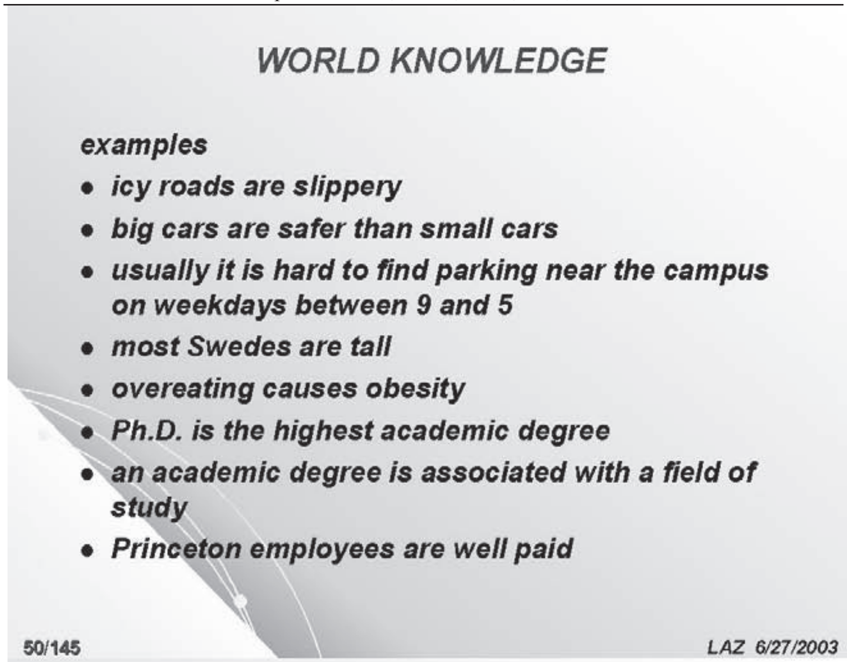
- *P: usually Robert returns from work at about 6 pm*
- *P*: Prob {(Time(Return(Robert)) is 6 pm} is usually*
- *PF(p): Prob {X is A} is B*
- *X: Time (Return (Robert))*
- *A: 6 pm*
- *B: usually*

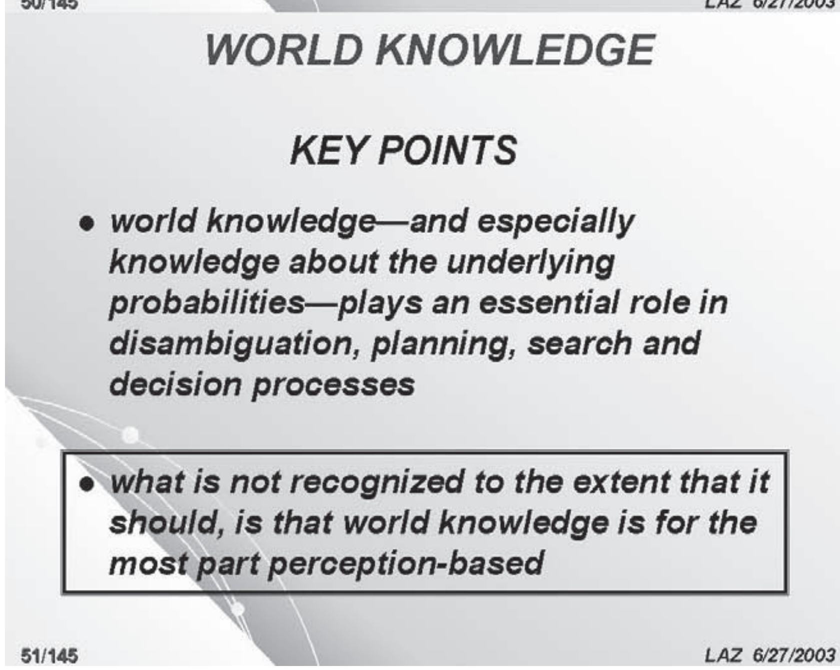
$p \in NL$
$p^* \in GCL$
$PF(p) \in PFL$

47/145 LAZ 6/27/2003


TRANSLATION FROM NL TO PFL
examples
Eva is young → A (B) is C
Age Eva young
Eva is much younger than Pat → (A (B), A (C)) is R
Age Eva Age Pat much younger
usually Robert returns from work at about 6pm →
Prob {A is B} is C
usually
about 6 pm
Time (Robert returns from work)
48/145
LAZ 6/27/2003


BASIC STRUCTURE OF PNL
NL
GCL
PFL
p
precisiation
p*
GC(p)
p**
PF(p)
precisiation (a)
abstraction (b)
WKDB
DDB
world knowledge database
deduction database
•In PNL, deduction=generalized constraint propagation
DDB: deduction database=collection of protoformal rules governing generalized constraint propagation
WKDB: PNL-based
49/145
LAZ 6/27/2003


WORLD KNOWLEDGE
examples
• icy roads are slippery
• big cars are safer than small cars
• usually it is hard to find parking near the campus on weekdays between 9 and 5
• most Swedes are tall
• overeating causes obesity
• Ph.D. is the highest academic degree
• an academic degree is associated with a field of study
• Princeton employees are well paid
50/145
LAZ 6/27/2003


WORLD KNOWLEDGE
KEY POINTS
• world knowledge—and especially knowledge about the underlying probabilities—plays an essential role in disambiguation, planning, search and decision processes
• what is not recognized to the extent that it should, is that world knowledge is for the most part perception-based
51/145
LAZ 6/27/2003

WORLD KNOWLEDGE: EXAMPLES

specific:

- ***if Robert works in Berkeley then it is likely that Robert lives in or near Berkeley***
- ***if Robert lives in Berkeley then it is likely that Robert works in or near Berkeley***

generalized:

if A/Person works in B/City then it is likely that A lives in or near B

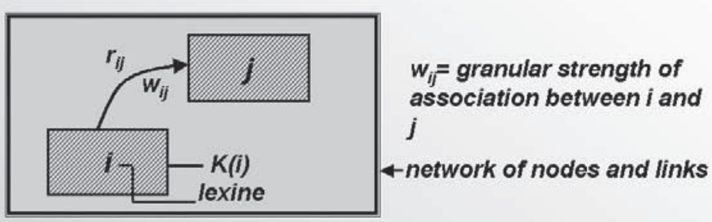
precisiated:

Distance (Location (Residence (A/Person), Location (Work (A/Person) isu near

protoform: ***F (A (B (C)), A (D (C))) isu R***

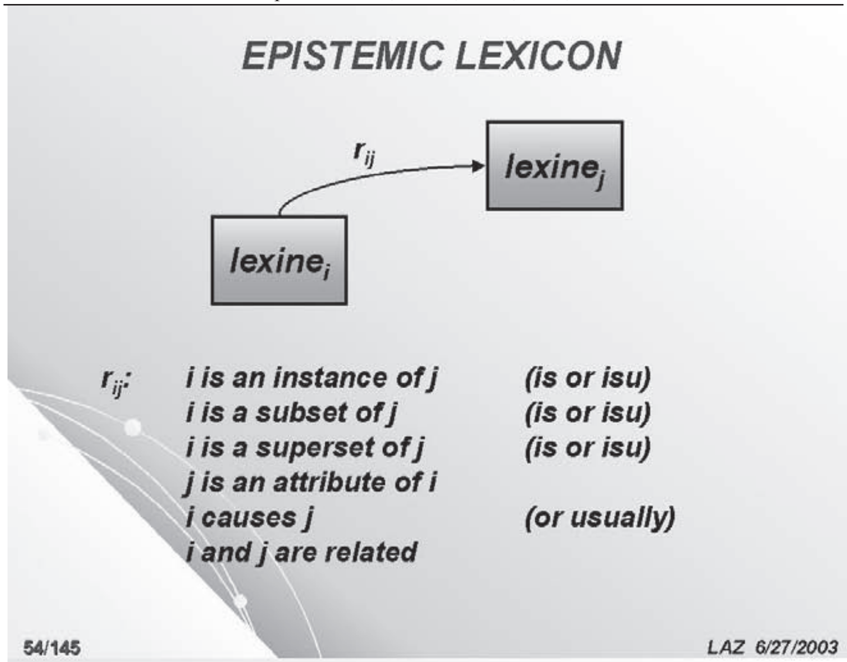
52/145 LAZ 6/27/2003

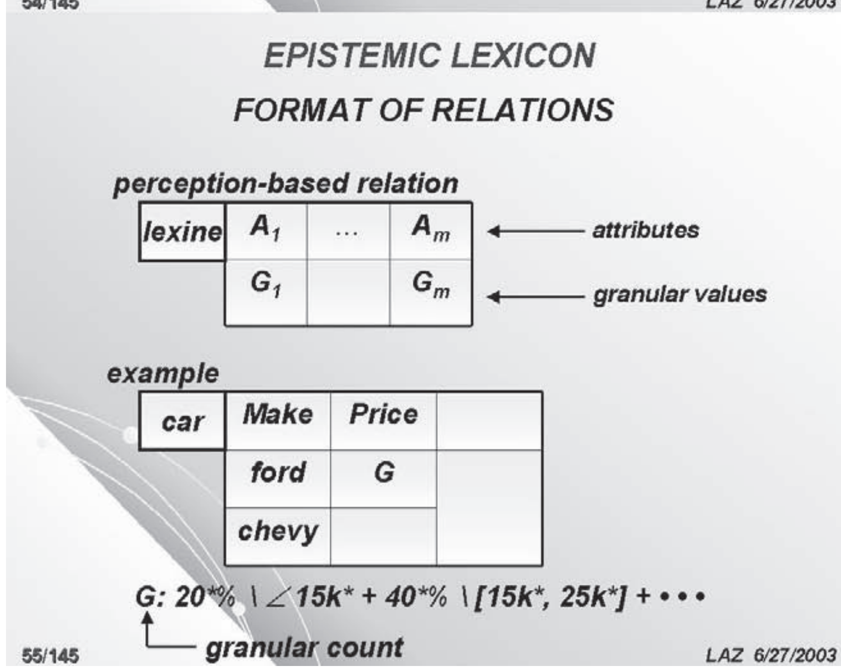
ORGANIZATION OF WORLD KNOWLEDGE EPISTEMIC (KNOWLEDGE-DIRECTED) LEXICON (EL) (ONTOLOGY-RELATED)

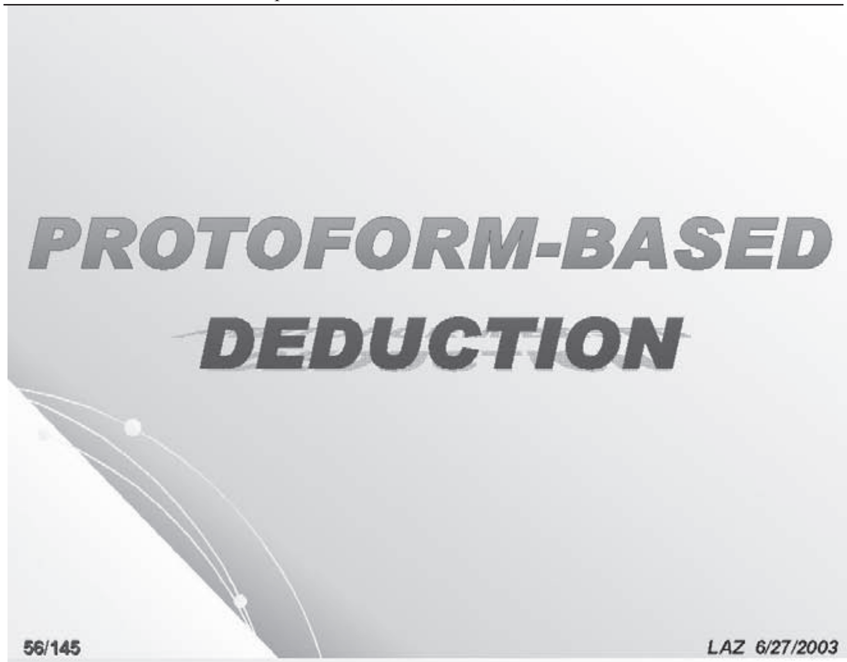


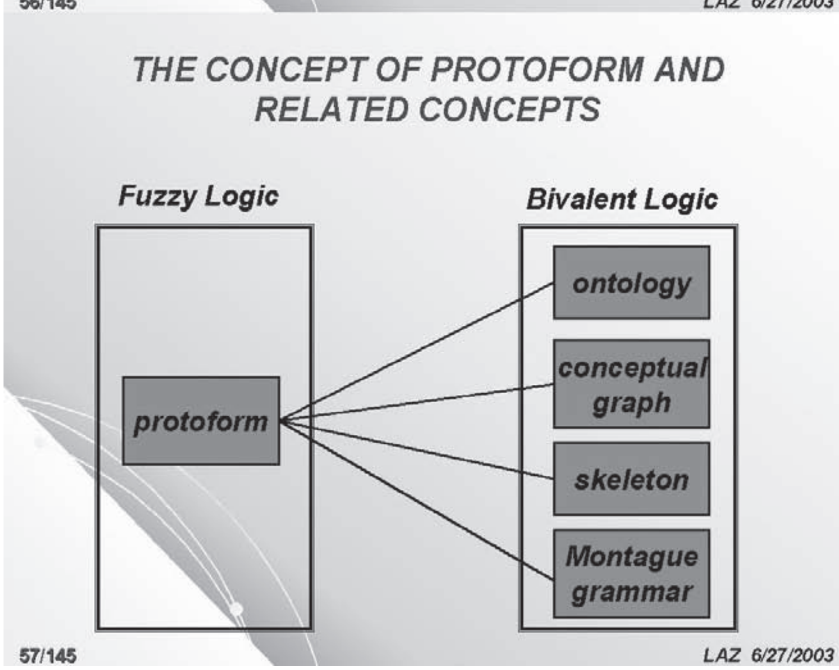
- ***i (lexine): object, construct, concept (e.g., car, Ph.D. degree)***
- ***K(i): world knowledge about i (mostly perception-based)***
- ***K(i) is organized into n(i) relations $R_{ii}, \ldots, R_{in}$***
- ***entries in R_{ij} are bimodal-distribution-valued attributes of i***
- ***values of attributes are, in general, granular and context-dependent***

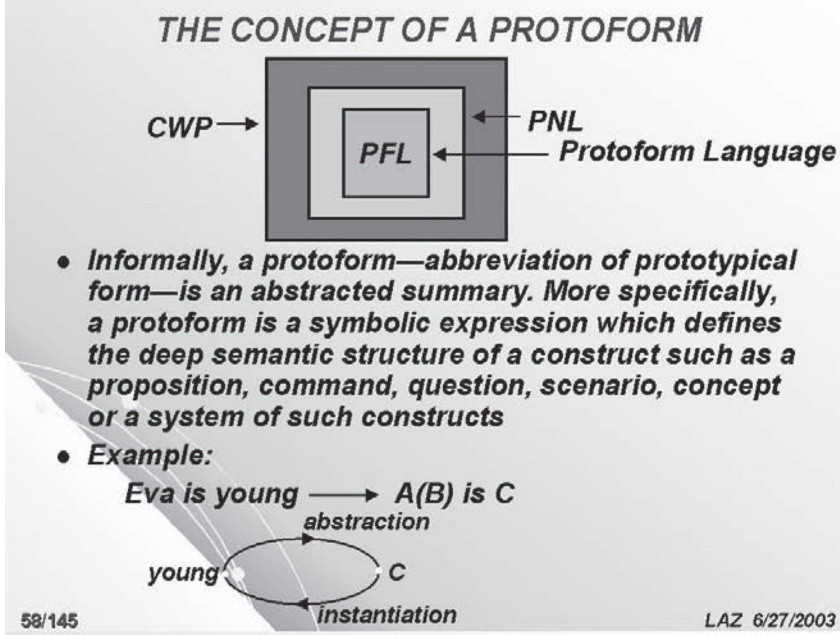
53/145 LAZ 6/27/2003

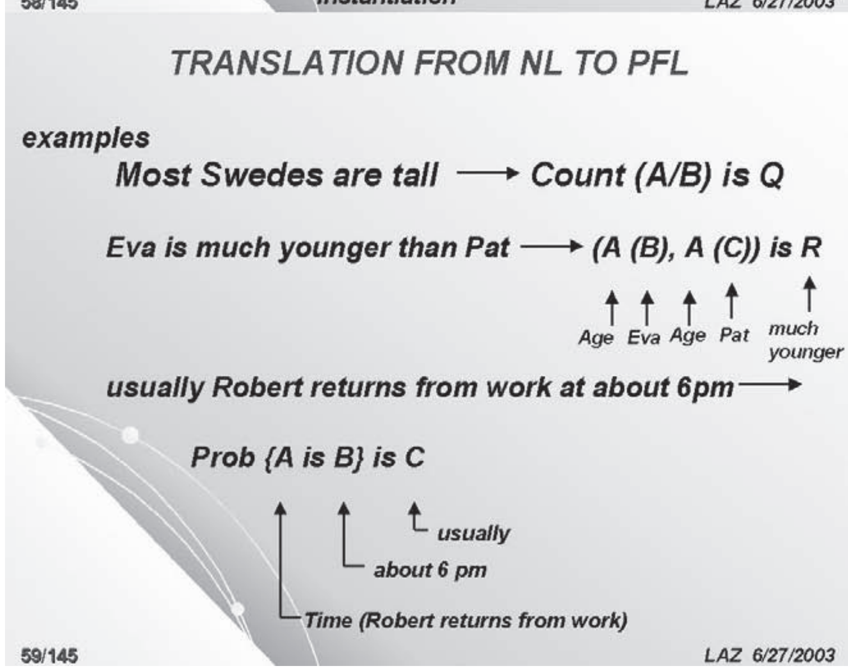

EPISTEMIC LEXICON
r_ij
lexine_j
lexine_i
r_ij:
i is an instance of j (is or isu)
i is a subset of j (is or isu)
i is a superset of j (is or isu)
j is an attribute of i
i causes j (or usually)
i and j are related
54/145
LAZ 6/27/2003

