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FROM SEARCH ENGINES TO QUESTION-ANSWERING SYSTEMS – THE PROBLEMS OF WORLD KNOWLEDGE, RELEVANCE, DEDUCTION AND PRECISIATION

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CURRICULUM VITAE

Lotfi A. Zadeh

Lotfi A. Zadeh joined the Department of Electrical Engineering at the University of California, Berkeley, in 1959, and served as its chairman from 1963 to 1968. Earlier, he was a member of the electrical engineering faculty at Columbia University. In 1956, he was a visiting member of the Institute for Advanced Study in Princeton, New Jersey. In addition, he held a number of other visiting appointments, among them a visiting professorship in Electrical Engineering at MIT in 1962 and 1968; a visiting scientist appointment at IBM Research Laboratory, San Jose, CA, in 1968, 1973, and 1977; and visiting scholar appointments at the AI Center, SRI International, in 1981, and at the Center for the Study of Language and Information, Stanford University, in 1987-1988. Currently he is a Professor in the Graduate School, and is serving as the Director of BISC (Berkeley Initiative in Soft Computing).

Until 1965, Dr. Zadeh's work had been centered on system theory and decision analysis. Since then, his research interests have shifted to the theory of fuzzy sets and its applications to artificial intelligence, linguistics, logic, decision analysis, control theory, expert systems and neural networks. Currently, his research is focused on fuzzy logic, soft computing, computing with words, and the newly developed computational theory of perceptions and precisiated natural language.

An alumnus of the University of Tehran, MIT, and Columbia University, Dr. Zadeh is a fellow of the IEEE, AAAS, ACM, AAAI and IFSA, and a member of the National Academy of Engineering. He held NSF Senior Postdoctoral Fellowships in 1956-57 and 1962-63, and was a Guggenheim Foundation Fellow in 1968. Dr. Zadeh was the recipient of the IEEE Education Medal in 1973 and a recipient of the IEEE Centennial Medal in 1984. In 1989, Dr. Zadeh was awarded the Honda Prize

by the Honda Foundation, and in 1991 received the Berkeley Citation, University of California.

In 1992, Dr. Zadeh was awarded the IEEE Richard W. Hamming Medal "For seminal contributions to information science and systems, including the conceptualization of fuzzy sets." He became a Foreign Member of the Russian Academy of Natural Sciences (Computer Sciences and Cybernetics Section) in 1992, and received the Certificate of Commendation for AI Special Contributions Award from the International Foundation for Artificial Intelligence. Also in 1992, he was awarded the Kampe de Fieriet Prize and became an Honorary Member of the Austrian Society of Cybernetic Studies.

In 1993, Dr. Zadeh received the Rufus Oldenburger Medal from the American Society of Mechanical Engineers "For seminal contributions in system theory, decision analysis, and theory of fuzzy sets and its applications to AI, linguistics, logic, expert systems and neural networks." He was also awarded the Grigore Moisil Prize for Fundamental Researches, and the Premier Best Paper Award by the Second International Conference on Fuzzy Theory and Technology. In 1995, Dr. Zadeh was awarded the IEEE Medal of Honor "For pioneering development of fuzzy logic and its many diverse applications." In 1996, Dr. Zadeh was awarded the Okawa Prize "For outstanding contribution to information science through the development of fuzzy logic and its applications."

In 1997, Dr. Zadeh was awarded the B. Bolzano Medal by the Academy of Sciences of the Czech Republic "For outstanding achievements in fuzzy mathematics." He also received the J.P. Wohl Career Achievement Award of the IEEE Systems, Science and Cybernetics Society. He served as a Lee Kuan Yew Distinguished Visitor, lecturing at the National University of Singapore and the Nanyang Technological University in Singapore, and as the Gulbenkian Foundation Visiting Professor at the New University of Lisbon in Portugal. In 1998, Dr. Zadeh was awarded the Edward Feigenbaum Medal by the International Society for Intelligent Systems and the

Richard E. Bellman Control Heritage Award by the American Council on Automatic Control. In addition, he received the Information Science Award from the Association for Intelligent Machinery and the SOFT Scientific Contribution Memorial Award from the Society for Fuzzy Theory in Japan. In 1999, he was elected to membership in Berkeley Fellows and received the Certificate of Merit from IFSA (International Fuzzy Systems Association). In 2000, he received the IEEE Millennium Medal; the IEEE Pioneer Award in Fuzzy Systems; the ASPIH 2000 Lifetime Distinguished Achievement Award; and the ACIDCA 2000 Award for the paper, "From Computing with Numbers to Computing with Words – From Manipulation of Measurements to Manipulation of Perceptions." In addition, he received the Chaos Award from the Center of Hyperincursion and Anticipation in Ordered Systems for his outstanding scientific work on foundations of fuzzy logic, soft computing, computing with words and the computational theory of perceptions. In 2001, Dr. Zadeh received the ACM 2000 Allen Newell Award for seminal contributions to AI through his development of fuzzy logic. In addition, he received a Special Award from the Committee for Automation and Robotics of the Polish Academy of Sciences for his significant contributions to systems and information science, development of fuzzy sets theory, fuzzy logic control, possibility theory, soft computing, computing with words and computational theory of perceptions. In 2003, Dr. Zadeh was elected as a foreign member of the Finnish Academy of Sciences, and received the Norbert Wiener Award of the IEEE Society of Systems, Man and Cybernetics "For pioneering contributions to the development of system theory, fuzzy logic and soft computing." In 2004, Dr. Zadeh was awarded Civitate Honoris Causa by Budapest Tech (BT) Polytechnical Institution, Budapest, Hungary. Also in 2004, he was awarded the V. Kaufmann Prize, International Association for Fuzzy-Set Management and Economy (SIGEF).

Dr. Zadeh is a recipient of twenty-three honorary doctorates from: Paul-Sabatier University, Toulouse, France;

State University of New York, Binghamton, NY; University of Dortmund, Dortmund, Germany; University of Oviedo, Oviedo, Spain; University of Granada, Granada, Spain; Lakehead University, Canada; University of Louisville, KY; Baku State University, Azerbaijan; the Silesian Technical University, Gliwice, Poland; the University of Toronto, Toronto, Canada; the University of Ostrava, the Czech Republic; the University of Central Florida, Orlando, FL; the University of Hamburg, Hamburg, Germany; the University of Paris(6), Paris, France; Johannes Kepler University, Linz, Austria; University of Waterloo, Canada; and the University of Aurel Vlaicu, Arad, Romania; Lappeenranta University of Technology, Lappeenranta, Finland; Muroran Institute of Technology, Muroran, Japan; Hong Kong Baptist University, Hong Kong, China.

Dr. Zadeh has single-authored over two hundred papers and serves on the editorial boards of over fifty journals. He is a member of the Advisory Committee, Center for Education and Research in Fuzzy Systems and Artificial Intelligence, Iasi, Romania; Senior Advisory Board, International Institute for General Systems Studies; the Board of Governors, International Neural Networks Society; and is the Honorary President of the Biomedical Fuzzy Systems Association of Japan and the Spanish Association for Fuzzy Logic and Technologies. In addition, he is a member of the Advisory Board of the National Institute of Informatics, Tokyo; a member of the Governing Board, Knowledge Systems Institute, Skokie, IL; and an honorary member of the Academic Council of NAISO-IAAC.