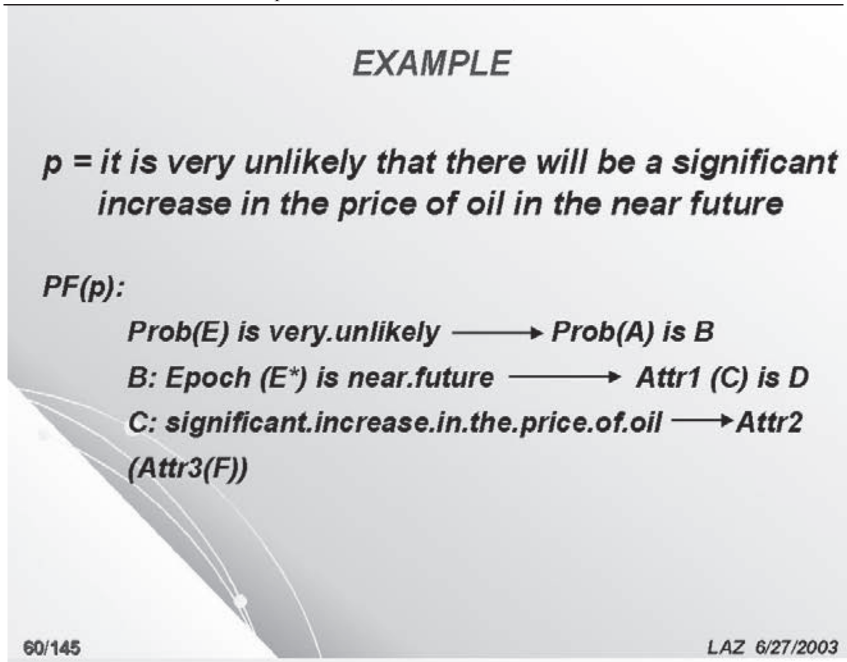

EPISTEMIC LEXICON
FORMAT OF RELATIONS
perception-based relation
lexine
A_1 … A_m
G_1 G_m
attributes
granular values
example
car
Make Price
ford G
chevy
G: 20*% \ ∠ 15k* + 40*% \ [15k*, 25k*] + • • •
granular count
55/145
LAZ 6/27/2003

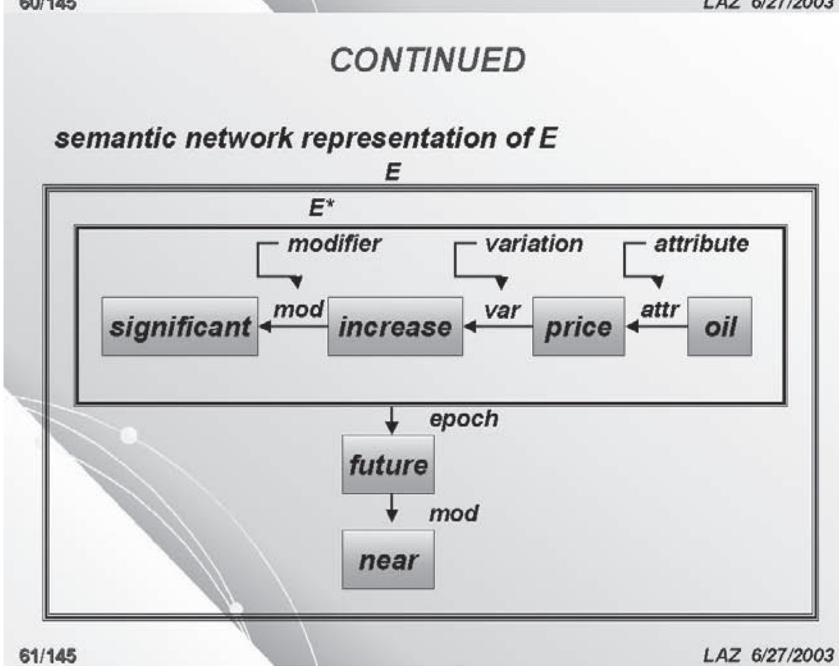

PROTOFORM-BASED
DEDUCTION
56/145
LAZ 6/27/2003

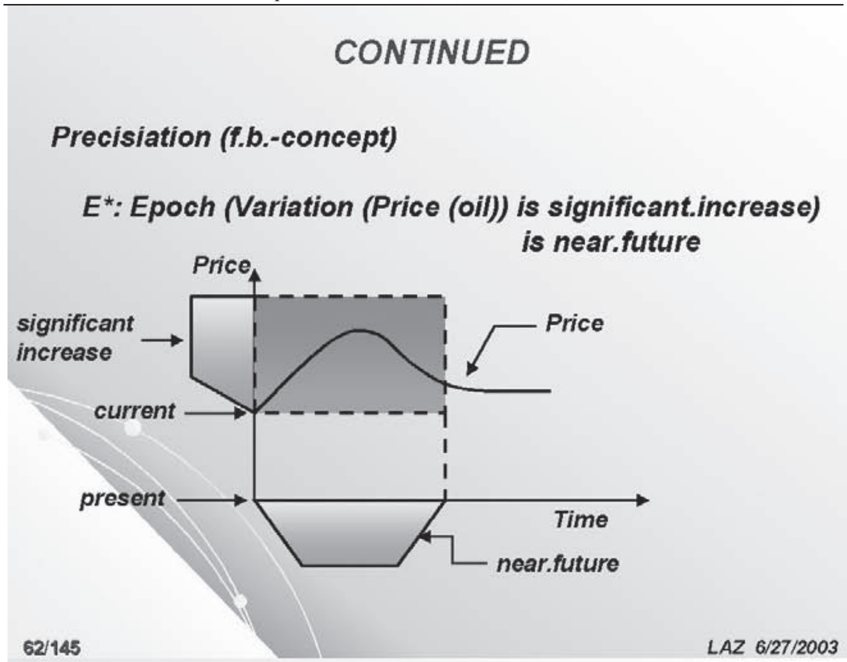

THE CONCEPT OF PROTOFORM AND RELATED CONCEPTS
Fuzzy Logic
Bivalent Logic
protoform
ontology
conceptual graph
skeleton
Montague grammar
57/145
LAZ 6/27/2003

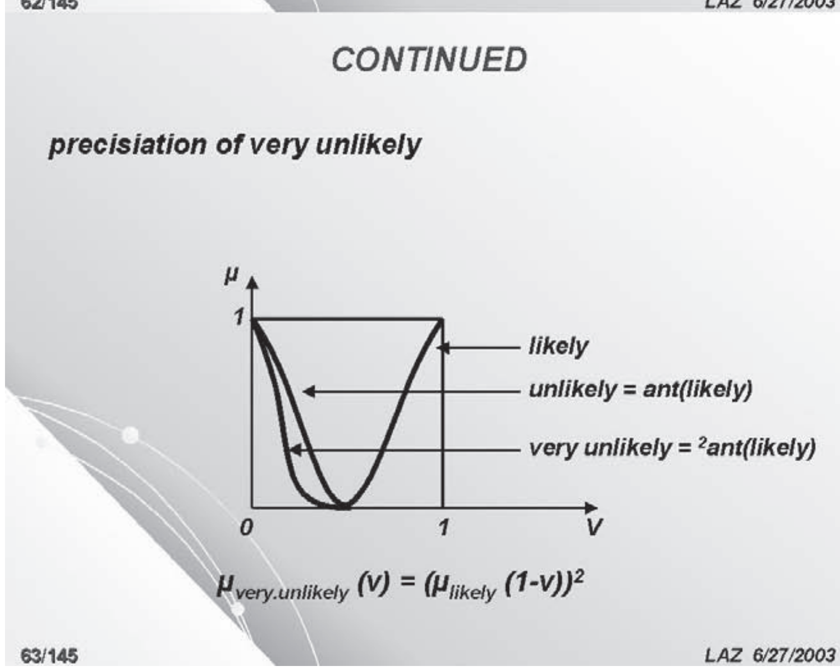

THE CONCEPT OF A PROTOFORM
CWP
PNL
PFL
Protoform Language
Informally, a protoform—abbreviation of prototypical form—is an abstracted summary. More specifically, a protoform is a symbolic expression which defines the deep semantic structure of a construct such as a proposition, command, question, scenario, concept or a system of such constructs
Example:
Eva is young → A(B) is C
abstraction
young
C
instantiation
58/145
LAZ 6/27/2003

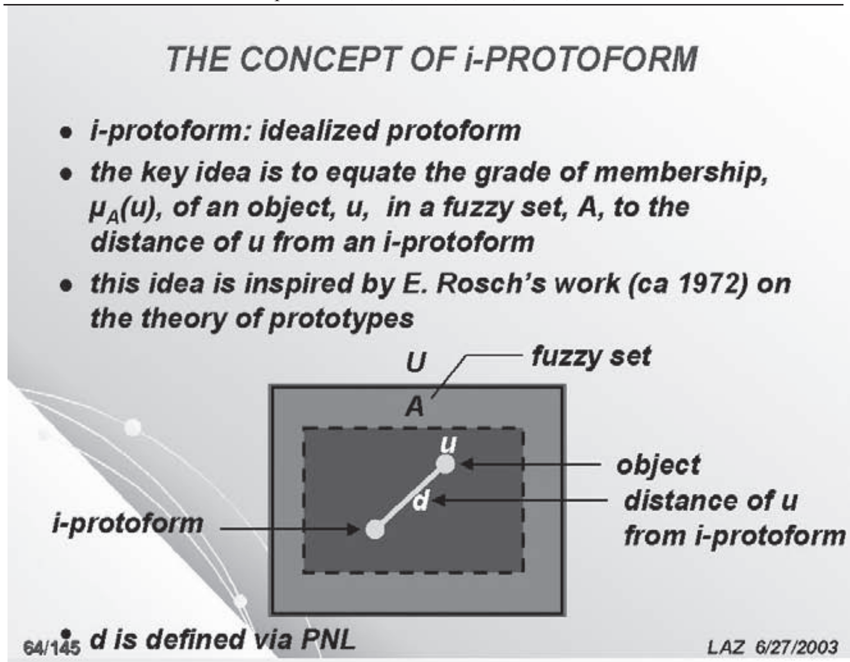

TRANSLATION FROM NL TO PFL
examples
Most Swedes are tall → Count (A/B) is Q
Eva is much younger than Pat → (A (B), A (C)) is R
Age
Eva
Age
Pat
much younger
usually Robert returns from work at about 6pm →
Prob {A is B} is C
usually
about 6 pm
Time (Robert returns from work)
59/145
LAZ 6/27/2003

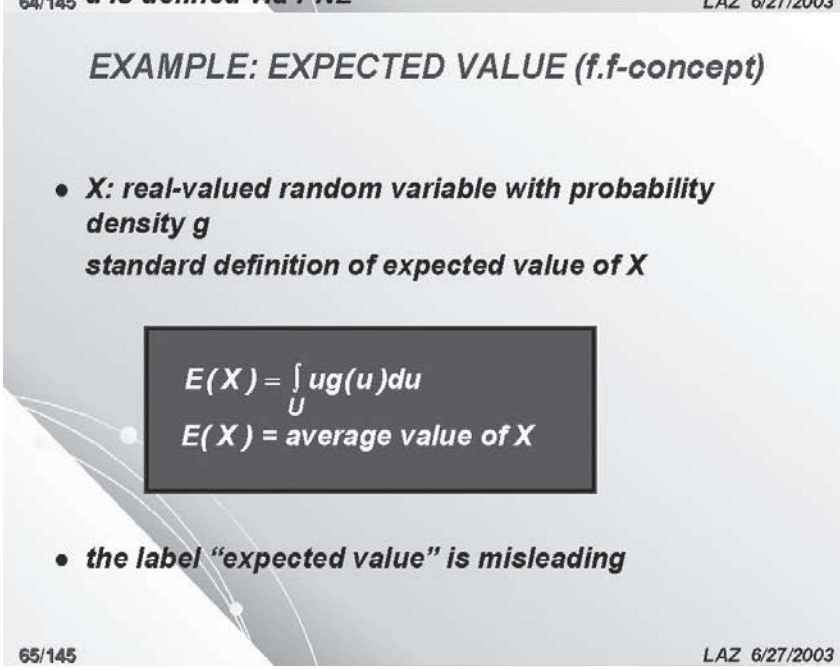

EXAMPLE
p = it is very unlikely that there will be a significant increase in the price of oil in the near future
PF(p):
Prob(E) is very.unlikely → Prob(A) is B
B: Epoch (E*) is near.future → Attr1 (C) is D
C: significant.increase.in.the.price.of.oil → Attr2 (Attr3(F))
60/145
LAZ 6/27/2003

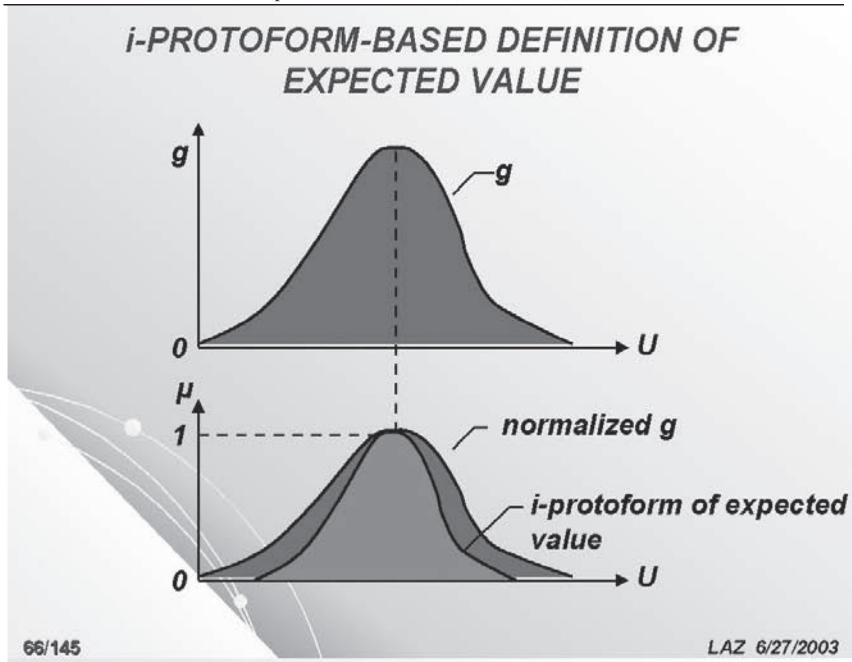

CONTINUED
semantic network representation of E
E
E*
modifier
variation
attribute
significant
mod
increase
var
price
attr
oil
epoch
future
mod
near
61/145
LAZ 6/27/2003

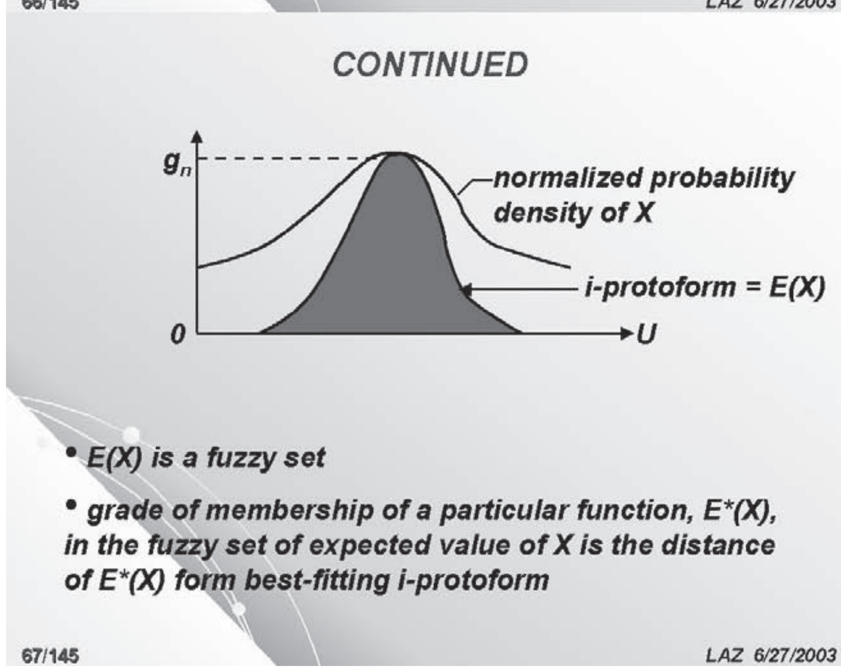

CONTINUED
Precisiation (f.b.-concept)
E*: Epoch (Variation (Price (oil)) is significant.increase) is near.future
Price
significant increase
Price
current
present
Time
near.future
62/145
LAZ 6/27/2003


CONTINUED
precisiation of very unlikely
μ
1
likely
unlikely = ant(likely)
very unlikely = ²ant(likely)
0
1
V
$\mu_{very.unlikely}(v) = (\mu_{likely}(1-v))^2$
63/145
LAZ 6/27/2003

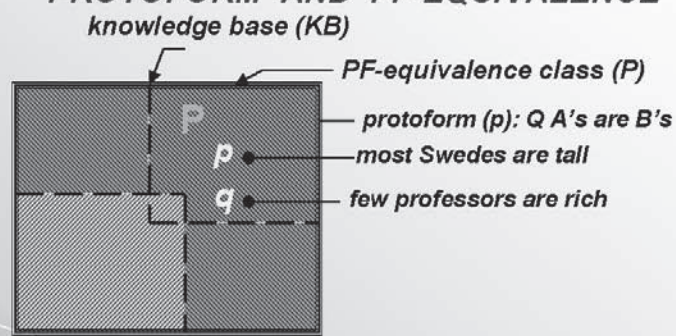

THE CONCEPT OF i-PROTOFORM
• i-protoform: idealized protoform
• the key idea is to equate the grade of membership, $\mu_A(u)$, of an object, u, in a fuzzy set, A, to the distance of u from an i-protoform
• this idea is inspired by E. Rosch's work (ca 1972) on the theory of prototypes
U
fuzzy set
A
u
object
d
distance of u from i-protoform
i-protoform
• d is defined via PNL
64/145
LAZ 6/27/2003


EXAMPLE: EXPECTED VALUE (f.f-concept)
• X: real-valued random variable with probability density g
standard definition of expected value of X
$E(X) = \int_U ug(u)du$
E(X) = average value of X
• the label "expected value" is misleading
65/145
LAZ 6/27/2003


i-PROTOFORM-BASED DEFINITION OF
EXPECTED VALUE
g
g
0
U
μ
1
normalized g
i-protoform of expected
value
0
U
66/145
LAZ 6/27/2003


CONTINUED
g_n
normalized probability
density of X
i-protoform = E(X)
0
U
• E(X) is a fuzzy set
• grade of membership of a particular function, E*(X),
in the fuzzy set of expected value of X is the distance
of E*(X) form best-fitting i-protoform
67/145
LAZ 6/27/2003

PROTOFORM AND PF-EQUIVALENCE



- P is the class of PF-equivalent propositions
- P does not have a prototype
- P has an abstracted prototype: Q A's are B's
- P is the set of all propositions whose protoform is: Q A's are B's

68/145 LAZ 6/27/2003

PF-EQUIVALENCE

Scenario A:

Alan has severe back pain. He goes to see a doctor. The doctor tells him that there are two options: (1) do nothing; and (2) do surgery. In the case of surgery, there are two possibilities: (a) surgery is successful, in which case Alan will be pain free; and (b) surgery is not successful, in which case Alan will be paralyzed from the neck down. Question: Should Alan elect surgery?

69/145 LAZ 6/27/2003

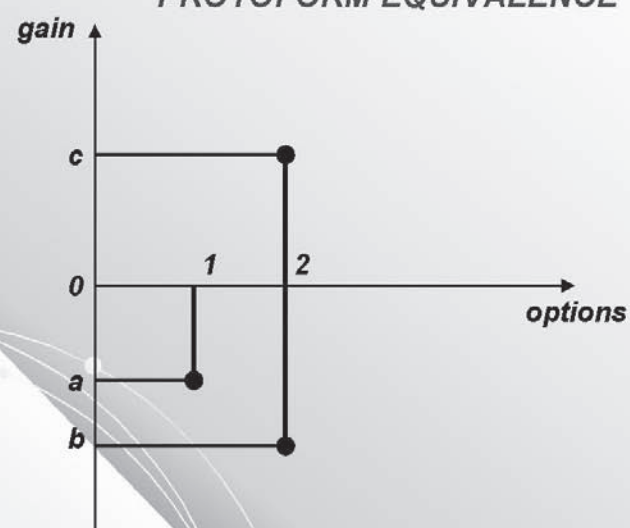
PF-EQUIVALENCE

Scenario B:

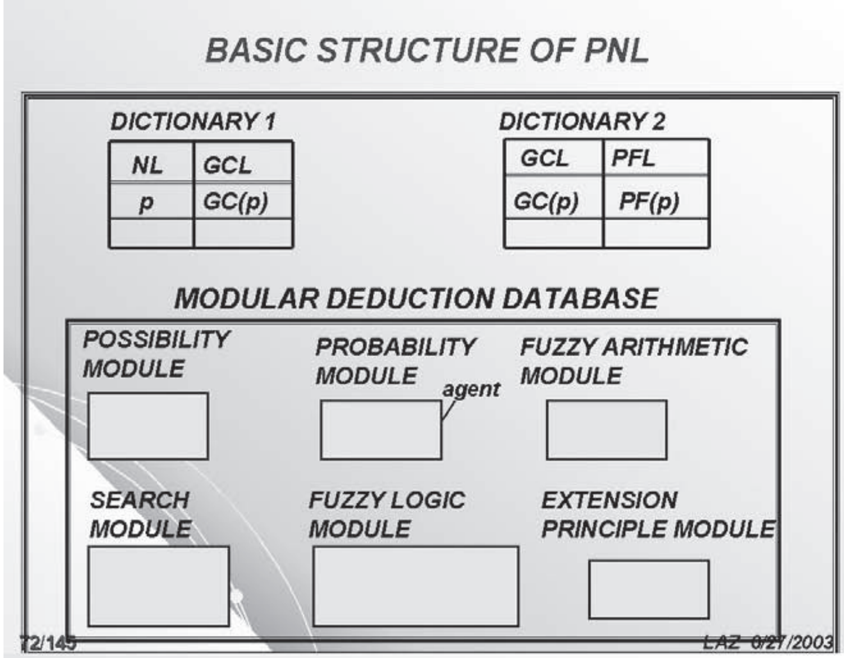
Alan needs to fly from San Francisco to St. Louis and has to get there as soon as possible. One option is fly to St. Louis via Chicago and the other through Denver. The flight via Denver is scheduled to arrive in St. Louis at time a. The flight via Chicago is scheduled to arrive in St. Louis at time b, with a<b. However, the connection time in Denver is short. If the flight is missed, then the time of arrival in St. Louis will be c, with c>b. Question: Which option is best?

70/145 LAZ 6/27/2003

PROTOFORM EQUIVALENCE



71/145 LAZ 6/27/2003



PROTOFORMAL SEARCH RULES

example

query: What is the distance between the largest city in Spain and the largest city in Portugal?

protoform of query: ?Attr (Desc(A), Desc(B))

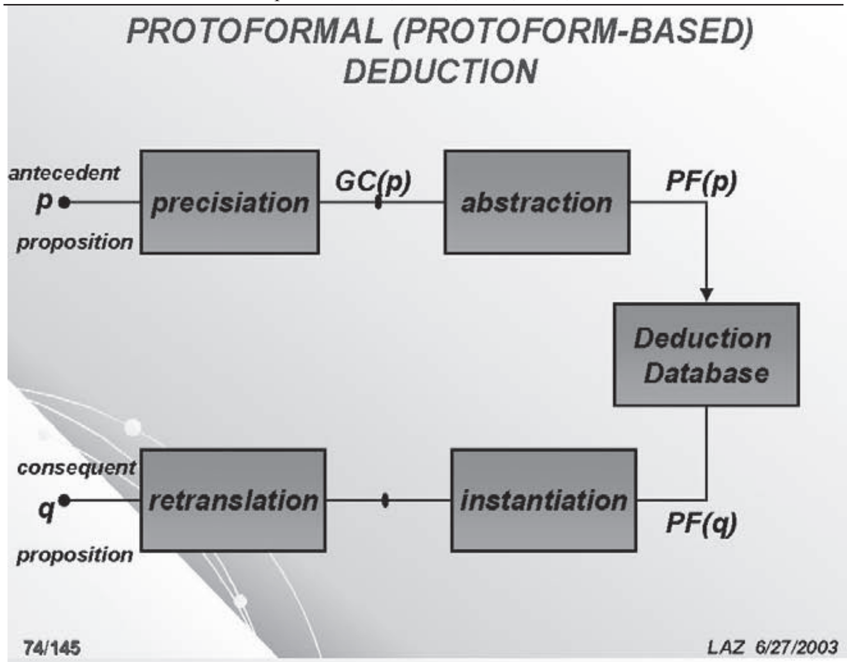
procedure

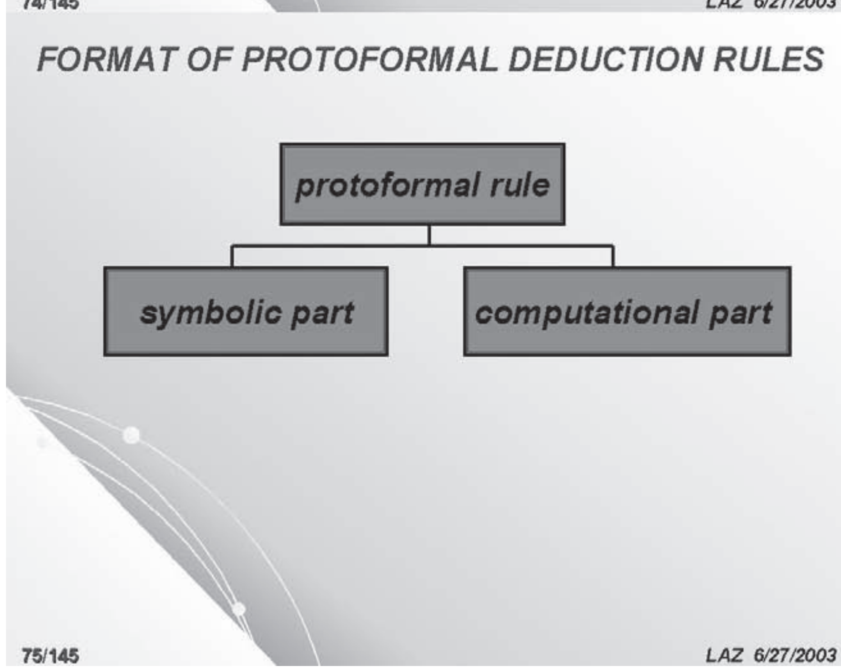
query: ?Name (A)|Desc (A)

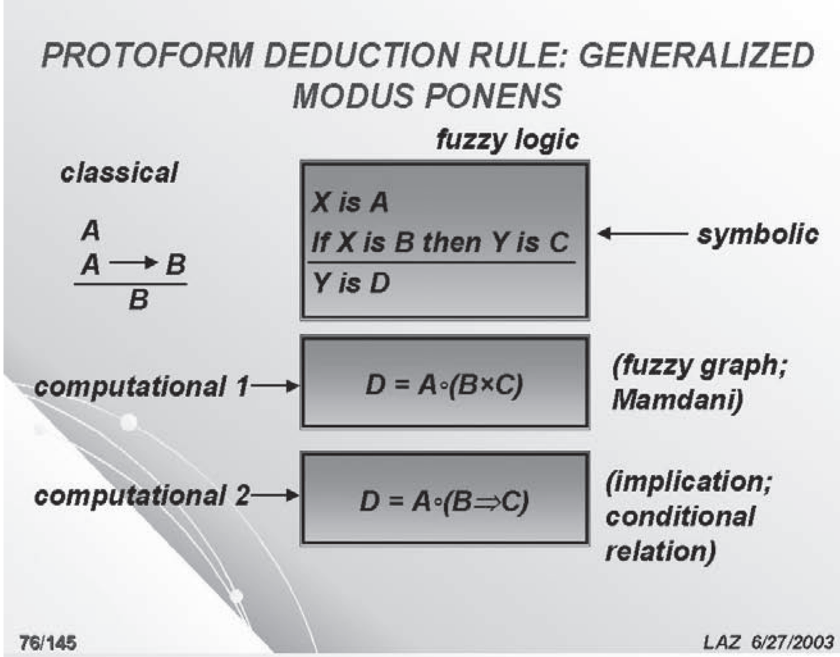
query: Name (B)|Desc (B)

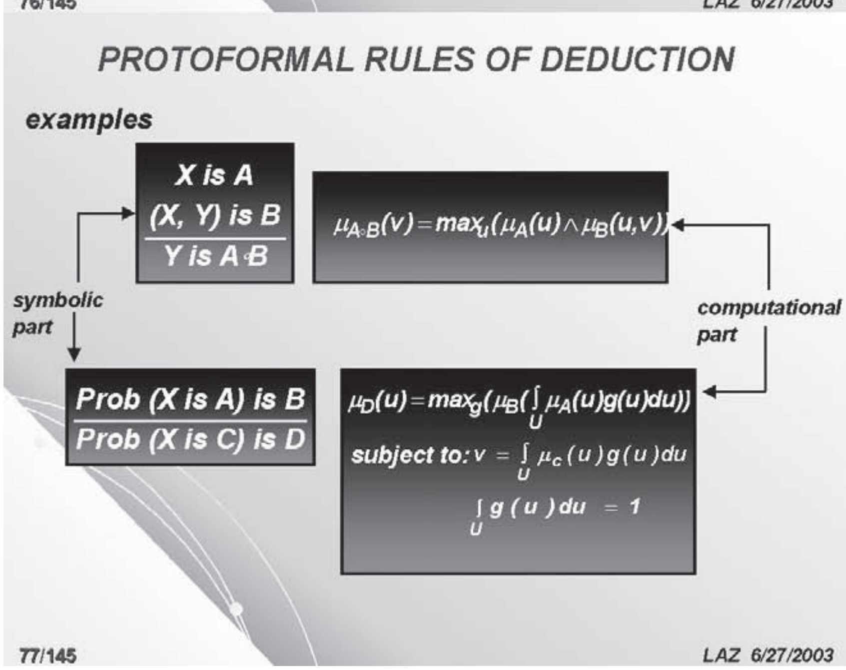
query: ?Attr (Name (A), Name (B))

73/145 LAZ 6/27/2003


PROTOFORMAL (PROTOFORM-BASED) DEDUCTION
antecedent
p
proposition
precisiation
GC(p)
abstraction
PF(p)
Deduction Database
consequent
q
proposition
retranslation
instantiation
PF(q)
74/145
LAZ 6/27/2003


FORMAT OF PROTOFORMAL DEDUCTION RULES
protoformal rule
symbolic part
computational part
75/145
LAZ 6/27/2003


PROTOFORM DEDUCTION RULE: GENERALIZED MODUS PONENS
fuzzy logic
classical
A
A → B
B
X is A
If X is B then Y is C
Y is D
symbolic
computational 1
D = A∘(B×C)
(fuzzy graph; Mamdani)
computational 2
D = A∘(B⇒C)
(implication; conditional relation)
76/145
LAZ 6/27/2003


PROTOFORMAL RULES OF DEDUCTION
examples
X is A
(X, Y) is B
Y is A∘B
$\mu_{A \circ B}(v) = max_u(\mu_A(u) \wedge \mu_B(u,v))$
symbolic part
computational part
Prob (X is A) is B
Prob (X is C) is D
$\mu_D(u) = max_g(\mu_B(\int_U \mu_A(u)g(u)du))$
subject to: $v = \int_U \mu_c(u)g(u)du$
$\int_U g(u)du = 1$
77/145
LAZ 6/27/2003

PROTOFORM-BASED (PROTOFORMAL) DEDUCTION

- Rules of deduction in the Deduction Database (DDB) are protoformal

  examples: (a) compositional rule of inference

symbolic →

$$\frac{\begin{array}{c}X \text{ is } A \\ (X, Y) \text{ is } B\end{array}}{Y \text{ is } A \circ B}$$

$$\mu_B(v) = \sup(\mu_A(u) \wedge \mu_B(u,v))$$

computational

(b) extension principle

$$\frac{\begin{array}{c}X \text{ is } A \\ Y = f(X)\end{array}}{Y = f(A)}$$

symbolic

$$\mu_y(v) = \sup_u(\mu_A(u))$$

Subject to: $v = f(u)$

computational

78/145 LAZ 6/27/2003

CONTINUED

- Rules of deduction are basically rules governing generalized constraint propagation
- The principal rule of deduction is the extension principle

$$\frac{X \text{ is } A}{f(X,) \text{ is } B}$$

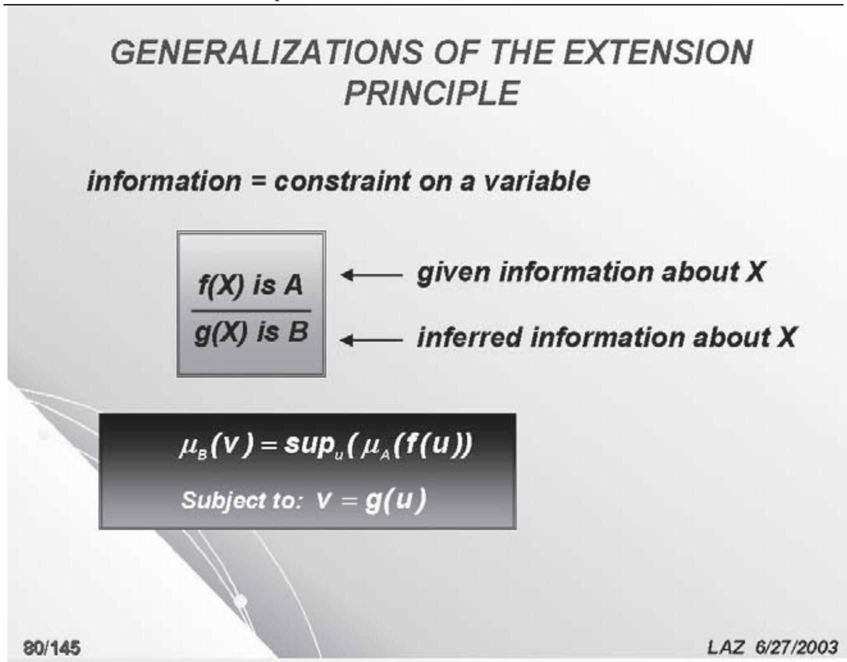
symbolic

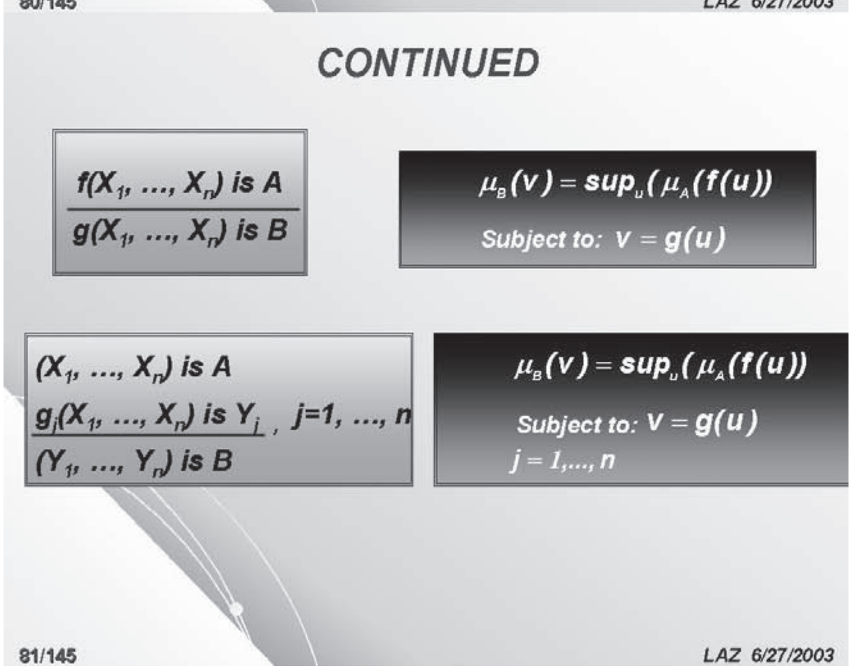
$$\mu_B(v) = \sup_u(\mu_A(u)$$

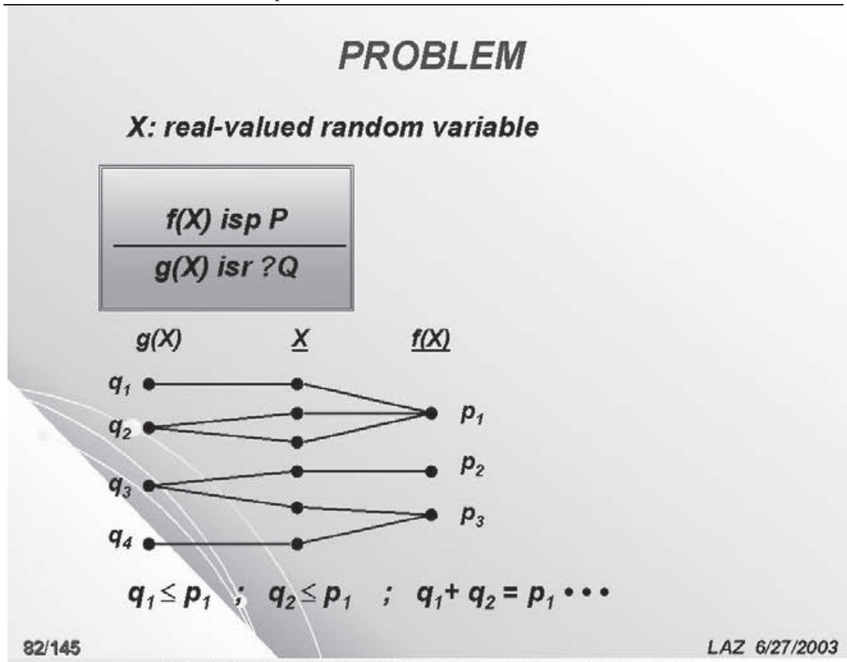
Subject to: $v = f(u)$

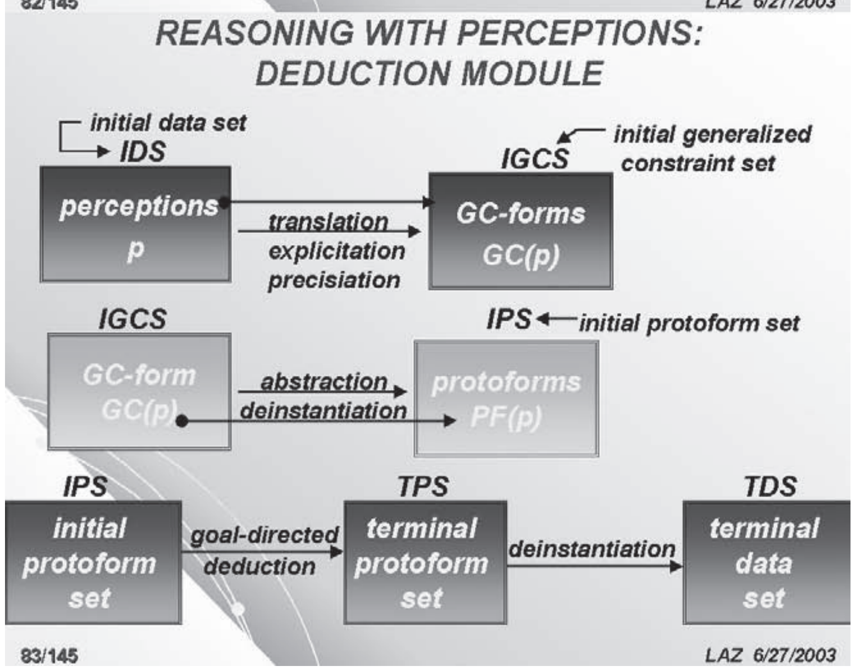
computational

79/145 LAZ 6/27/2003


GENERALIZATIONS OF THE EXTENSION PRINCIPLE
information = constraint on a variable
f(X) is A
g(X) is B
given information about X
inferred information about X
$\mu_B(v) = sup_u(\mu_A(f(u))$
Subject to: $v = g(u)$
80/145
LAZ 6/27/2003