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Abstract

FROM SEARCH ENGINES TO QUESTION-ANSWERING SYSTEMS – THE PROBLEMS OF WORLD KNOWLEDGE, RELEVANCE, DEDUCTION AND PRECISIATION

Lotfi A. Zadeh*

Existing search engines, with Google at the top, have many truly remarkable capabilities. Furthermore, constant progress is being made in improving their performance. But what is not widely recognized is that there is a basic capability which existing search engines do not have: deduction capability – the capability to synthesize an answer to a query by drawing on bodies of information which reside in various parts of the knowledge base. By definition, a question-answering system, or a Q/A system for short, is a system which has deduction capability. Can a search engine be upgraded to a question-answering system through the use of existing tools – tools which are based on bivalent logic and probability theory? A view which is articulated in the following is that the answer is: No.

The first obstacle is world knowledge – the knowledge which humans acquire through experience, communication and education. Simple examples are: “Icy roads are slippery,” “Princeton usually means Princeton University,” “Paris is the capital of France,” and “There are no honest politicians.” World knowledge plays a central role in search, assessment of relevance

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and deduction. The problem with world knowledge is that it is, for the most part, perception-based. Perceptions—and especially perceptions of probabilities—are intrinsically imprecise, reflecting the fact that human sensory organs, and ultimately the brain, have a bounded ability to resolve detail and store information. Imprecision of perceptions stands in the way of using conventional techniques – techniques which are based on bivalent logic and probability theory – to deal with perception-based information. A further complication is that much of world knowledge is negative knowledge in the sense that it relates to what is impossible and/or non-existent. For example, “A person cannot have two fathers,” and “Netherlands has no mountains.”

The second obstacle centers on the concept of relevance. There is an extensive literature on relevance, and every search engine deals with relevance in its own way, some at a high level of sophistication. But what is quite obvious is that the problem of assessment of relevance is quite complex and far from solution.

There are two kinds of relevance: (a) question relevance and (b) topic relevance. Both are matters of degree. For example, on a very basic level, if the question is q : “Number of cars in California?” and the available information is p : “Population of California is 37,000,000,” then what is the degree of relevance of p to q ? Another example: To what degree is a paper entitled “A New Approach to Natural Language Understanding” of relevance to the topic of machine translation.

Basically, there are two ways of approaching assessment of relevance: (a) semantic; and (b) statistical. To illustrate, in the number of cars example, relevance of p to q is a matter of semantics and world knowledge. In existing search engines, relevance is largely a matter of statistics, involving counts of links and words, with little if any consideration of semantics. Assessment of semantic relevance presents difficult problems whose solutions lie beyond the reach of bivalent logic and probability theory. What should be noted is that assessment of topic relevance is more amendable to the use of statistical techniques, which explains why existing search engines are

much better at assessment of topic relevance than question relevance.

The third obstacle is deduction from perception-based information. As a basic example, assume that the question is q : What is the average height of Swedes?, and the available information is p : Most adult Swedes are tall. Another example is: Usually Robert returns from work at about 6 pm. What is the probability that Robert is at home at 6:15 pm? Neither bivalent logic nor probability theory provide effective tools for dealing with problems of this type. The difficulty is centered on deduction from premises which are both uncertain and imprecise.

Underlying the problems of world knowledge, relevance and deduction is a very basic problem – the problem of natural language understanding. Much of world knowledge and web knowledge is expressed in a natural language. A natural language is basically a system for describing perceptions. Since perceptions are intrinsically imprecise, so are natural languages.

A prerequisite to mechanization of question-answering is mechanization of natural language understanding, and a prerequisite to mechanization of natural language understanding is precisiation of meaning of concepts and proposition drawn from a natural language. To deal effectively with world knowledge, relevance, deduction and precisiation, new tools are needed. The principal new tools are: Precisiated Natural Language (PNL); Protoform Theory (PFT); and the Generalized Theory of Uncertainty (GTU). These tools are drawn from fuzzy logic—a logic in which everything is, or is allowed to be, a matter of degree.

The centerpiece of the new tools is the concept of a generalized constraint. The importance of the concept of a generalized constraint derives from the fact that in PNL and GTU it serves as a basis for generalizing the universally accepted view that information is statistical in nature. More specifically, the point of departure in PNL and GTU is the fundamental premise that, in general, information is representable as a system

of generalized constraints, with statistical information constituting a special case. This, much more general, view of information is needed to deal effectively with world knowledge, relevance, deduction, precisiation and related problems.

In summary, the principal objectives of this paper are: (a) to make a case for the view that a quantum jump in search engine IQ cannot be achieved through the use of methods based on bivalent logic and probability theory; and (b) to introduce and outline a collection of non-standard concepts, ideas and tools which are needed to achieve a quantum jump in search engine IQ.

***From Search Engines to Question-
Answering Systems—The Problems of
World Knowledge, Relevance, Deduction
and Precisation***

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BACKDROP



KEY ISSUE—DEDUCTION CAPABILITY

Existing search engines, with Google at the top, have many truly remarkable capabilities. Furthermore, constant progress is being made in improving their performance. But what should be realized is that existing search engines do not have an important capability—deduction capability—the capability to synthesize an answer to a query by drawing on bodies of information which reside in various parts of the knowledge base.

3

CONTINUED

- ***What should be noted, however, is that there are many widely used special purpose question-answering systems which have limited deduction capability. Examples of such systems are driving direction systems, reservation systems, diagnostic systems and specialized expert systems, especially in the domain of medicine.***

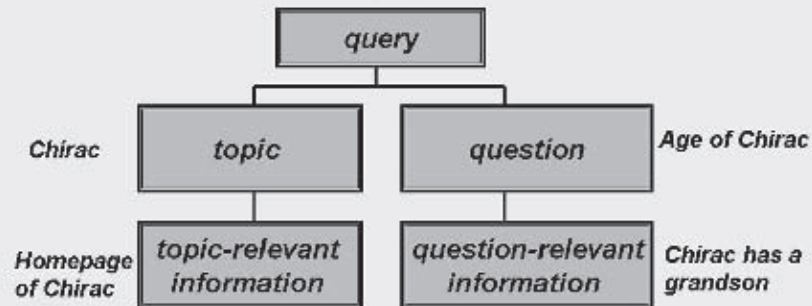
4

SEARCH VS. QUESTION-ANSWERING

- *A question-answering system may be viewed as a system which mechanizes question-answering*
- *A search engine in a system which partially mechanizes question-answering*

5

PARTIAL MECHANIZATION



- *A search engine is primarily a provider of topic-relevant information*
- *User of a search engine exploits this capability to derive an answer to a question*