CONTINUED
$f(X_1, \ldots, X_n)$ is A
$g(X_1, \ldots, X_n)$ is B
$\mu_B(v) = sup_u(\mu_A(f(u))$
Subject to: $v = g(u)$
$(X_1, \ldots, X_n)$ is A
$g_j(X_1, \ldots, X_n)$ is Y_j , j=1, …, n
$(Y_1, \ldots, Y_n)$ is B
$\mu_B(v) = sup_u(\mu_A(f(u))$
Subject to: $v = g(u)$
$j = 1, \ldots, n$
81/145
LAZ 6/27/2003

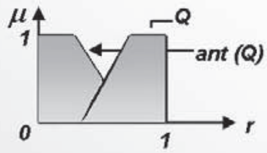

PROBLEM
X: real-valued random variable
f(X) isp P
g(X) isr ?Q
g(X)
X
f(X)
q1
q2
q3
q4
p1
p2
p3
q1 ≤ p1 ; q2 ≤ p1 ; q1 + q2 = p1 • • •
82/145
LAZ 6/27/2003


REASONING WITH PERCEPTIONS: DEDUCTION MODULE
initial data set
IDS
IGCS
initial generalized constraint set
perceptions p
translation
explicitation
precisiation
GC-forms GC(p)
IGCS
IPS
initial protoform set
GC-form GC(p)
abstraction
deinstantiation
protoforms PF(p)
IPS
TPS
TDS
initial protoform set
goal-directed deduction
terminal protoform set
deinstantiation
terminal data set
83/145
LAZ 6/27/2003

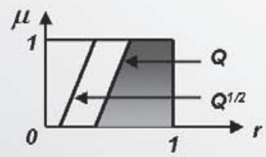
COUNT-AND MEASURE-RELATED RULES

crisp

Q A's are B's
____
ant (Q) A's are not B's



Q A's are B's
____
$Q^{1/2}$ A's are ^{2}B's



most Swedes are tall
____
ave (height) Swedes is ?h

$\longrightarrow$

Q A's are B's
____
ave (B|A) is ?C

$$\mu_{ave}(v) = sup_a \, \mu_Q\left(\frac{1}{N}\sum\nolimits_i \mu_B(a_i)\right) \quad , \; a = (a_1, \ldots, a_N)$$

$$v = \frac{1}{N}\left(\sum\nolimits_i a_i\right)$$

84/145 LAZ 6/27/2003

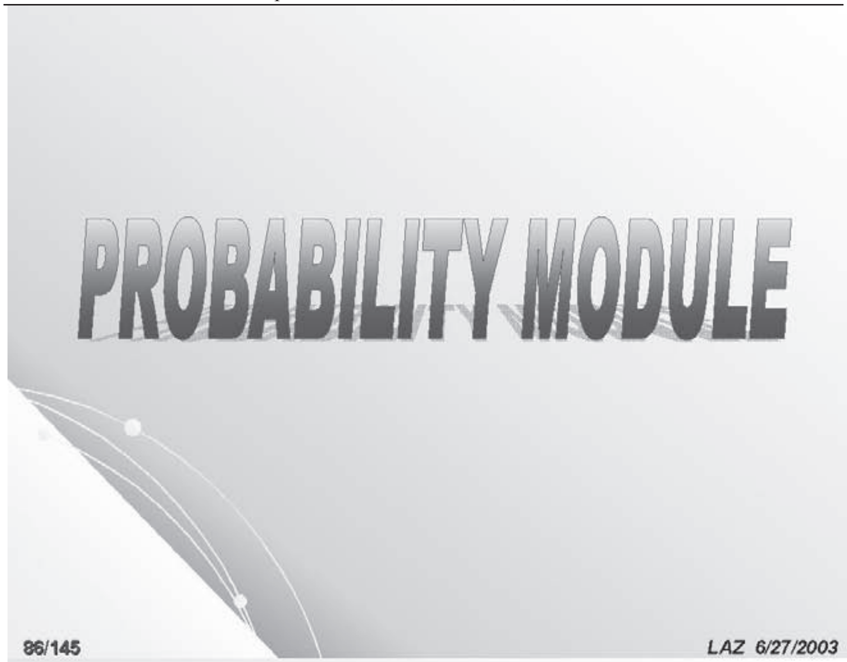
CONTINUED

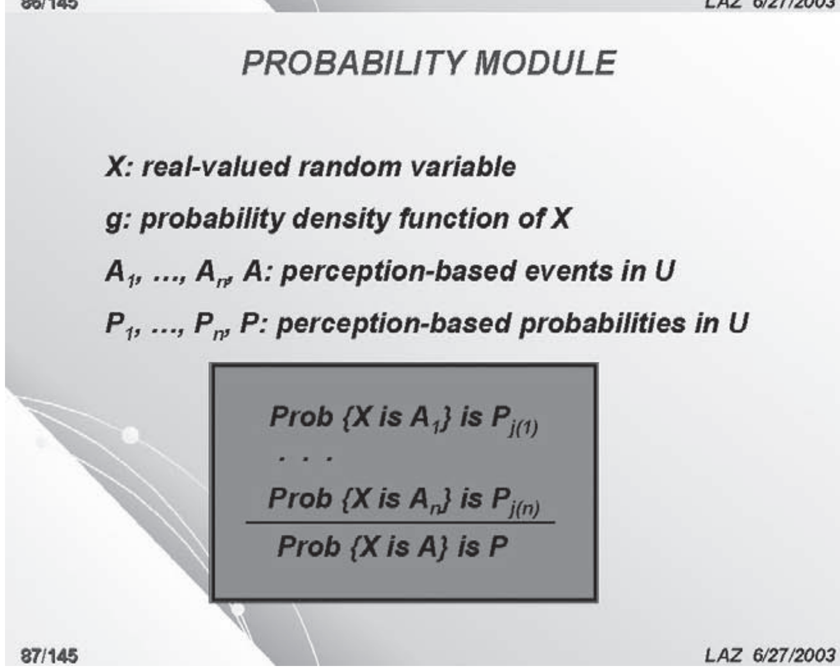
not(QA's are B's) $\longleftrightarrow$ (not Q) A's are B's

Q_1 A's are B's
Q_2 (A&B)'s are C's
____
Q_1 Q_2 A's are (B&C)'s

Q_1 A's are B's
Q_2 A's are C's
____
(Q_1 +Q_2 -1) A's are (B&C)'s

85/145 LAZ 6/27/2003


PROBABILITY MODULE
86/145
LAZ 6/27/2003


PROBABILITY MODULE
X: real-valued random variable
g: probability density function of X
A_1, …, A_n, A: perception-based events in U
P_1, …, P_n, P: perception-based probabilities in U
Prob {X is A_1} is $P_{j(1)}$
. . .
Prob {X is A_n} is $P_{j(n)}$
Prob {X is A} is P
87/145
LAZ 6/27/2003

CONTINUED

$$\mu_p(v) = sup_g(\mu_{P_1}(\int_U g(u)\mu_{A_1}(u)du) \wedge \cdots$$

$$\wedge \mu_{P_n}(\int_U g(u)\mu_{A_n}(u)du)$$

subject to:

$$v = \int_U g(u)\mu_A(u)du$$

88/145 LAZ 6/27/2003

PROBABILITY MODULE (CONTINUED)

$$\frac{\begin{array}{c} X \text{ isp } P \\ Y = f(X) \end{array}}{Y \text{ isp } f(P)}$$

$$\frac{Prob\ \{X \text{ is } A\} \text{ is } P}{Prob\ \{f(X) \text{ is } B\} \text{ is } Q}$$

$$\frac{\begin{array}{c} X \text{ isp } P \\ (X,Y) \text{ is } R \end{array}}{Y \text{ isrs } S}$$

$$\frac{\begin{array}{c} X \text{ isu } A \\ Y = f(X) \end{array}}{Y \text{ isu } f(A)}$$

89/145 LAZ 6/27/2003

PROBABILISTIC CONSTRAINT PROPAGATION RULE
(a special version of the generalized extension principle)

$$\frac{\int_U g(u)\mu_A(u)du \quad \text{is } R}{\int_U g(u)\mu_B(u)du \quad \text{is } ?S}$$

$$\mu_S(v) = \sup_g(\mu_R(\int_U g(u)\mu_A(u)du))$$

subject to

$$v = \int_U g(u)\mu_B(u)du$$

$$\int_U g(u)du = 1$$

90/145 LAZ 6/27/2003

PROTOFORMAL DEDUCTION RULES

possibilistic extension principle

$$\frac{\begin{array}{l} X \text{ is } (\Sigma_i \mu_i/u_i) \\ Y = f(X) \end{array}}{Y \text{ is } (\Sigma_i \mu_i / f(u_i))}$$

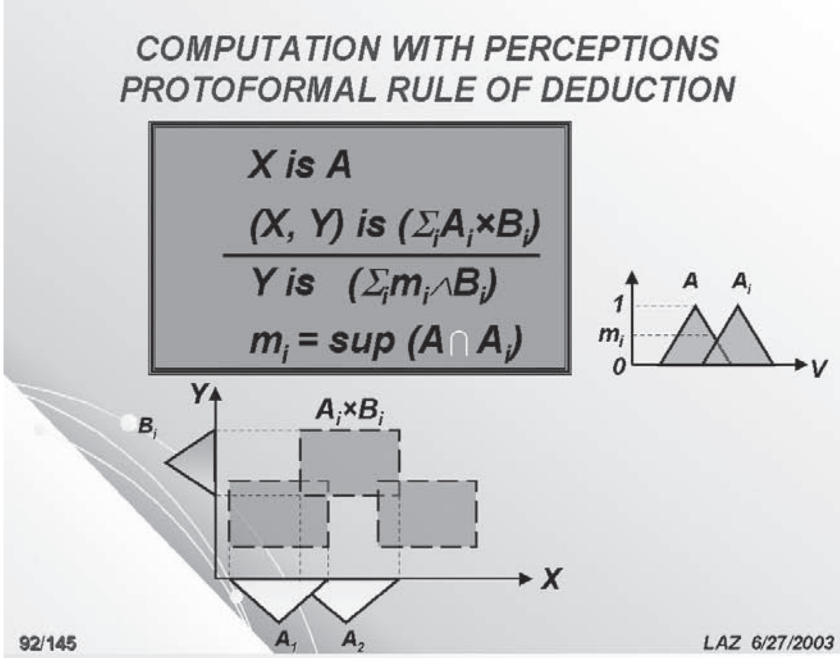
$$\mu_i / u_i + \mu_j / u_i = (\mu_i \vee \mu_j) / u_i$$

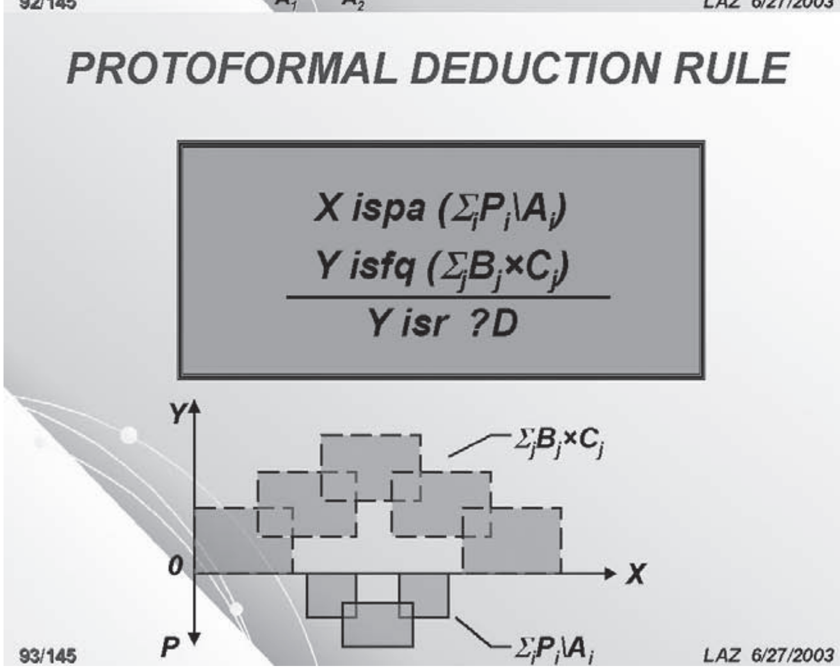
probabilistic extension principle

$$\frac{\begin{array}{l} X \text{ isp } (\Sigma_i p_i \backslash u_i) \\ Y = f(X) \end{array}}{Y \text{ isp } (\Sigma_i p_i \backslash f(u_i))}$$

$$p_i \backslash u_i + p_j \backslash u_i = (p_i + p_j) \backslash u_i$$

91/145 LAZ 6/27/2003


COMPUTATION WITH PERCEPTIONS
PROTOFORMAL RULE OF DEDUCTION
X is A
(X, Y) is $(\Sigma_i A_i \times B_i)$
Y is $(\Sigma_i m_i \wedge B_i)$
$m_i = \sup (A \cap A_i)$
A
A_i
1
m_i
0
V
Y
B_i
$A_i \times B_i$
X
A_1
A_2
92/145
LAZ 6/27/2003


PROTOFORMAL DEDUCTION RULE
X ispa $(\Sigma_i P_i \backslash A_i)$
Y isfq $(\Sigma_j B_j \times C_j)$
Y isr ?D
Y
$\Sigma_j B_j \times C_j$
0
X
P
$\Sigma_i P_i \backslash A_i$
93/145
LAZ 6/27/2003

PROTOFORMAL DEDUCTION RULE

$$\frac{\begin{array}{l} X \textit{ ispa } (\Sigma_i P_i \backslash A_i) \\ Y = f(X) \end{array}}{Y \textit{ isr } ?B}$$

$$\frac{\begin{array}{l} X \textit{ ispb } (\Sigma_i P_i \backslash\backslash A_i) \\ Y = f(X) \end{array}}{Y \textit{ isr } ?C}$$

94/145 LAZ 6/27/2003

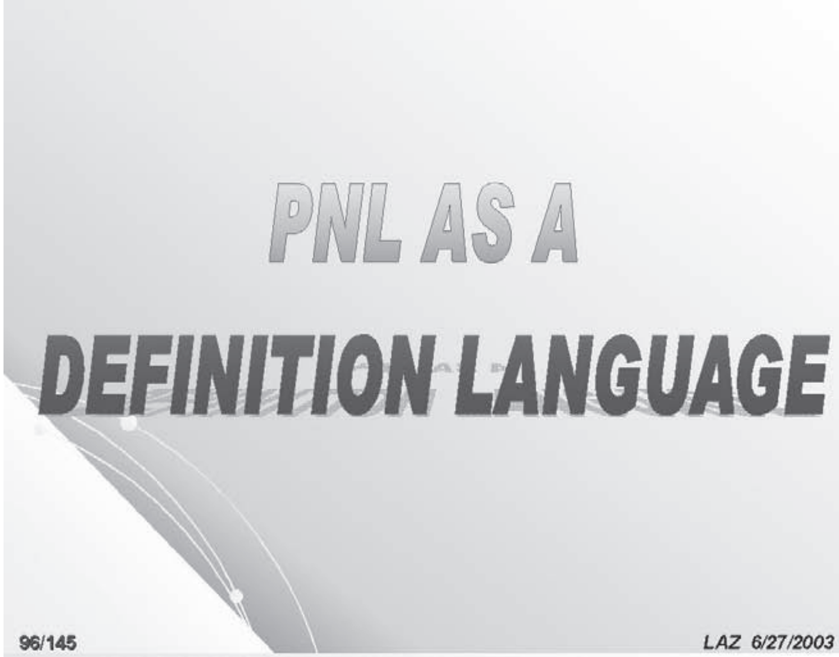
PROTOFORMAL CONSTRAINT PROPAGATION

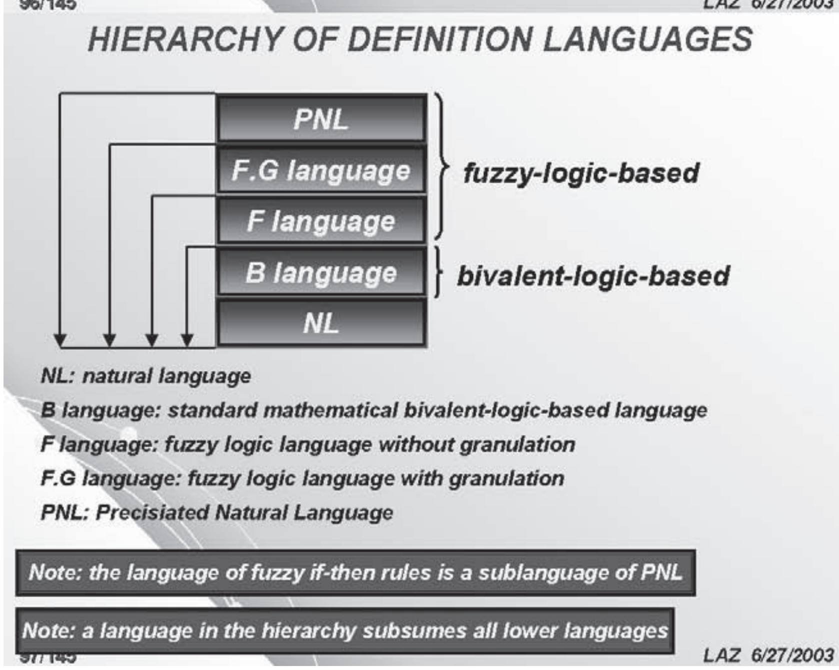
p	GC(p)	PF(p)
Dana is young	Age (Dana) is young	X is A
Tandy is a few years older than Dana	Age (Tandy) is (Age (Dana)) +few	Y is (X+B)

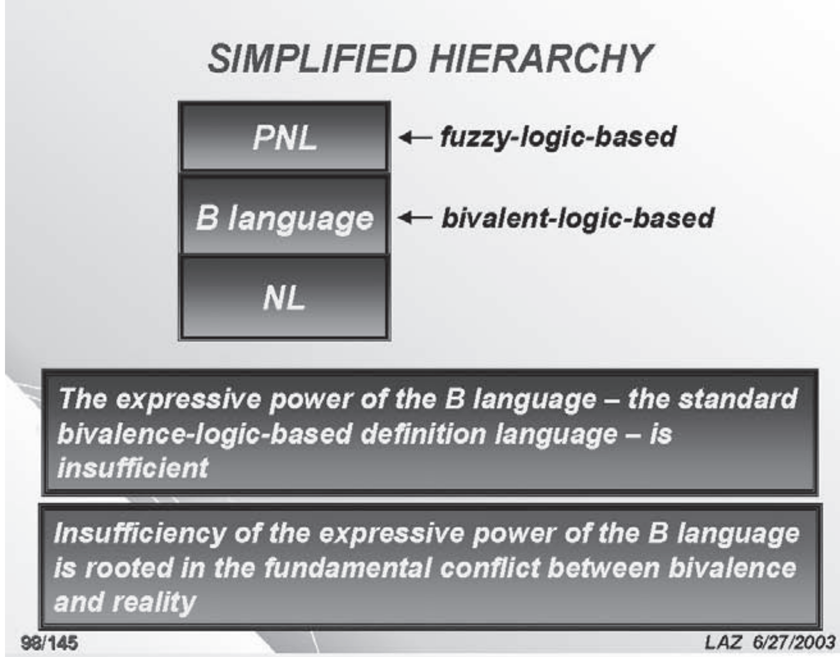
$$\frac{\begin{array}{l} X \text{ is } A \\ Y \text{ is } (X+B) \end{array}}{Y \text{ is } A+B} \longrightarrow \text{Age (Tandy) is (young+few)}$$

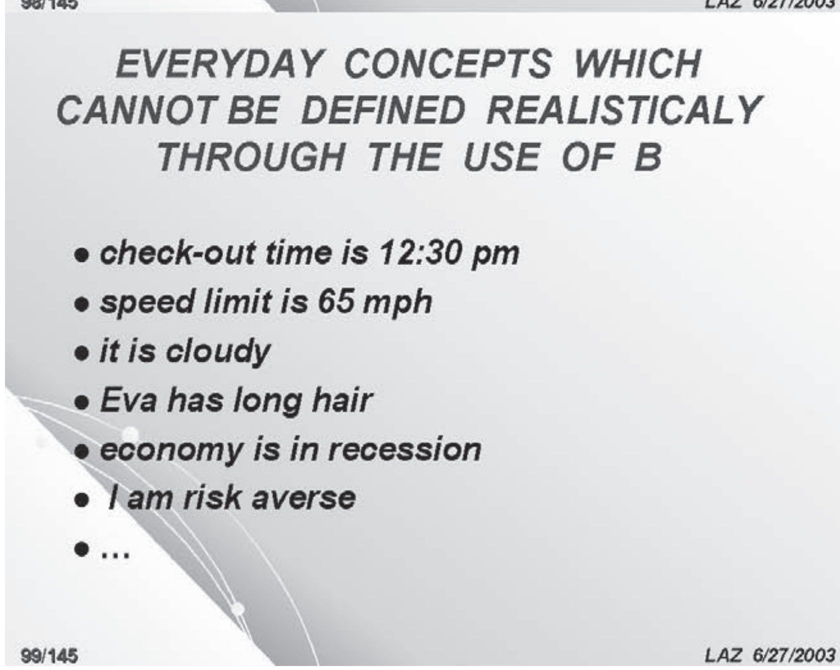
$$\mu_{A+B}(v) = sup_u(\mu_A(u) \wedge \mu_B(v-u))$$

95/145 LAZ 6/27/2003

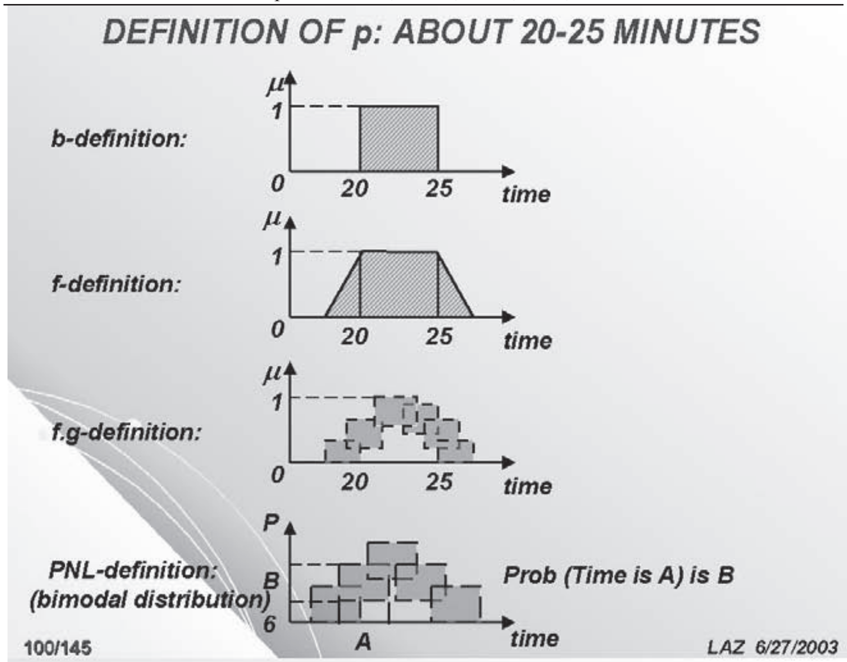

PNL AS A
DEFINITION LANGUAGE
96/145
LAZ 6/27/2003

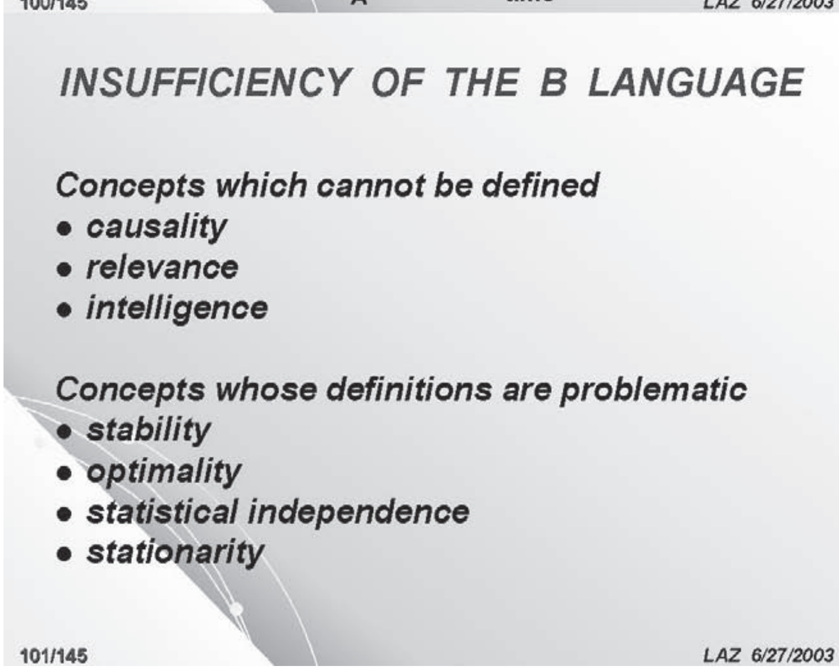

HIERARCHY OF DEFINITION LANGUAGES
PNL
F.G language
F language
B language
NL
fuzzy-logic-based
bivalent-logic-based
NL: natural language
B language: standard mathematical bivalent-logic-based language
F language: fuzzy logic language without granulation
F.G language: fuzzy logic language with granulation
PNL: Precisiated Natural Language
Note: the language of fuzzy if-then rules is a sublanguage of PNL
Note: a language in the hierarchy subsumes all lower languages
LAZ 6/27/2003

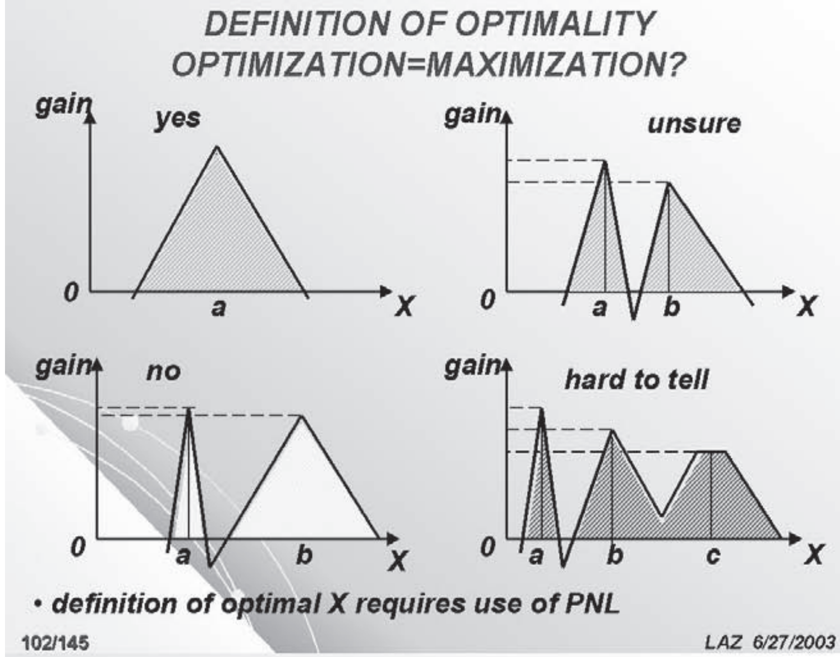

SIMPLIFIED HIERARCHY
PNL
← fuzzy-logic-based
B language
← bivalent-logic-based
NL
The expressive power of the B language – the standard bivalence-logic-based definition language – is insufficient
Insufficiency of the expressive power of the B language is rooted in the fundamental conflict between bivalence and reality
98/145
LAZ 6/27/2003

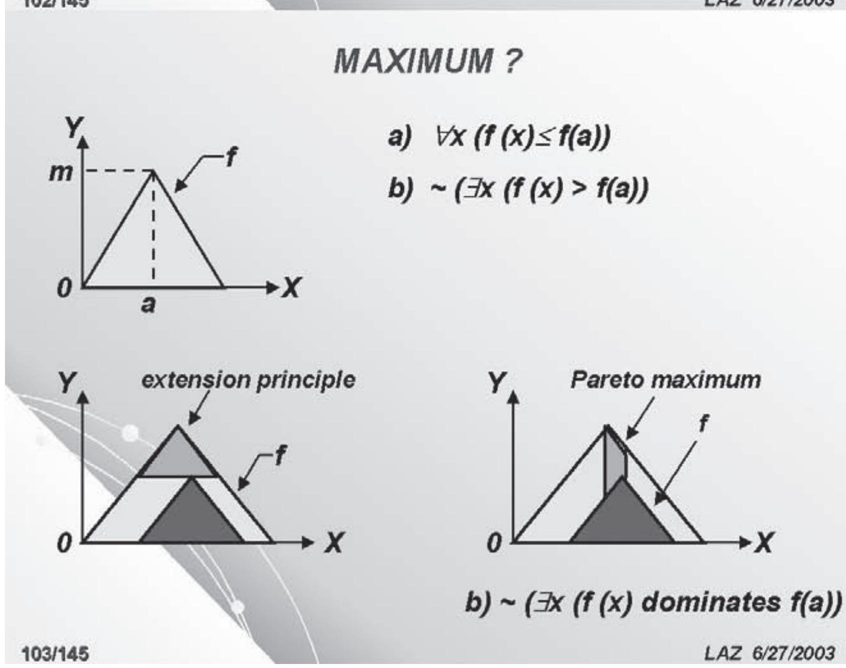

EVERYDAY CONCEPTS WHICH CANNOT BE DEFINED REALISTICALY THROUGH THE USE OF B
• check-out time is 12:30 pm
• speed limit is 65 mph
• it is cloudy
• Eva has long hair
• economy is in recession
• I am risk averse
• …
99/145
LAZ 6/27/2003

Scientific and Technical Bulletin Series: Electrotechnics, Electronics, Automatic Control and Computer Science, Vol. 2, No. 1, 2005**,** ISSN 1584-9198

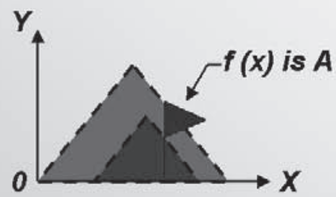


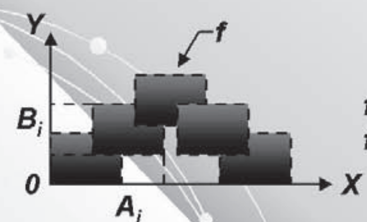



DEFINITION OF OPTIMALITY
OPTIMIZATION=MAXIMIZATION?
gain
yes
0
a
X
gain
unsure
0
a
b
X
gain
no
0
a
b
X
gain
hard to tell
0
a
b
c
X
• definition of optimal X requires use of PNL
102/145
LAZ 6/27/2003


MAXIMUM ?
Y
m
f
0
a
X
a) ∀x (f (x)≤ f(a))
b) ~ (∃x (f (x) > f(a))
Y
extension principle
f
0
X
Y
Pareto maximum
f
0
X
b) ~ (∃x (f (x) dominates f(a))
103/145
LAZ 6/27/2003

MAXIMUM ?





$f = \Sigma_i A_i \times B_i$

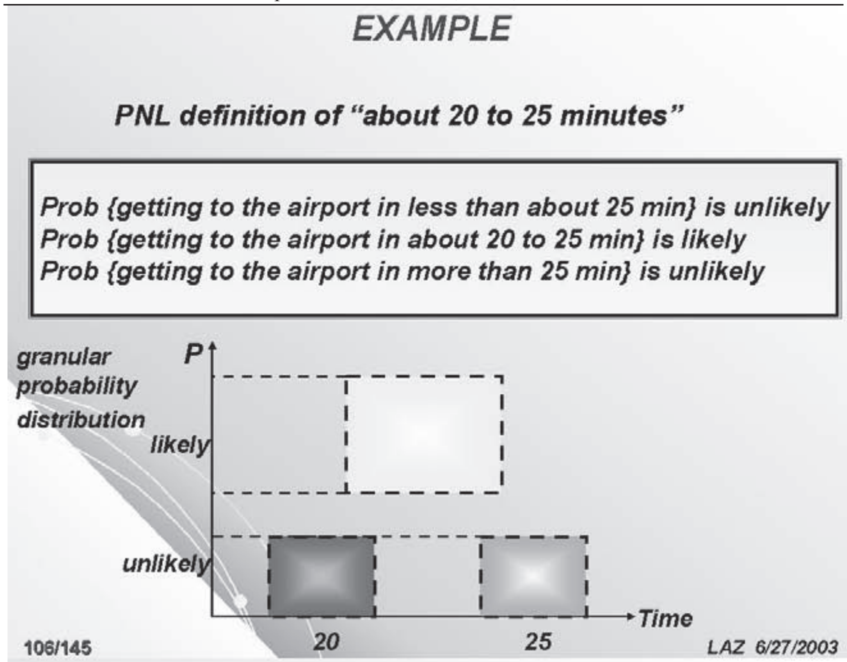
f: if X is A_i then Y is B_i, i=1, ..., n