6

COMPLEXITY OF UPGRADING

- *Addition of deduction capability to a search engine is a highly complex problem—a problem which is a major challenge to computer scientists and logicians*
- *A view which is articulated in the following is that the challenge cannot be met through the use of existing methods—methods which are based on bivalent logic and probability theory*
- *To add deduction capability to a search engine it is necessary to (a) generalize bivalent logic; (b) generalize probability theory*

7

HISTORICAL NOTE

- *1970-1980 was a period of intense interest in question-answering and expert systems*
- *There was no discussion of search engines*
Example: L.S. Coles, "Techniques for Information Retrieval Using an Inferential Question-Answering System with Natural Language Input," SRI Report, 1972
- *M. Nagao, J. Tsujii: Mechanism of Deduction in a Question-Answering System with Natural Language Inputd. IJCAI 1973: 285-290.*
- *J. R. McSkimin, J. Minker: The Use of a Semantic Network in a Deductive Question- Answering System. IJCAI 1977: 50-58.*
- *A. R. Aronson, B. E. Jacobs, J. Minker: A Note on Fuzzy Deduction. J. ACM 27(4): 599-603 (1980)*
- *W.J.H.J. Bronnenberg, H.C. Bunt, S.P.J. Lendsbergen, R.J.H. Scha, W.J. Schoenmakers and E.P.C. van Utteren. The Question Answering System PHLIQA1. In L. Bolc (editor), Natural Language Question Answering Systems. Macmillan, 1980.*

8

TEST QUERIES: GOOGLE

t: *precisiation*

q: *What is precisiation?*

r:

[UAI] The concept of cointensive precisiation

... from data expressed in a natural language is precisiation of meaning. ...

In this perspective, the problem of precisiation is that of picking a ...

AI Magazine: Precisiated natural language

... The Concepts of Precisiability and

Precisiation Language ... p is precisiable

if it can be translated into what may be called a precisiation language, ...

9

SIMPLE EXAMPLES OF DEDUCTION INCAPABILITY

q: *What is precisiation?*

r: *same as for t*

10

CONTINUED

q_1 : *What is the capital of New York?*

q_2 : *What is the population of the capital of New York?*

r_1 :

Web definitions for capital of new york Albany: state capital of New York; located in eastern New York State on the west bank of the Hudson river

News results for what is the capital of New York - View today's top stories After the twin-tower nightmare, New York is back on form, says ... - Economist - 3 hours ago
The New Raiders - BusinessWeek - 14 hours ago
Brascan acquires New York-based Hyperion Capital for \$50M US

11

CONTINUED

q_2 : *What is the population of the capital of New York?*

r_2 :

News results for population of New York - View today's top stories After the twin-tower nightmare, New York is back on form, says ...

UN: World's population is aging rapidly - New, deadly threat from AIDS virus

12

CONTINUED

**q: What is the distance between the largest city in Spain
and the largest city in Portugal?**

r:

[Porto - Oporto - Portugal Travel Planner](#)

[Munich Germany Travel Planner - Hotels Restaurants
Language ...](#)



CONTINUED

q: Age of Chirac

r:

Jacques Chirac

Date of Birth: 29 November 1932



CONTINUED

q: Age of son of Chirac

r:

*... Albert, their only son, becomes Monaco's de facto ruler
until a formal investiture*

*... French President Jacques Chirac hailed the prince's
"courage and ...*



15

CONTINUED

**q: How many Ph.D. degrees in mathematics were granted
by European Universities in 1986?**

r:

A History of the University of Podlasie

Annual Report 1996

A Brief Report on Mathematics in Iran



16

UPGRADING

- *There are three major problems in upgrading a search engine to a question-answering system*
 - *World knowledge*
 - *Relevance*
 - *Deduction*
 - *Precisiation*
- *These problems are beyond the reach of existing methods based on bivalent logic and probability theory*
- *A basic underlying problem is mechanization of natural language understanding. A prerequisite to mechanization of natural language understanding is precisiation of meaning*

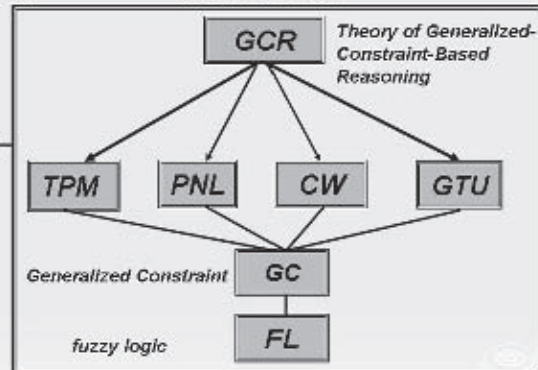
17

NEED FOR NEW TOOLS

Tools in current use



New Tools



PT: standard bivalent-logic-based probability theory
 TPM: Theory of Precisiation of Meaning
 PNL: Precisiated Natural Language
 CW: Computing with Words
 GTU: Generalized Theory of Uncertainty
 GCR: Theory of Generalized-Constraint-Based Reasoning

18

KEY CONCEPT

- *The concept of a generalized constraint is the centerpiece of new tools—the tools that are needed to upgrade a search engine to a question-answering system*
 - *The concept of a generalized constraint serves as a bridge between linguistics and mathematics by providing a means of precisiation of propositions and concepts drawn from a natural language*
- *Information = generalized constraint*

19

WORLD KNOWLEDGE

- *World knowledge is the knowledge acquired through experience, education and communication*
 - *Few professors are rich*
 - *There are no honest politicians*
 - *It is not likely to rain in San Francisco in midsummer*
 - *Most Swedes are tall*
 - *There are no mountains in Holland*
 - *Usually Princeton means Princeton University*
 - *A person can have only one father*

20

A QUICK EXAMPLE OF LACK OF WORLD KNOWLEDGE

Query: *number of fathers of Bush*

Google:

Results 1 - 10 of about 803,000 for *number of fathers of Bush*. (0.20 seconds)

- **Sins of the Fathers -- In These Times**
American Dynasty impressively describes an America where a small number of key ... Phillips traces the origins of the Bush Dynasty to two of Dubya's great ...
www.inthesetimes.com/site/main/article/142/
- **George Bush Jr. And the Number 13**
George Bush Jr. And The Number 13. by Robert Howard ... We have President George Bush Jr. who is 13th cousin of Britain's Queen Mother, and of her daughter ...
www.theforbiddenknowledge.com/hardtruth/george_bushjr_13.htm

21

COMPONENTS OF WORLD KNOWLEDGE

- **Propositional**
 - *Paris is the capital of France*
- **Conceptual**
 - *Climate*
- **Ontological**
 - *Rainfall is related to climate*
- **Existential**
 - *A person cannot have more than one father*
- **Contextual**
 - *Tall*

22

CONTINUED

- *Much of world knowledge is perception-based*
 - *Most Swedes are tall*
 - *Most Swedes are much taller than most Italians*
 - *Usually a large house costs more than a small house*
- *Much of world knowledge is negative, i.e., relates to impossibility or nonexistence*
 - *A person cannot have more than one father*
 - *There are no honest politicians*
- *Much of world knowledge is expressed in a natural language*

23

PROBLEM

- *Existing methods cannot deal with deduction from perception-based knowledge*
 - *Most Swedes are tall*
What is the average height of Swedes?
How many are not tall?
How many are short?
 - *A box contains about 20 black and white balls. Most are black. There are several times as many black balls as white balls.*
How many balls are white?