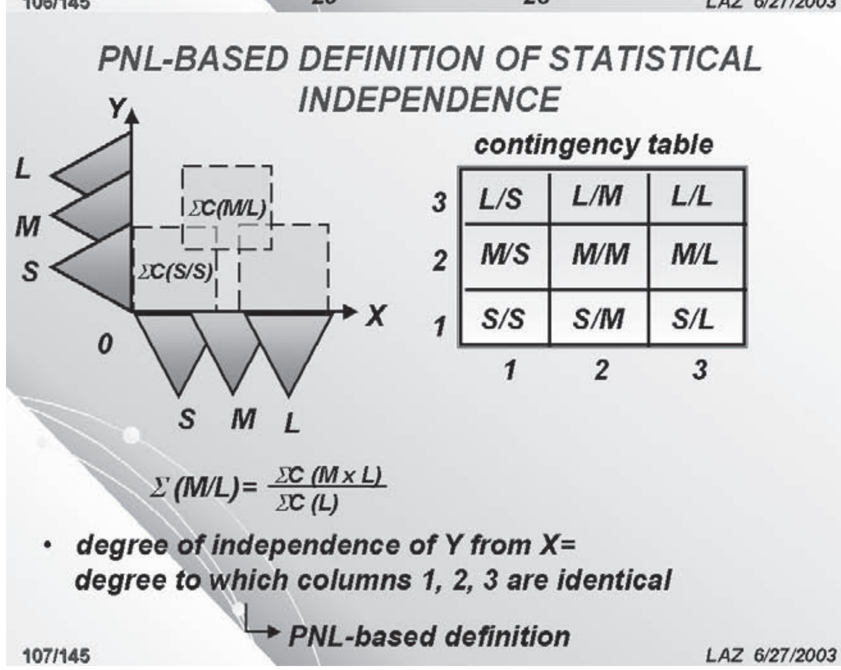
104/145 LAZ 6/27/2003

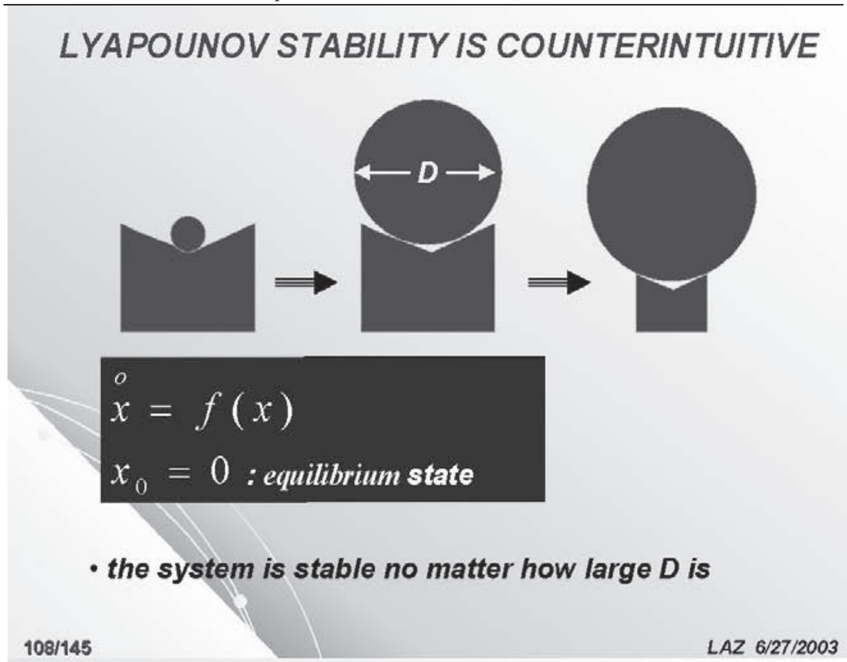
EXAMPLE

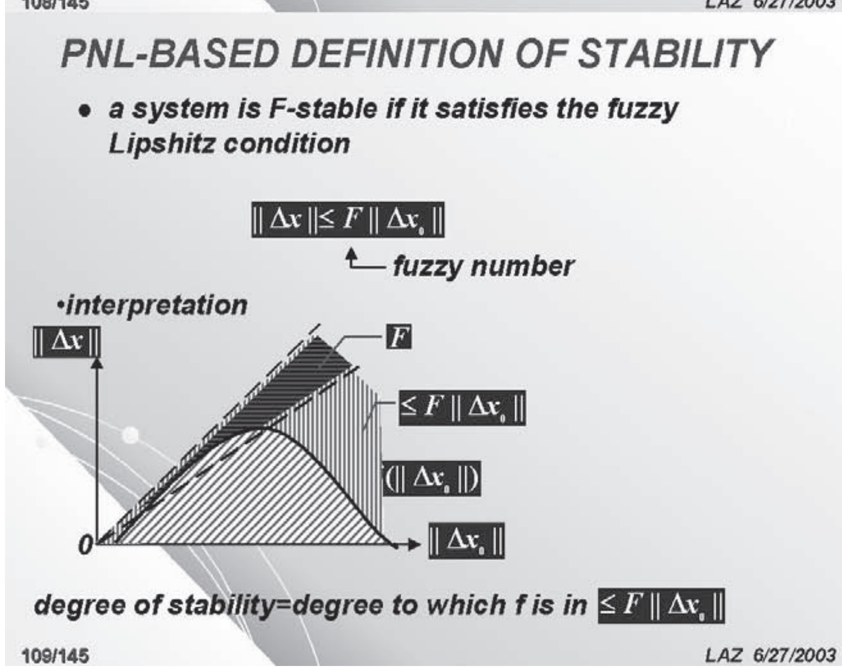
- *I am driving to the airport. How long will it take me to get there?*
- *Hotel clerk's perception-based answer: about 20-25 minutes*
- *"about 20-25 minutes" cannot be defined in the language of bivalent logic and probability theory*
- *To define "about 20-25 minutes" what is needed is PNL*

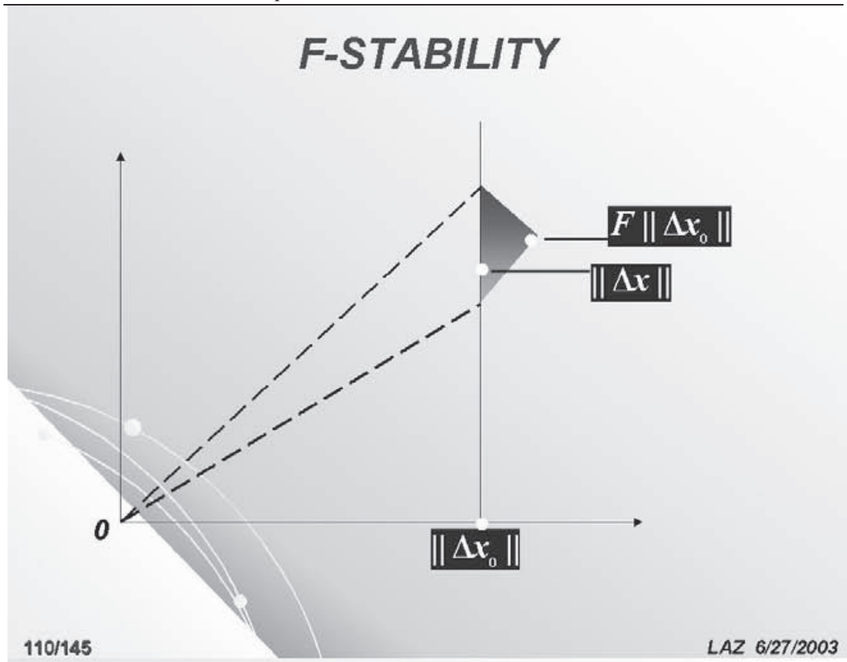
105/145 LAZ 6/27/2003


EXAMPLE
PNL definition of "about 20 to 25 minutes"
Prob {getting to the airport in less than about 25 min} is unlikely
Prob {getting to the airport in about 20 to 25 min} is likely
Prob {getting to the airport in more than 25 min} is unlikely
granular
probability
distribution
P
likely
unlikely
Time
20
25
106/145
LAZ 6/27/2003


PNL-BASED DEFINITION OF STATISTICAL INDEPENDENCE
Y
L
M
S
ΣC(M/L)
ΣC(S/S)
X
0
S M L
contingency table
3 L/S L/M L/L
2 M/S M/M M/L
1 S/S S/M S/L
1 2 3
Σ (M/L)= ΣC (M x L) / ΣC (L)
• degree of independence of Y from X=
degree to which columns 1, 2, 3 are identical
PNL-based definition
107/145
LAZ 6/27/2003


LYAPOUNOV STABILITY IS COUNTERINTUITIVE
D
$\overset{o}{x} = f(x)$
$x_0 = 0$: equilibrium state
• the system is stable no matter how large D is
108/145
LAZ 6/27/2003


PNL-BASED DEFINITION OF STABILITY
• a system is F-stable if it satisfies the fuzzy Lipshitz condition
$\| \Delta x \| \leq F \| \Delta x_0 \|$
fuzzy number
•interpretation
$\| \Delta x \|$
F
$\leq F \| \Delta x_0 \|$
$(\| \Delta x_0 \|)$
0
$\| \Delta x_0 \|$
degree of stability=degree to which f is in $\leq F \| \Delta x_0 \|$
109/145
LAZ 6/27/2003



HIGHER-ORDER CONCEPTS

- ***What is not widely recognized is that some seemingly simple concepts, e.g., cluster and edge, are hard to define because they are higher-order concepts.***
- ***Informally, a concept is of order (level) k if its denotation is a set of order k. A set whose elements are points is of order one. A set whose elements are sets of order one is of order two, etc.***

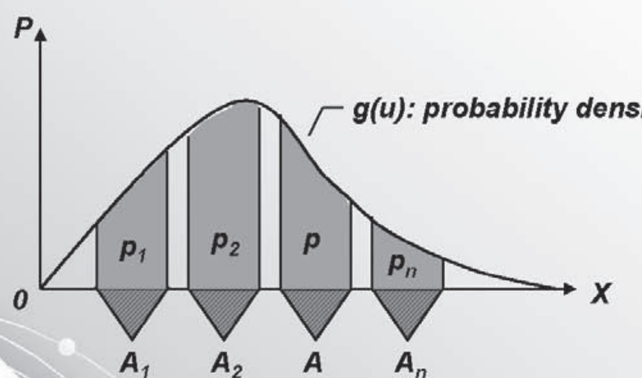
111/145 LAZ 6/27/2003

CONTINUED

- ***There are four categories of second-order concepts: (1) b.b-concepts, I.e., bivalent (crisp) concepts whose instances are bivalent sets, e.g., convex set; (2) b.f-concepts, I.e., bivalent concepts whose instances are fuzzy sets, e.g., convex fuzzy sets; (3) f.b-concepts, I.e., fuzzy concepts whose instances are bivalent sets, e.g., small squares; and (4) f.f-concepts, I.e., fuzzy concepts whose instances are fuzzy sets. The concepts of cluster and edge are examples of f.f-concepts. That is why they are hard to define.***

112/145 LAZ 6/27/2003

INTERPOLATION OF BIMODAL DISTRIBUTIONS



p_i is P_i : granular value of p_i , i=1, ..., n
(P_i , A_i) , i=1, ..., n are given
A is given
(?P, A)

113/145 LAZ 6/27/2003

INTERPOLATION MODULE AND PROBABILITY MODULE

$$\frac{\textit{Prob }\{X \textit{ is } A_i\} \textit{ is } P_i \quad , i = 1, \ldots, n}{\textit{Prob }\{X \textit{ is } A\} \textit{ is } Q}$$

$$\mu_Q(v) = sup_g(\mu_{P_1}(\int_U \mu_{A_1}(u)g(u)du) \wedge \cdots \wedge$$

$$\mu_{P_n} \int_U \mu_{P_n}(\int_U \mu_{A_n}(u)g(u)du))$$

subject to

$$U = \int_U \mu_A(u)g(u)du$$

114/145 LAZ 6/27/2003

PROBABILISTIC CONSTRAINT PROPAGATION RULE
(a special version of the generalized extension principle)

$$\frac{\int_U g(u)\mu_A(u)du \quad \textbf{is } R}{\int_U g(u)\mu_B(u)du \quad \textbf{is } ?S}$$

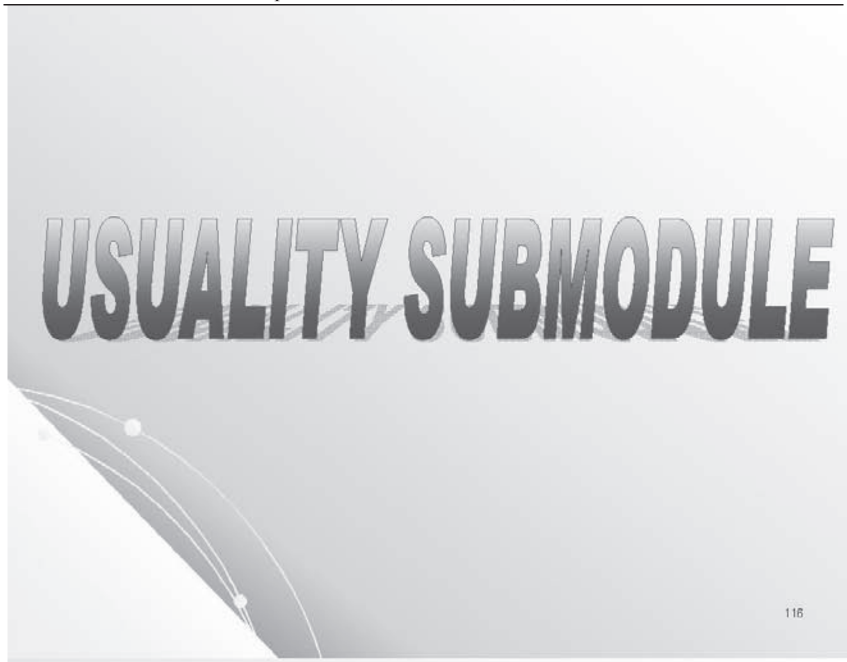
$$\mu_S(v) = \sup_g(\mu_R(\int_U g(u)\mu_A(u)du))$$