24

THE PROBLEM OF DEDUCTION

- p_1 : usually temperature is not very low
 p_2 : usually temperature is not very high
?average value of temperature
- most students are young
most young students are single
?students are young and single
- Brian is much older than most of his close friends
How old is Brian?

25

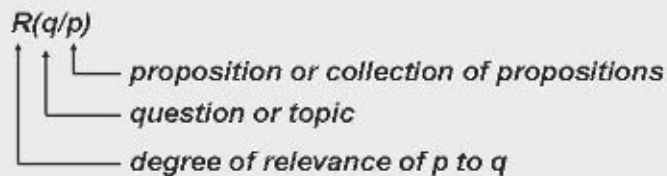
THE PROBLEM OF RELEVANCE

- A major obstacle to upgrading is the concept of relevance. There is an extensive literature on relevance, and every search engine deals with relevance in its own way, some at a high level of sophistication. But what is quite obvious is that the problem of assessment of relevance is very complex and far from solution
- What is relevance?
- Relevance is not bivalent
- Relevance is a matter of degree, i.e., is a fuzzy concept
- There is no cointensive definition of relevance in the literature

26

CONTINUED

Definition of relevance function



- q : number of cars in California?
- p : population of California is 37,000,000
- To what degree is p relevant to q ?

• assessment of degree of relevance requires world knowledge

27

A SERIOUS COMPLICATION— NONCOMPOSITIONALITY

- $R(q/p, r) = ?$
- $R(q/p) = 0; R(q/r) = 0; R(q/p, r) \neq 0$

Example

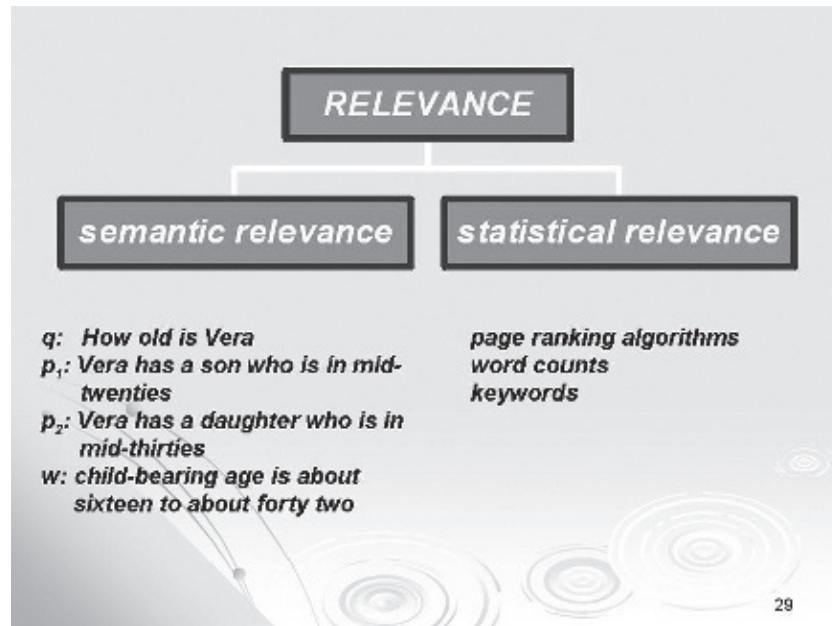
q : How old is Mary?

p : Mary's age is the same as Carol's age

r : Carol is 32 $R(q/p) = 0; R(q/r) = 0; R(q/p, r) = 1$

- Conclusion: relevance cannot be assessed in isolation
- Definition
- p is i -relevant to q if p is relevant to q in isolation
- p is i -irrelevant to q if p is not relevant to q in isolation

28



MECHANIZATION OF QUESTION ANSWERING

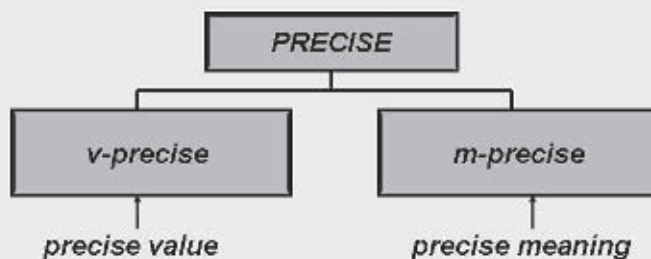
- *Much of world knowledge and web knowledge is expressed in a natural language*
- *Natural language understanding is a prerequisite to question-answering*
- *Precisiation of meaning is a prerequisite to mechanization of natural language understanding*
- *Human natural language understanding is a prerequisite to precisiation*
- *Machines do not have the human ability to understand what has imprecise meaning*

Example: Take a few steps

THE CONCEPTS OF PRECISIATION AND COINTENSIVE PRECISIATION

31

WHAT IS PRECISE?

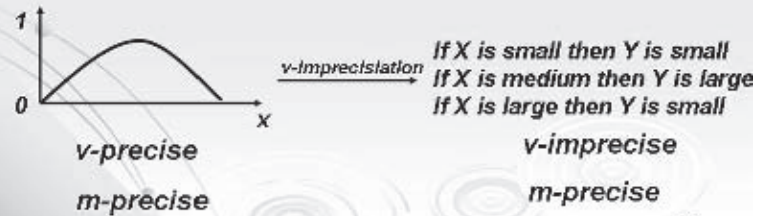
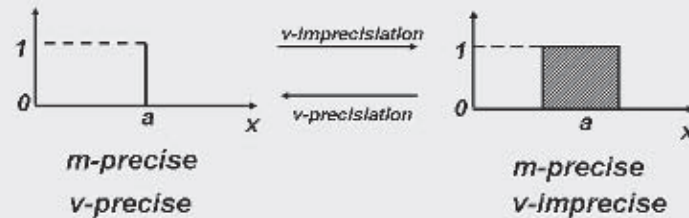


- p : X is a Gaussian random variable with mean m and variance σ^2 . m and σ^2 are precisely defined real numbers
- p is v -imprecise and m -precise
- p : X is in the interval $[a, b]$. a and b are precisely defined real numbers
- p is v -imprecise and m -precise

m -precise = mathematically well-defined

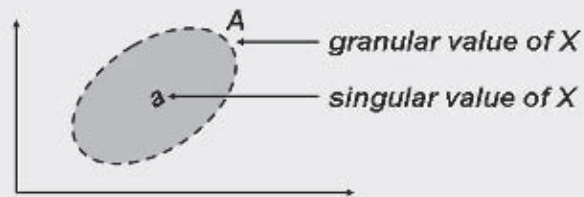
32

PRECISIATION AND IMPRECISIATION



33

DIGRESSION SINGULAR AND GRANULAR VALUATION



- singular: X is a \longrightarrow singleton
- granular: X is A \longleftarrow granule
- a granule is defined by a generalized constraint

example:

X : unemployment

a : 7.3%

A : high

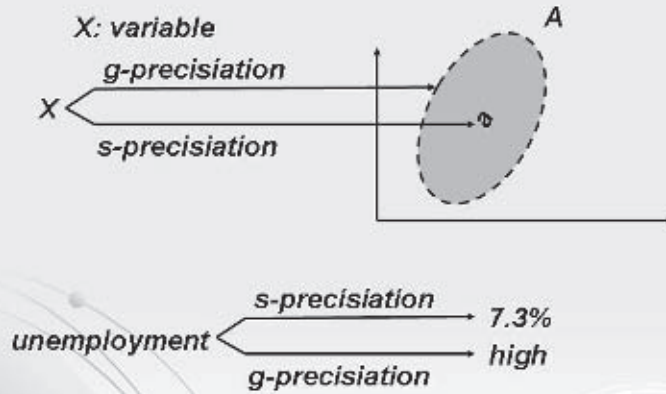
34

ATTRIBUTES OF A GRANULE

- *Probability measure*
- *Possibility measure*
- *Verity measure*
- ...

35

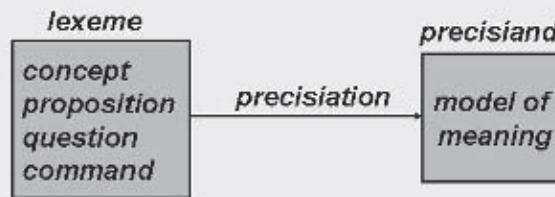
V-PRECISIATION



• *s-precision is used routinely in scientific theories and especially in probability theory*

36

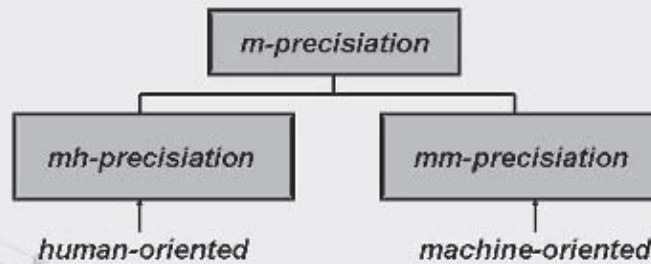
m-PRECISIATION AND COINTENSIVE PRECISIATION



- To be useful, precisiation must be cointensive
- Cointension = goodness of model of meaning

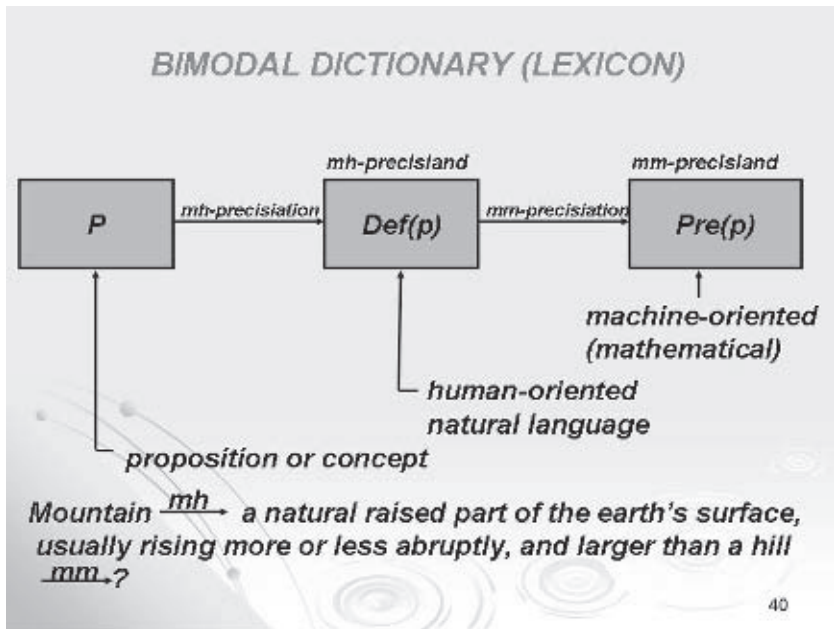
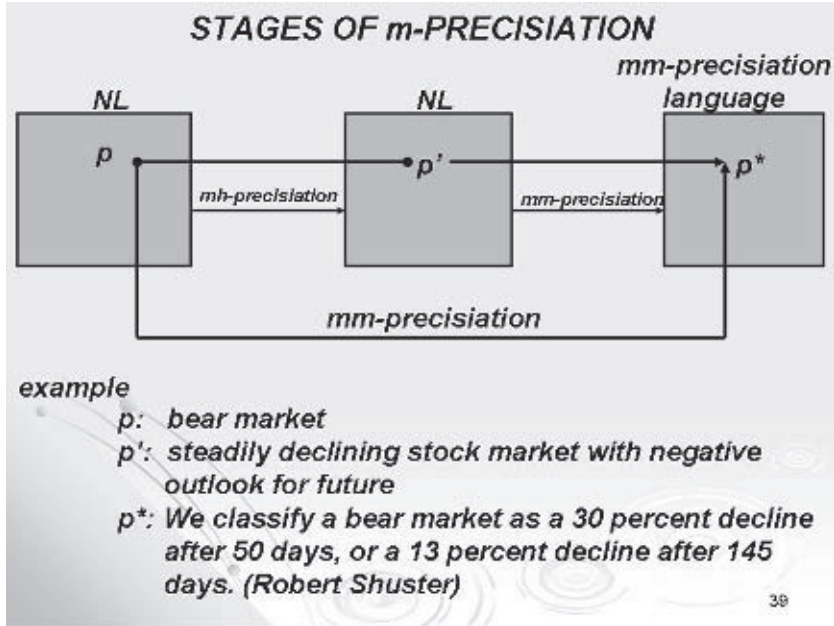
37

MODALITIES OF *m*-PRECISIATION



$p \xrightarrow{m\text{-precisiation}} \text{precisand of } p \text{ (Pre}(p))$

38



RATIONALE FOR IMPRECISIATION IMPRECISIATION PRINCIPLE

$p: X \text{ is } V$
└── value of X
└── variable

X : real-valued variable

X : (X_1, \dots, X_n)

X : function

X : relation

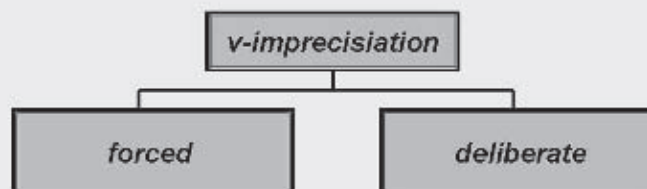
...