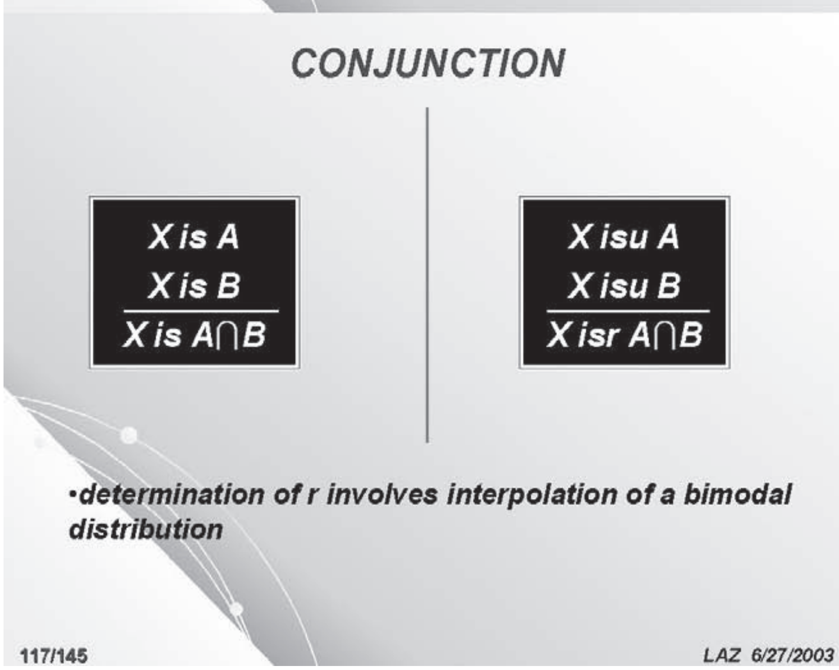
subject to

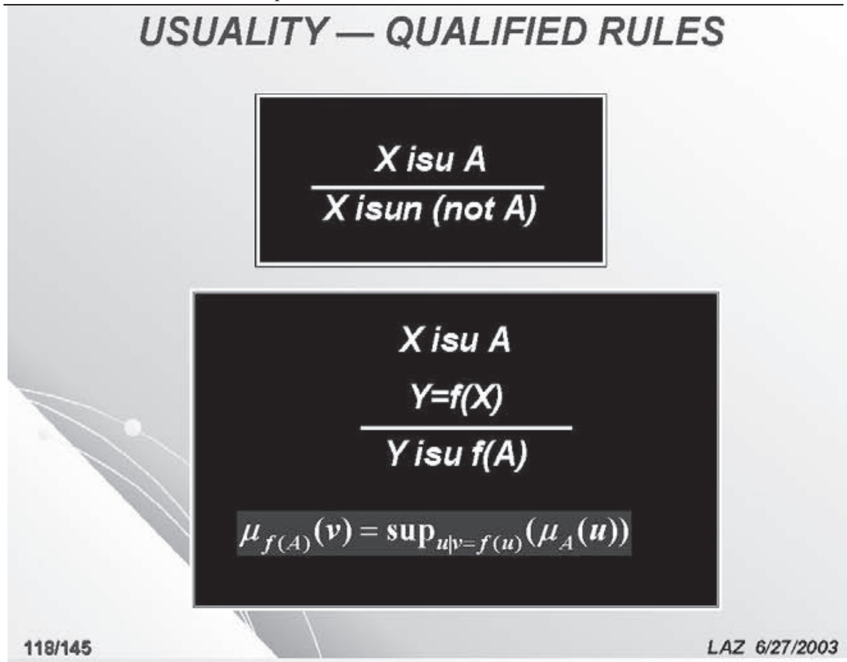
$$v = \int_U g(u)\mu_B(u)du$$

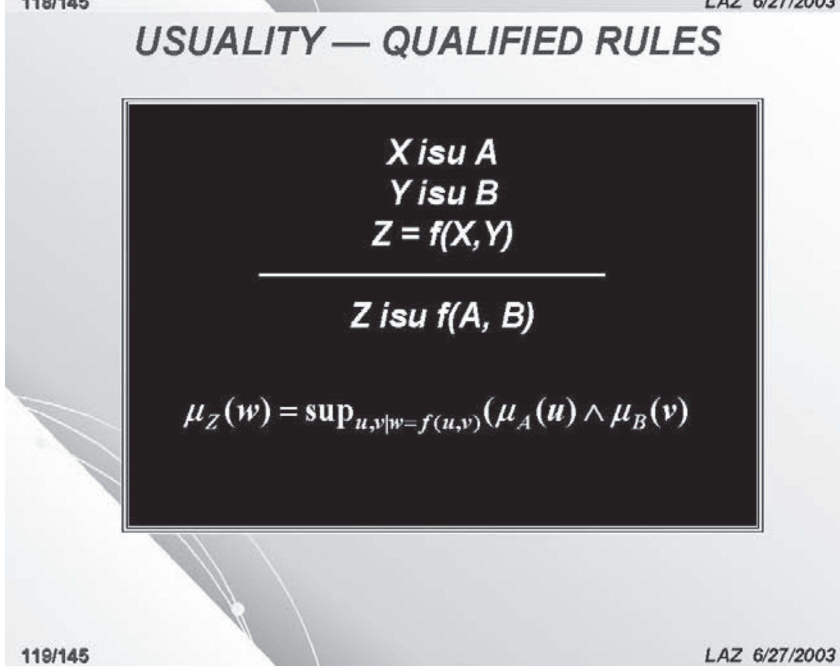
$$\int_U g(u)du = 1$$

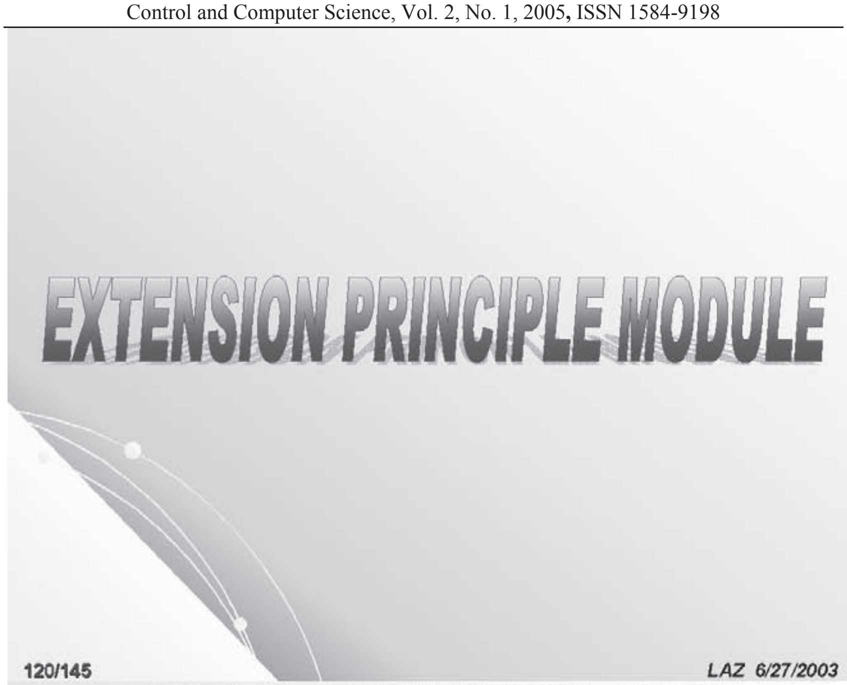
115/145 LAZ 6/27/2003

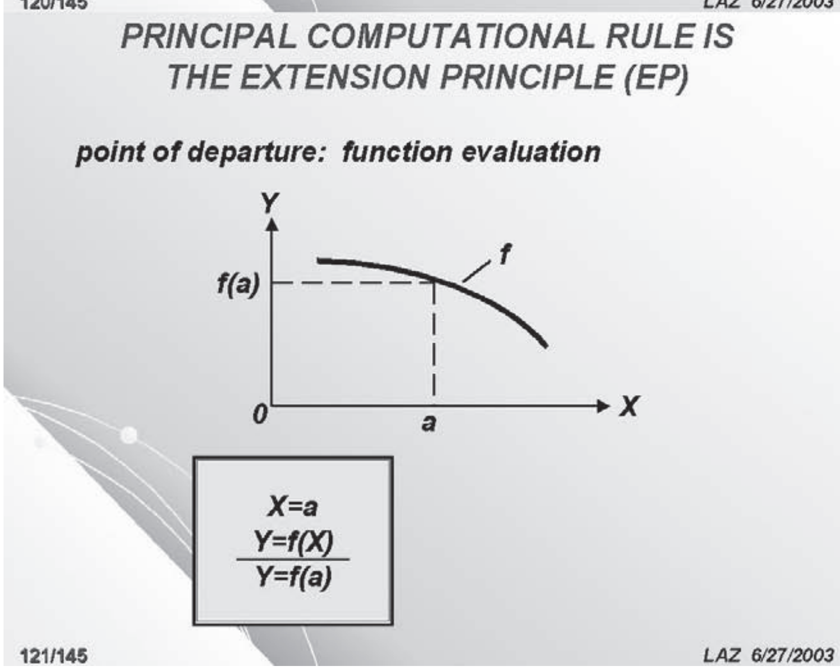

USUALITY SUBMODULE
116

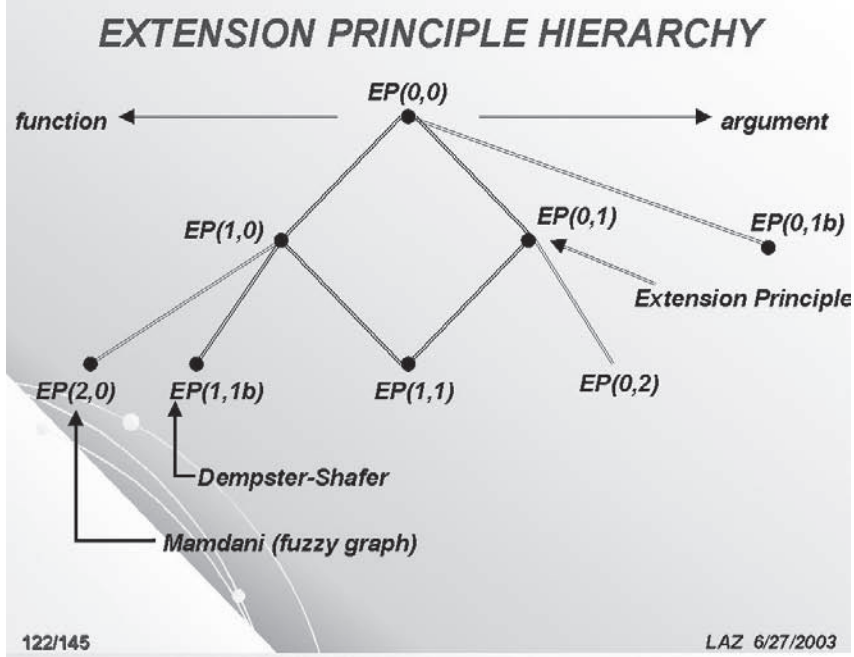

CONJUNCTION
X is A
X is B
X is A∩B
X isu A
X isu B
X isr A∩B
•determination of r involves interpolation of a bimodal distribution
117/145
LAZ 6/27/2003

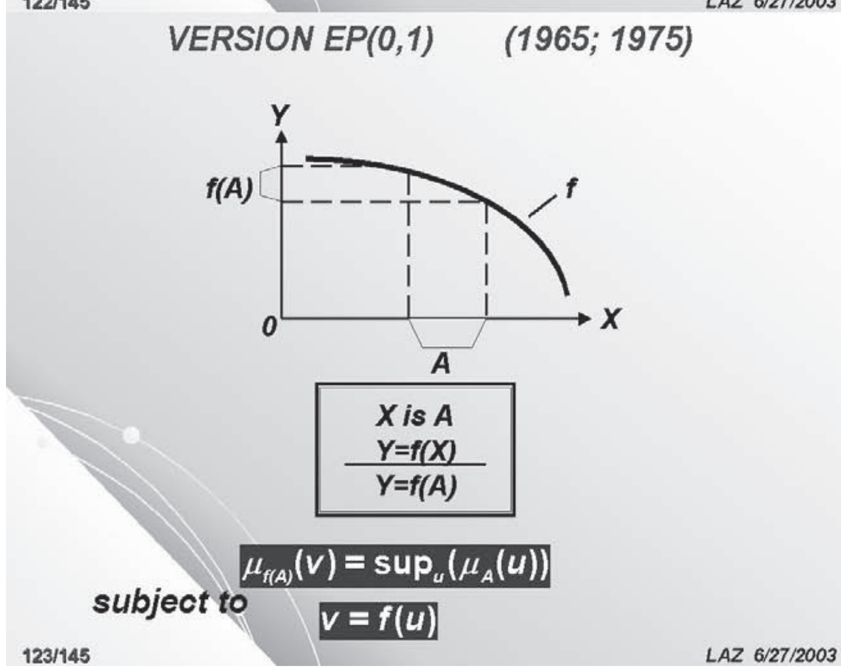

USUALITY — QUALIFIED RULES
X isu A
X isun (not A)
X isu A
Y=f(X)
Y isu f(A)
$\mu_{f(A)}(v) = \sup_{u|v=f(u)}(\mu_A(u))$
118/145
LAZ 6/27/2003

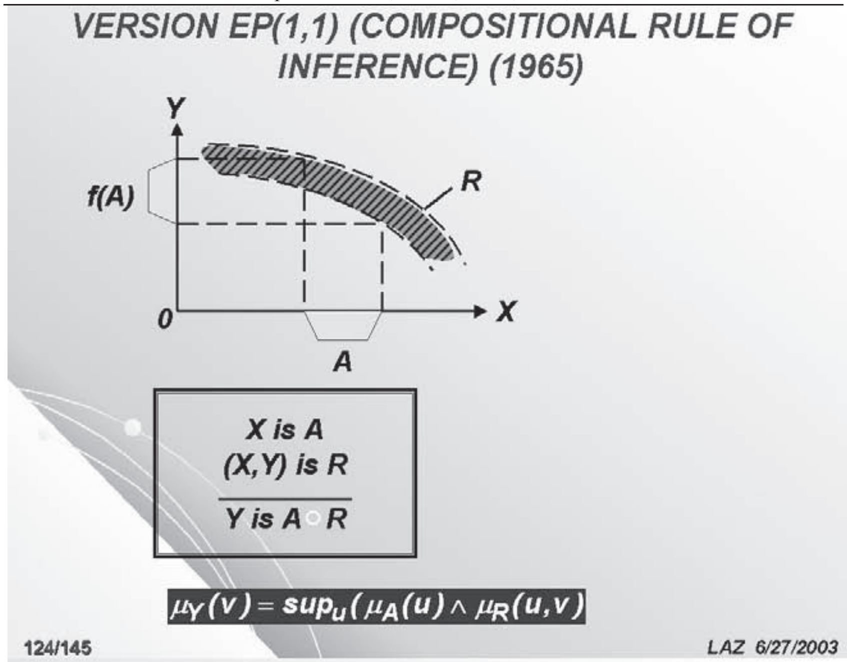

USUALITY — QUALIFIED RULES
X isu A
Y isu B
Z = f(X,Y)
Z isu f(A, B)
$\mu_Z(w) = \sup_{u,v|w=f(u,v)}(\mu_A(u) \wedge \mu_B(v))$
119/145
LAZ 6/27/2003

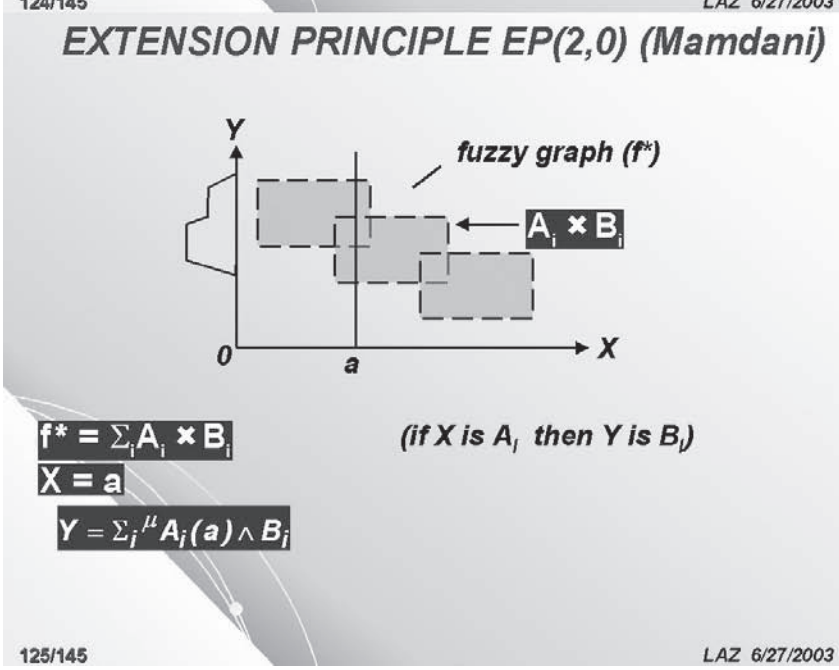

EXTENSION PRINCIPLE MODULE
120/145
LAZ 6/27/2003


PRINCIPAL COMPUTATIONAL RULE IS
THE EXTENSION PRINCIPLE (EP)
point of departure: function evaluation
Y
f
f(a)
0
a
X
X=a
Y=f(X)
Y=f(a)
121/145
LAZ 6/27/2003

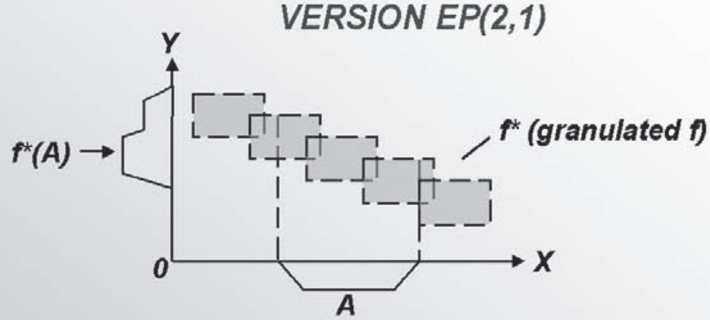

EXTENSION PRINCIPLE HIERARCHY
EP(0,0)
function
argument
EP(1,0)
EP(0,1)
EP(0,1b)
Extension Principle
EP(2,0)
EP(1,1b)
EP(1,1)
EP(0,2)
Dempster-Shafer
Mamdani (fuzzy graph)
122/145
LAZ 6/27/2003


VERSION EP(0,1) (1965; 1975)
Y
f(A)
f
0
X
A
X is A
Y=f(X)
Y=f(A)
$\mu_{f(A)}(v) = \sup_u(\mu_A(u))$
subject to
$v = f(u)$
123/145
LAZ 6/27/2003


VERSION EP(1,1) (COMPOSITIONAL RULE OF INFERENCE) (1965)
Y
f(A)
R
0
X
A
X is A
(X,Y) is R
Y is A ∘ R
$\mu_Y(v) = sup_u(\mu_A(u) \wedge \mu_R(u,v))$
124/145
LAZ 6/27/2003


EXTENSION PRINCIPLE EP(2,0) (Mamdani)
Y
fuzzy graph (f*)
$A_i \times B_i$
0
a
X
$f^* = \Sigma_i A_i \times B_i$
(if X is A_i then Y is B_i)
X = a
$Y = \Sigma_i \mu A_i(a) \wedge B_i$
125/145
LAZ 6/27/2003

VERSION EP(2,1)



$$\frac{\begin{array}{c} X \text{ is } A \\ (X, Y) \text{ is } R \end{array}}{Y \text{ is } \Sigma_i m_i \wedge B_i}$$

$R = \Sigma_i A_i \times B_i$

$m_i = sup_u (\mu_A(u) \wedge \mu_{Ai}(u))$: matching coefficient

126/145 LAZ 6/27/2003

VERSION EP(1,1b) (DEMPSTER-SHAFER)

$$\frac{\begin{array}{c} X \text{ isp } (p_1 \backslash u_1 + \ldots + p_u \backslash u_n) \\ (X,Y) \text{ is } R \end{array}}{Y \text{ isp } (p_1 \backslash R(u_1) + \ldots + p_n \backslash R(u_n))}$$

Y is a fuzzy-set-valued random variable

$\mu_{R(ui)}(v) = \mu_R(u_i, v)$

127/145 LAZ 6/27/2003

VERSION GEP(0,0)

$$\frac{f(X) \text{ is } A}{g(X) \text{ is } g(f^{-1}(A))}$$

$$\mu_{g(f^{-1}(A))}(v) = \sup_u(\mu_A(f(u)))$$

subject to

$$v = g(u)$$

128/145 LAZ 6/27/2003

GENERALIZED EXTENSION PRINCIPLE

$$\frac{\begin{array}{l} f(X) \text{ is } A \\ g(Y) \text{ is } B \\ Z=h(X,Y) \end{array}}{Z \text{ is } h\,(f^{-1}(A),\, g^{-1}\,(B))}$$

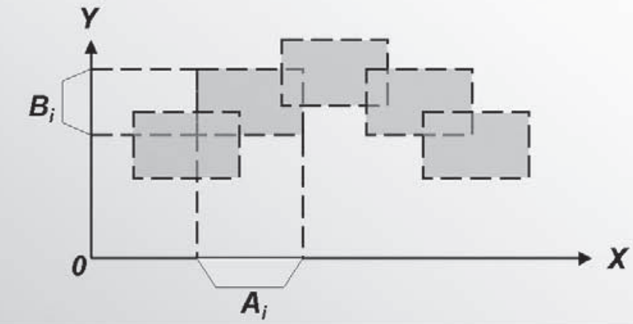
$$\mu_Z(w) = \sup_{u,v}(\mu_A(f(u)) \wedge \mu_B(g(u)))$$

subject to

$$w = h(u,v)$$

129/145 LAZ 6/27/2003

U-QUALIFIED EXTENSION PRINCIPLE



If X is A_i then Y isu B_i, i=1,…, n
X isu A

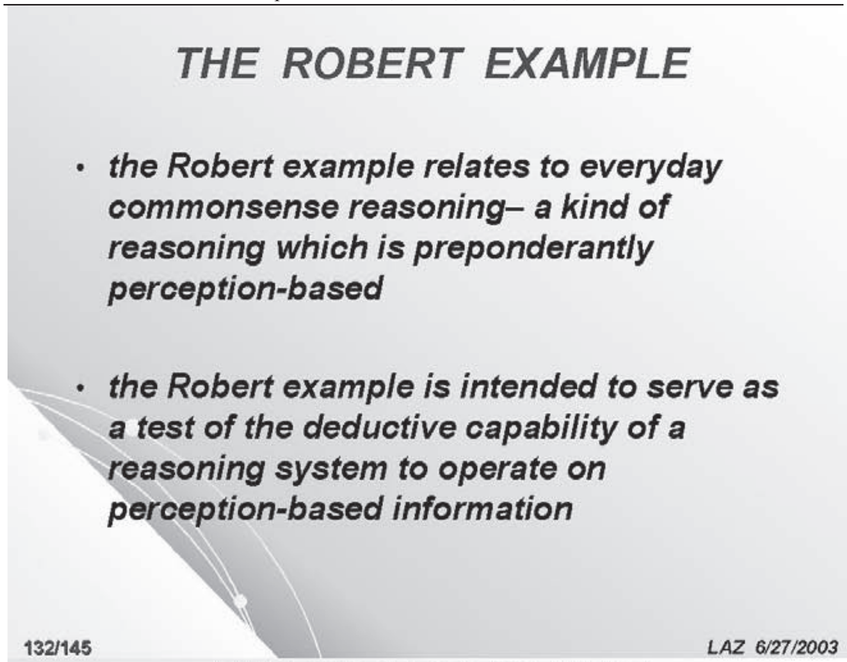
Y isu $\Sigma_i\, m_i \wedge B_i$

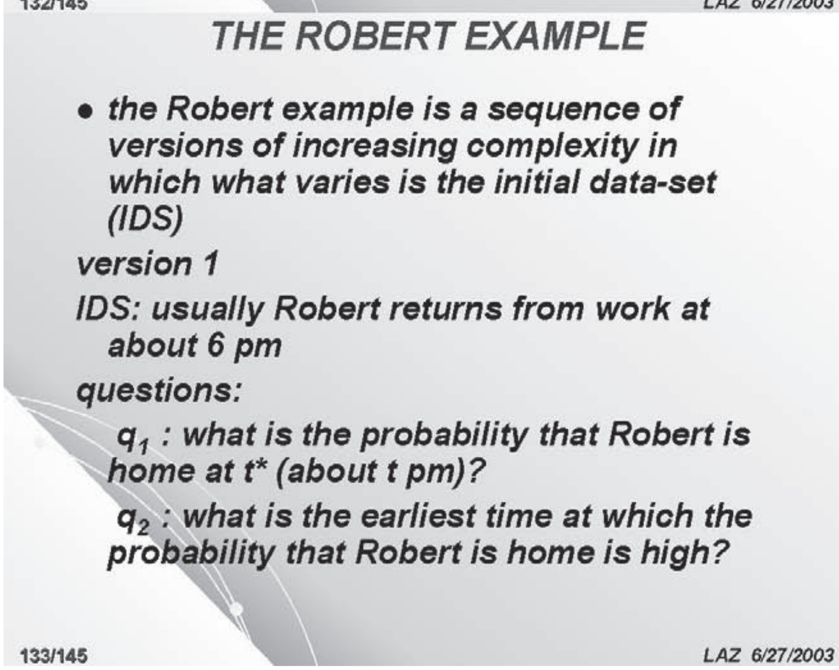
$m = \sup_u (\mu_A(u) \wedge \mu_{A_i}(u))$: matching coefficient