V is v -precise if V is a singleton (singular)

v -imprecisiation: singular \longrightarrow granular

41

v -IMPRECISIATION



forced: V is not known precisely

deliberate: V need not be known precisely

v -imprecisiation principle: Precision carries a cost. If there is a tolerance for imprecision, exploit it by employing v -imprecisiation to achieve lower cost, robustness, tractability, decision-relevance and higher level of confidence

42

CONFIDENCE

$p = X \text{ is } V$
 value of X is uncertain

$Con(p) = \text{Prob that } (X \text{ is } V) \text{ is true}$

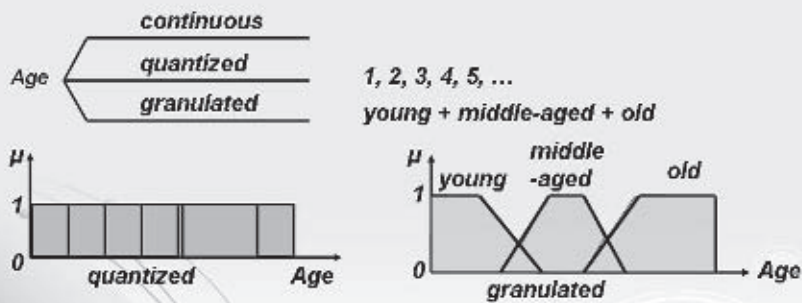
➤ Generally, v -imprecisiation of V serves to increase $Con(p)$

$Con(\text{Carol is young}) > Con(\text{Carol is 23})$
 lower specificity \longrightarrow higher confidence

43

GRANULATION REVISITED

➤ Granulation is a derivative of v -imprecisiation principle



$granulation = v\text{-imprecisiation} / m\text{-precision}$

44

KEY POINT

- *Granulation plays a key role in human cognition*
- *In human cognition, v-impreciation is followed by mh-preciation. Granulation is mh-preciation-based*
- *In fuzzy logic, v-impreciation is followed by mm-preciation. Granulation is mm-preciation-based*

• *mm-preciation-based granulation is a major contribution of fuzzy logic. No other logical system offers this capability*

KEY IDEAS

➤ *In TPM, a proposition, p , is precisiated by representing its meaning as a generalized constraint, $GC(p)$*

➤ *In TPM,*

preciation = m-preciation

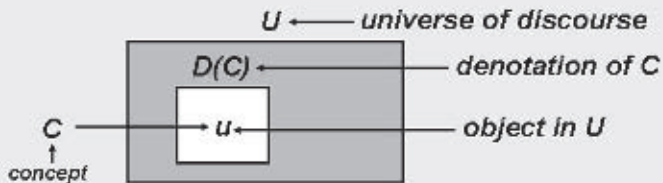
➤ *preciation of meaning does not imply precisiation of value*

➤ *A desideratum of precisiation is cointension*

➤ *Informally, p and q are cointensive if the intension (attribute-based-meaning) of p is approximately the same as the intension (attribute-based-meaning) of q*

46

DIGRESSION: EXTENSION VS. INTENSION



- $D(C)$ = set of all u 's which fit C
- **Extensional definition of $D(C)$:** instance-based, surface-structure-based
- **Intensional definition of $D(C)$:** attribute-based, deep-structure-based
- **Extension of $C = D(C)$ if $D(C)$ is defined extensionally**
- **Intension of C :** intensional definition of $D(C)$

Example:

young	{	$u = \text{Robert}$	extensional, Robert is young
		$u = \text{Robert}$	intensional, young \leftrightarrow Age < 25
			Robert is 23 \rightarrow Robert is young

47

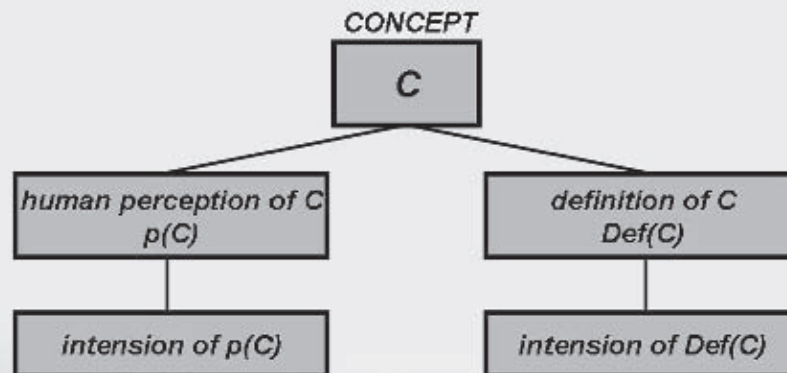
KEY POINT

- **Precisiation is necessary but not sufficient**
- **To serve its purpose, precisiation must be cointensive**

Cointensive precisiation is a key to mechanization of natural language understanding

48

COINTENSIVE DEFINITION



cointension: coincidence of the intended intension of the definiendum, C, and the intension of the definiens Def(C)

If C is a concept and Def(C) is its definition, then Def(C) is a valid definition if it is cointensive with C

49

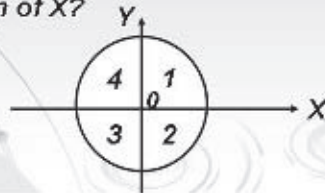
AN IMPORTANT IMPLICATION FOR SCIENCE

- *It is a deep-seated tradition in science to employ the conceptual structure of bivalent logic and probability theory as a basis for formulation of definitions of concepts. What is widely unrecognized is that, in reality, most concepts are fuzzy rather than bivalent, and that, in general, it is not possible to formulate a cointensive definition of a fuzzy concept within the conceptual structure of bivalent logic and probability theory.*

50

TEST PROBLEMS (PROBABILITY THEORY)

- *X is a real-valued random variable. What is known about X is: a(usually X is much larger than approximately a; b usually X is much smaller than approximately b, where a and b are real numbers with $a < b$. What is the expected value of X?*
- *X and Y are random variables. (X, Y) takes values in the unit circle. Prob(1) is approximately 0.1; Prob(2) is approximately 0.2; Prob(3) is approximately 0.3; Prob(4) is approximately 0.4. What is the marginal distribution of X?*



51

EXAMPLES OF FUZZY CONCEPTS WHOSE STANDARD, BIVALENT-LOGIC-BASED DEFINITIONS ARE NOT COINTENSIVE

- *stability*
- *causality*
- *relevance*
- *bear market*
- *recession*
- *mountain*
- *independence*
- *stationarity*
- *cluster*

52

EXAMPLE OF A BIVALENT DEFINITION OF A FUZZY CONCEPT

Robert Shuster

(Ned Davis Research)

We classify a bear market as a 30 percent decline after 50 days, or a 13 percent decline after 145 days.