130/145 LAZ 6/27/2003

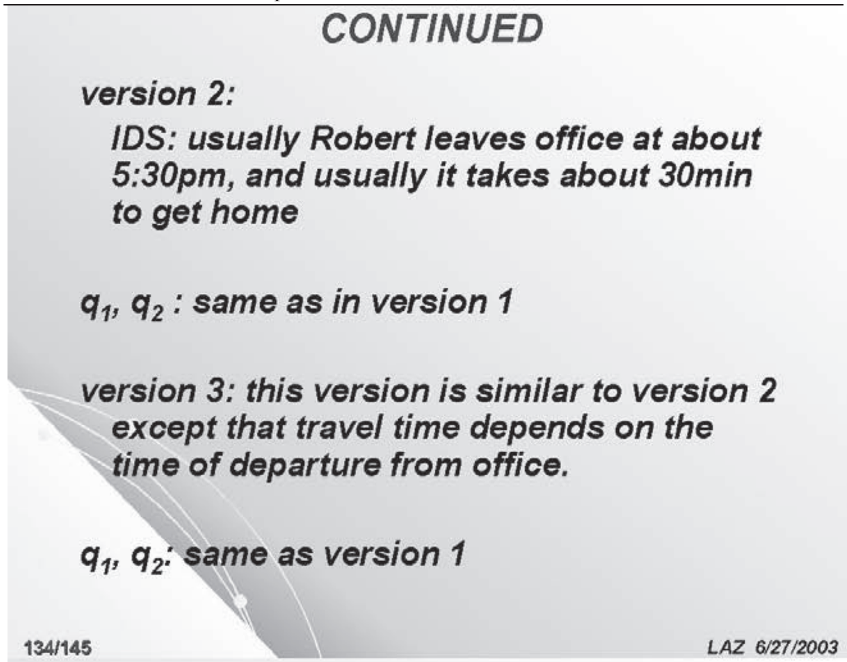
THE ROBERT EXAMPLE

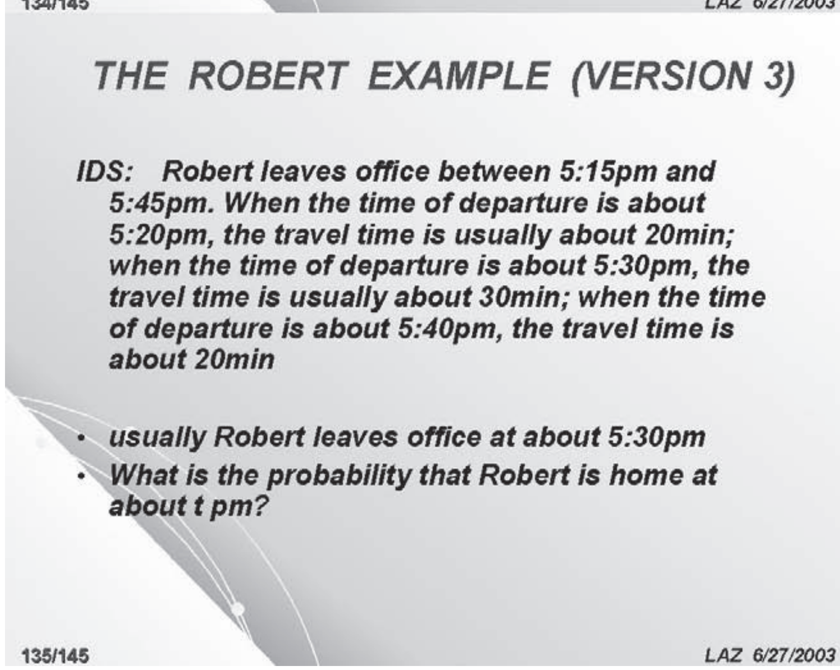
131/145 LAZ 6/27/2003

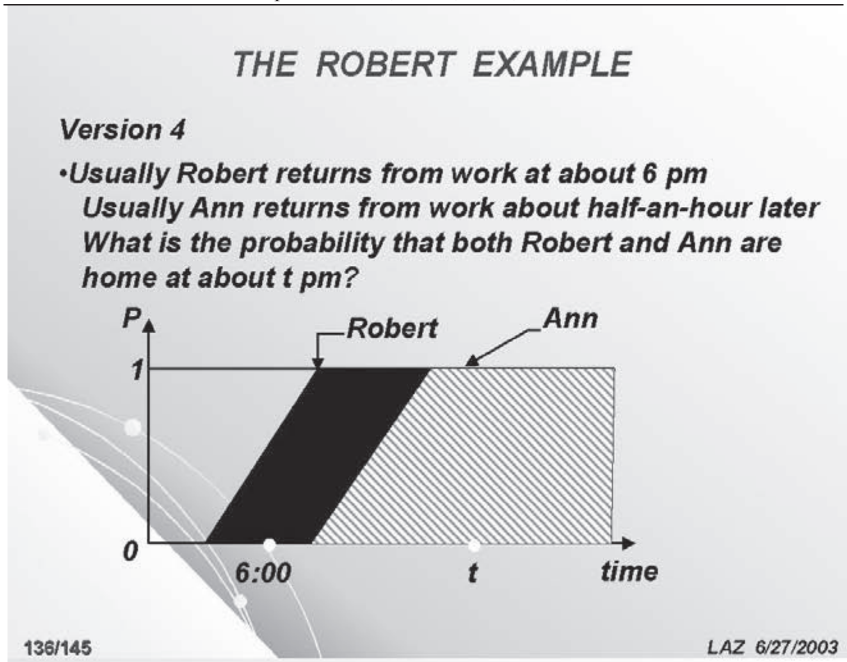

THE ROBERT EXAMPLE
• the Robert example relates to everyday commonsense reasoning– a kind of reasoning which is preponderantly perception-based
• the Robert example is intended to serve as a test of the deductive capability of a reasoning system to operate on perception-based information
132/145
LAZ 6/27/2003

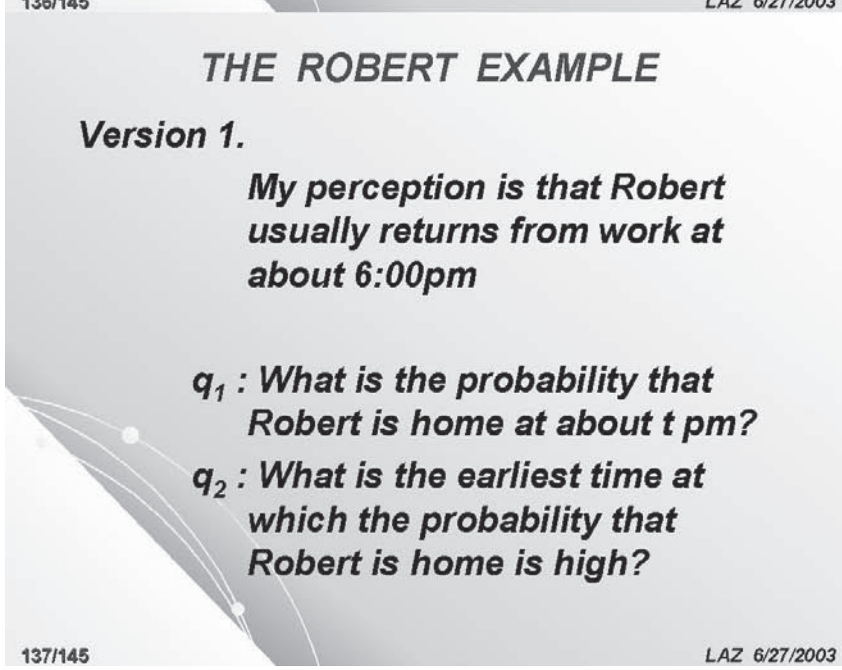

THE ROBERT EXAMPLE
● the Robert example is a sequence of versions of increasing complexity in which what varies is the initial data-set (IDS)
version 1
IDS: usually Robert returns from work at about 6 pm
questions:
q_1 : what is the probability that Robert is home at t* (about t pm)?
q_2 : what is the earliest time at which the probability that Robert is home is high?
133/145
LAZ 6/27/2003

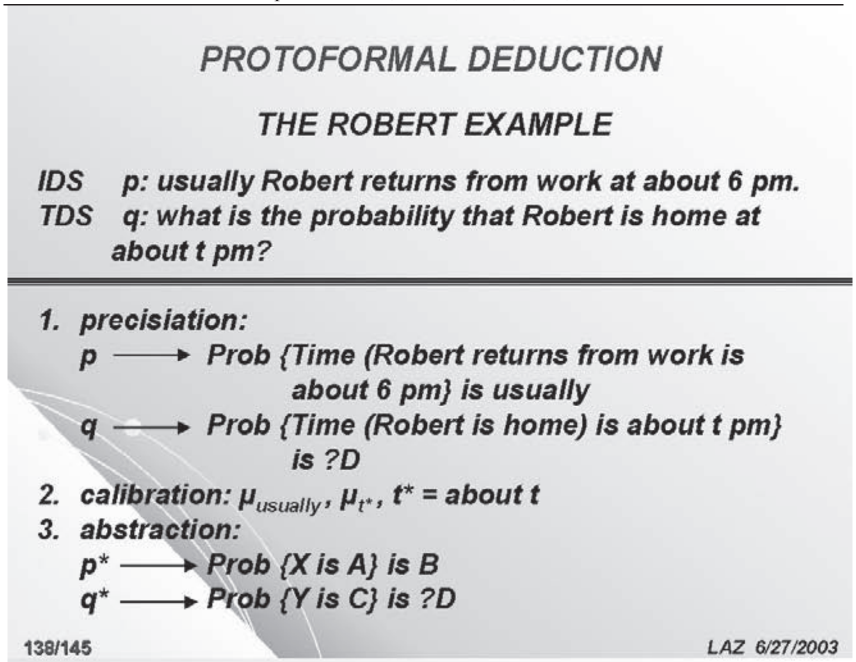
Scientific and Technical Bulletin Series: Electrotechnics, Electronics, Automatic Control and Computer Science, Vol. 2, No. 1, 2005, ISSN 1584-9198

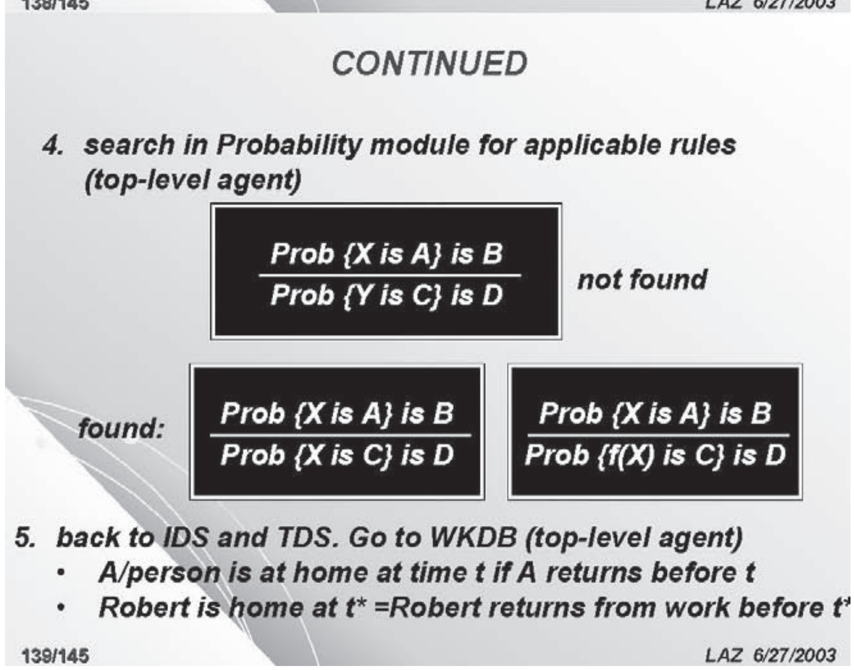


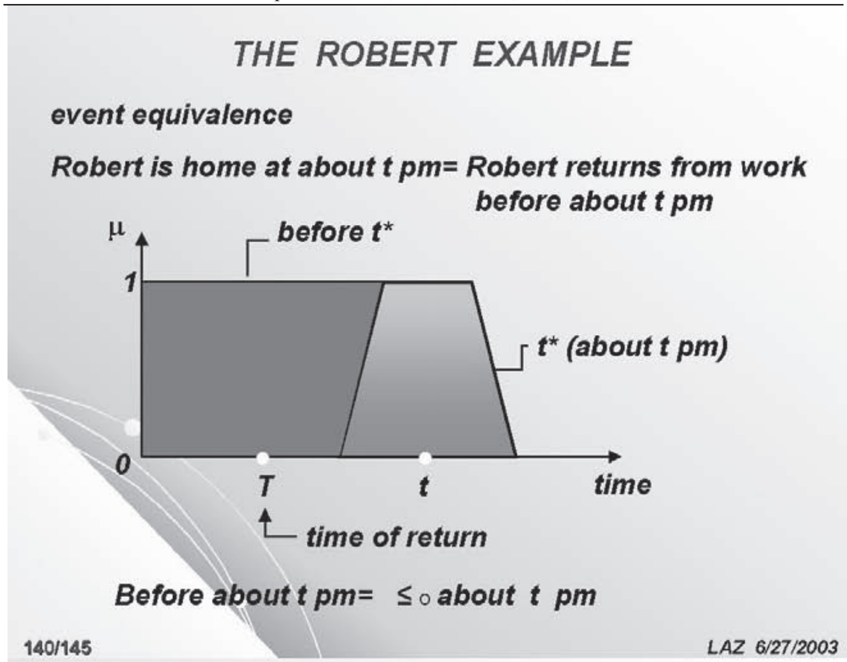


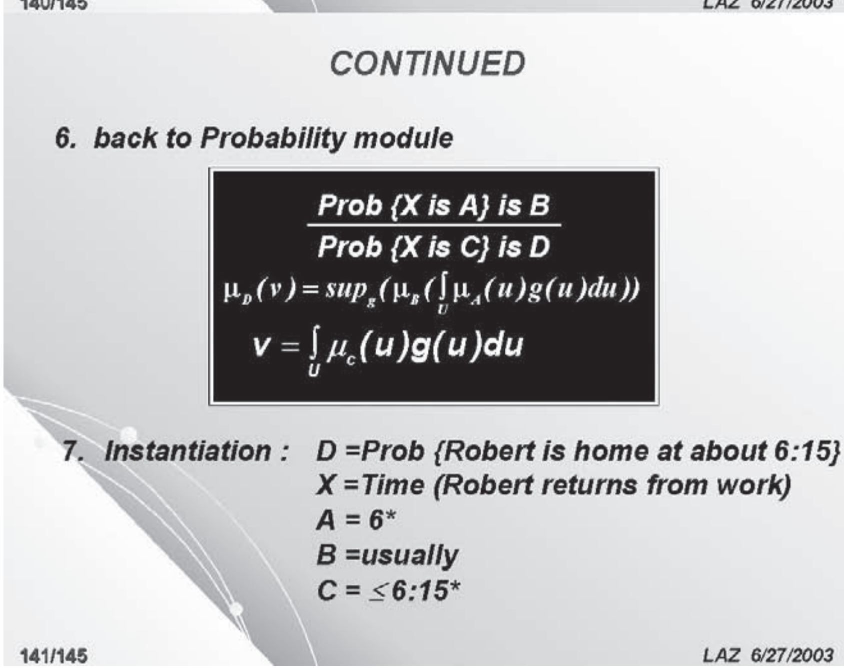

THE ROBERT EXAMPLE
Version 4
•Usually Robert returns from work at about 6 pm
Usually Ann returns from work about half-an-hour later
What is the probability that both Robert and Ann are home at about t pm?
P
1
0
Robert
Ann
6:00
t
time
136/145
LAZ 6/27/2003


THE ROBERT EXAMPLE
Version 1.
My perception is that Robert usually returns from work at about 6:00pm
q1 : What is the probability that Robert is home at about t pm?
q2 : What is the earliest time at which the probability that Robert is home is high?
137/145
LAZ 6/27/2003


PROTOFORMAL DEDUCTION
THE ROBERT EXAMPLE
IDS p: usually Robert returns from work at about 6 pm.
TDS q: what is the probability that Robert is home at about t pm?
1. precisiation:
p ⟶ Prob {Time (Robert returns from work is about 6 pm} is usually
q ⟶ Prob {Time (Robert is home) is about t pm} is ?D
2. calibration: $\mu_{usually}$, μ_{t^*}, t^* = about t
3. abstraction:
p* ⟶ Prob {X is A} is B
q* ⟶ Prob {Y is C} is ?D
138/145
LAZ 6/27/2003


CONTINUED
4. search in Probability module for applicable rules (top-level agent)
$\frac{\text{Prob \{X is A\} is B}}{\text{Prob \{Y is C\} is D}}$ not found
found: $\frac{\text{Prob \{X is A\} is B}}{\text{Prob \{X is C\} is D}}$ $\frac{\text{Prob \{X is A\} is B}}{\text{Prob \{f(X) is C\} is D}}$
5. back to IDS and TDS. Go to WKDB (top-level agent)
• A/person is at home at time t if A returns before t
• Robert is home at t* =Robert returns from work before t*
139/145
LAZ 6/27/2003


THE ROBERT EXAMPLE
event equivalence
Robert is home at about t pm= Robert returns from work before about t pm
μ
before t*
1
t* (about t pm)
0
T
t
time
time of return
Before about t pm= ≤ ∘ about t pm
140/145
LAZ 6/27/2003


CONTINUED
6. back to Probability module
Prob {X is A} is B
Prob {X is C} is D
$\mu_D(v) = sup_g(\mu_B(\int_U \mu_A(u)g(u)du))$
$v = \int_U \mu_c(u)g(u)du$
7. Instantiation : D =Prob {Robert is home at about 6:15}
X =Time (Robert returns from work)
A = 6*
B =usually
C = ≤6:15*
141/145
LAZ 6/27/2003

SUMMATION

KEY POINTS

- ***humans have a remarkable capability—a capability which machines do not have—to perform a wide variety of physical and mental tasks using only perceptions, with no measurements and no computations***

- ***perceptions are intrinsically imprecise, reflecting the bounded ability of sensory organs, and ultimately the brain, to resolve detail and store information***

142/145 LAZ 6/27/2003

CONTINUED

- ***imprecision of perceptions stands in the way of constructing a computational theory of perceptions within the conceptual structure of bivalent logic and bivalent-logic-based probability theory***

- ***this is why existing scientific theories—based as they are on bivalent logic and bivalent-logic-based probability theory—provide no tools for dealing with perception-based information***

143/145 LAZ 6/27/2003

CONTINUED

- *in computing with words and perceptions (CWP), the objects of computation are propositions drawn from a natural language and, in particular, propositions which are descriptors of perceptions*

- *computing with words and perceptions is a methodology which may be viewed as (a) a new direction for dealing with imprecision, uncertainty and partial truth; and (b) as a basis for the analysis and design of systems which are capable of operating on perception-based information*

144/145 LAZ 6/27/2003

STATISTICS

Count of papers containing the word "fuzzy" in the title, as cited in INSPEC and MATH.SCI.NET databases. (data for 2002 are not complete)

Compiled by Camille Wanat, Head, Engineering Library, UC Berkeley, April 17, 2003

INSPEC/fuzzy		Math.Sci.Net/fuzzy
1970-1979	569	443
1980-1989	2,404	2,466
1990-1999	23,207	5,472
2000-present	8,745	2,319
1970-present	34,925	10,700

145/145 LAZ 6/27/2003