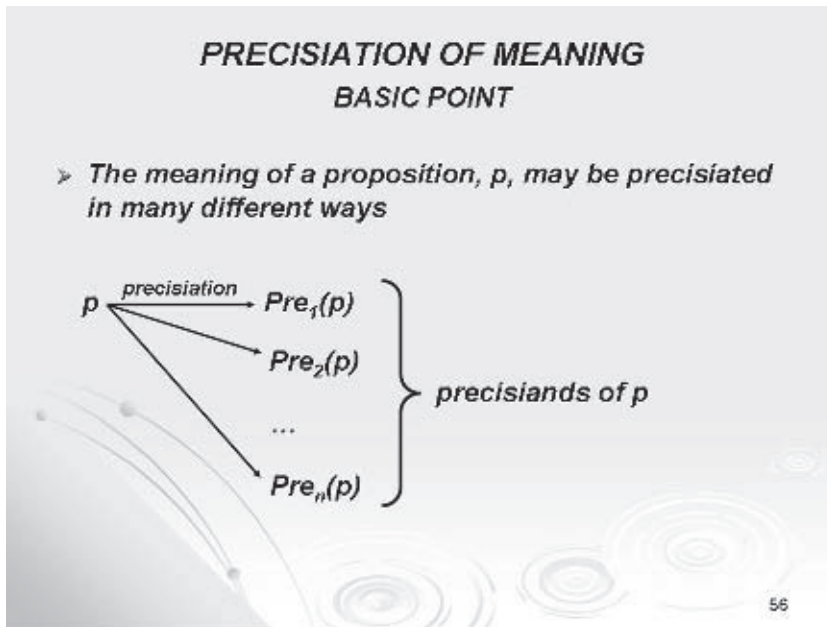
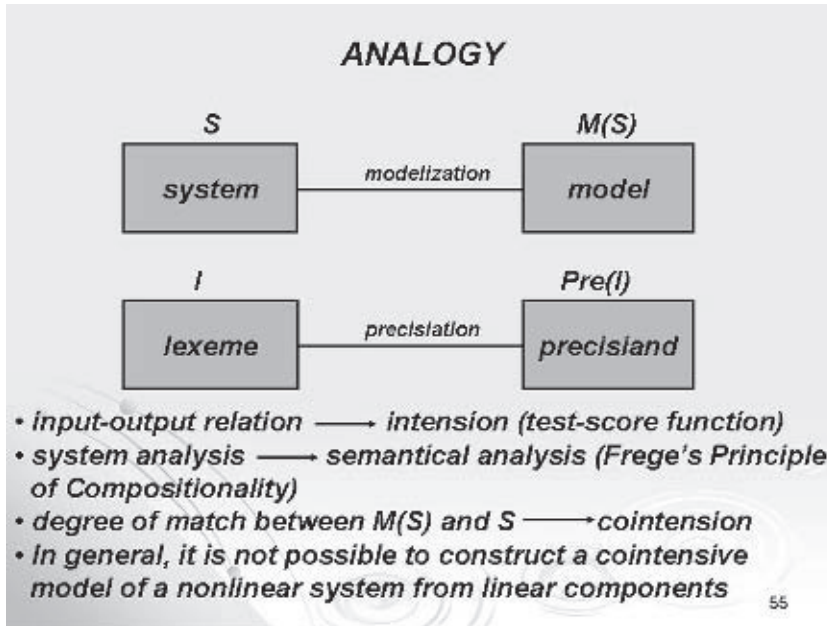
- ***A problem with this definition of bear market is that it is not cointensive***

53

COINTENSION

- ***In general, a cointensive definition of a fuzzy concept cannot be formulated within the conceptual structure of bivalent logic and probability theory***
- ***To understand the meaning of this cointension, an analogy is helpful***

54



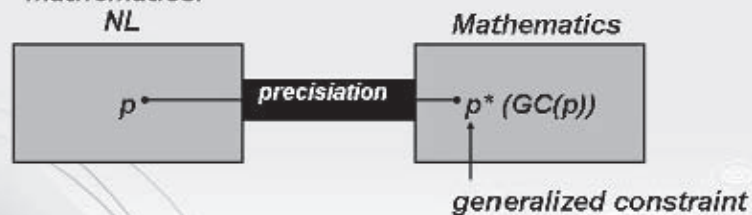
CHOICE OF PRECISIANDS

- In TPM, $Pre(p)$ is equated to $GC(p)$
- The concept of a generalized constraint opens the door to an unlimited enlargement of the number of ways in which a proposition may be precisiated
- An optimal choice is one which achieves a compromise between complexity and cointension

57

THE KEY IDEA

- In TPM, a proposition, p , is precisiated by expressing its meaning as a generalized constraint. In this sense, the concept of a generalized constraint serves as a bridge from natural languages to mathematics.



- The concept of a generalized constraint is the centerpiece of TPM

58

THE CONCEPT OF A GENERALIZED CONSTRAINT

59

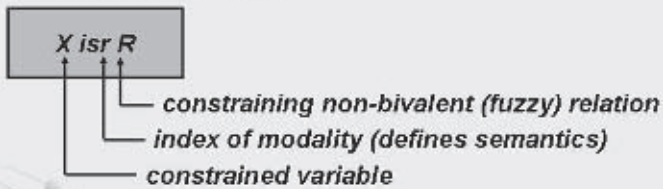
GENERALIZED CONSTRAINT (Zadeh 1986)

- **Bivalent constraint (hard, inelastic, categorical):**

$$X \in C$$

└─ constraining bivalent relation

- **Generalized constraint:**



$r: \in | = | \leq | \geq | < | \dots | \text{blank} | p | v | u | rs | fg | ps | \dots$

bivalent

primary

60

CONTINUED

• *constrained variable*

- *X is an n-ary variable, $X = (X_1, \dots, X_n)$*
- *X is a proposition, e.g., Leslie is tall*
- *X is a function of another variable: $X = f(Y)$*
- *X is conditioned on another variable, X/Y*
- *X has a structure, e.g., $X = \text{Location}$ (Residence(Carol))*
- *X is a generalized constraint, $X: Y \text{ is } R$*
- *X is a group variable. In this case, there is a group, $G[A]: (\text{Name}_1, \dots, \text{Name}_n)$, with each member of the group, Name_i , $i = 1, \dots, n$, associated with an attribute-value, A_i . A_i may be vector-valued. Symbolically*

$$G[A]: (\text{Name}_1/A_1 + \dots + \text{Name}_n/A_n)$$

Basically, X is a relation

61

SIMPLE EXAMPLES

- *“Check-out time is 1 pm,” is an instance of a generalized constraint on check-out time*
- *“Speed limit is 100km/h” is an instance of a generalized constraint on speed*
- *“Vera is a divorcee with two young children,” is an instance of a generalized constraint on Vera’s age*

62

GENERALIZED CONSTRAINT—MODALITY r

$$X \text{ is }_r R$$

- $r: =$ equality constraint: $X=R$ is abbreviation of $X \text{ is }=R$
 $r: \leq$ inequality constraint: $X \leq R$
 $r: c$ subethood constraint: $X \subset R$
 $r: \text{blank}$ possibilistic constraint; $X \text{ is } R$; R is the possibility distribution of X
 $r: v$ veristic constraint; $X \text{ is }_v R$; R is the verity distribution of X
 $r: p$ probabilistic constraint; $X \text{ is }_p R$; R is the probability distribution of X
Standard constraints: bivalent possibilistic, bivalent veristic and probabilistic

63

CONTINUED

- $r: bm$ bimodal constraint; X is a random variable; R is a bimodal distribution
 $r: rs$ random set constraint; $X \text{ is }_rs R$; R is the set-valued probability distribution of X
 $r: fg$ fuzzy graph constraint; $X \text{ is }_fg R$; X is a function and R is its fuzzy graph
 $r: u$ usuality constraint; $X \text{ is }_u R$ means usually ($X \text{ is } R$)
 $r: g$ group constraint; $X \text{ is }_g R$ means that R constrains the attribute-values of the group

64

PRIMARY GENERALIZED CONSTRAINTS

- **Possibilistic: X is R**
- **Probabilistic: X isp R**
- **Veristic: X isv R**

- **Primary constraints are formalizations of three basic perceptions: (a) perception of possibility; (b) perception of likelihood; and (c) perception of truth**

- **In this perspective, probability may be viewed as an attribute of perception of likelihood**

65

DIGRESSION—FOUNDATIONS OF SCIENTIFIC THEORIES

- **Scientific theories have their origins in perceptions**
- **A scientific theory may be viewed as a formalization of perceptions**

Basic perceptions and their formalizations

- **Likelihood —→ probability theory , probability is an attribute of likelihood**
- **Possibility —→ possibility theory**
- **Truth and consequence —→ logic**
- **Similarity —→ classification theory**
- **Causality —→ theory of causality**

66

EXAMPLES: VERISTIC

- **Robert is half German, quarter French and quarter Italian**
Ethnicity (Robert) isv $(0.5|German + 0.25|French + 0.25|Italian)$
- **Robert resided in London from 1985 to 1990**
Reside (Robert, London) isv $[1985, 1990]$

69

STANDARD CONSTRAINTS

- **Bivalent possibilistic: $X \varepsilon C$ (crisp set)**
- **Bivalent veristic: $Ver(p)$ is true or false**
- **Probabilistic: X isv R**
- **Standard constraints are instances of generalized constraints which underlie methods based on bivalent logic and probability theory**

70

GENERALIZED CONSTRAINT—SEMANTICS

A generalized constraint, GC, is associated with a test-score function, $ts(u)$, which associates with each object, u , to which the constraint is applicable, the degree to which u satisfies the constraint. Usually, $ts(u)$ is a point in the unit interval. However, if necessary, it may be an element of a semi-ring, a lattice, or more generally, a partially ordered set, or a bimodal distribution.

example: possibilistic constraint, X is R

$$X \text{ is } R \longrightarrow \text{Poss}(X=u) = \mu_R(u)$$

$$ts(u) = \mu_R(u)$$

71

TEST-SCORE FUNCTION

- $GC(X)$: generalized constraint on X
- X takes values in $U = \{u\}$
- test-score function $ts(u)$: degree to which u satisfies GC
- $ts(u)$ may be defined (a) directly (extensionally) as a function of u ; or indirectly (intensionally) as a function of attributes of u

intensional definition = attribute-based definition

- example (a) Andrea is tall 0.9
(b) Andrea's height is 175cm; $\mu_{\text{tall}}(175) = 0.9$;
Andrea is 0.9 tall

72

CONSTRAINT QUALIFICATION

➤ $p \text{ isr } R$ means r -value of p is R

in particular

$p \text{ isp } R \longrightarrow \text{Prob}(p) \text{ is } R$ (probability qualification)

$p \text{ isv } R \longrightarrow \text{Tr}(p) \text{ is } R$ (truth (verity) qualification)

$p \text{ is } R \longrightarrow \text{Poss}(p) \text{ is } R$ (possibility qualification)

examples

$(X \text{ is small}) \text{ isp likely} \longrightarrow \text{Prob}\{X \text{ is small}\} \text{ is likely}$

$(X \text{ is small}) \text{ isv very true} \longrightarrow \text{Ver}\{X \text{ is small}\} \text{ is very true}$

$(X \text{ is } R) \longrightarrow \text{Prob}\{X \text{ is } R\} \text{ is usually}$

73

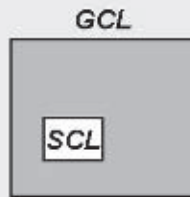
GENERALIZED CONSTRAINT LANGUAGE (GCL)

- GCL is an abstract language
 - GCL is generated by combination, qualification and propagation of generalized constraints
 - examples of elements of GCL
 - $(X \text{ isp } R)$ and $(X, Y) \text{ is } S$
 - $(X \text{ isr } R)$ is unlikely) and $(X \text{ iss } S)$ is likely
 - If X is A then Y is B
 - the language of fuzzy if-then rules is a sublanguage of GCL
- deduction= generalized constraint propagation
- the language of fuzzy if-then rules is a sublanguage of GCL

74

STANDARD CONSTRAINT LANGUAGE (SCL)

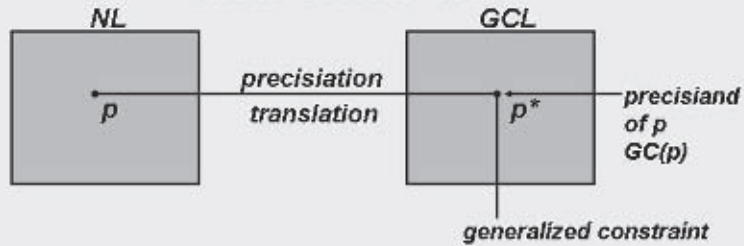
- SCL is a sublanguage of GCL



- SCL is generated by combination, qualification and propagation of standard constraints
- The language of differential equations is a sublanguage of SCL

75

PRECISION = TRANSLATION INTO GCL BASIC STRUCTURE



annotation

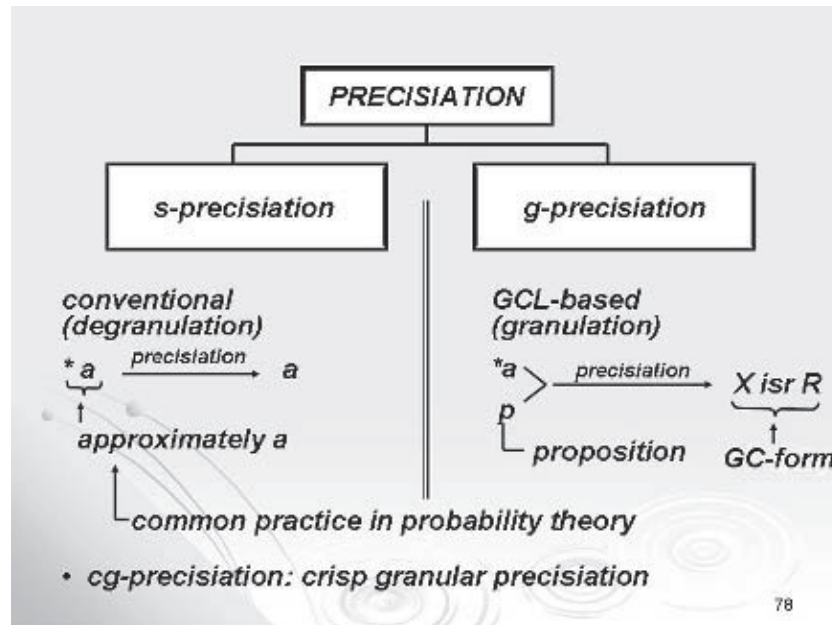
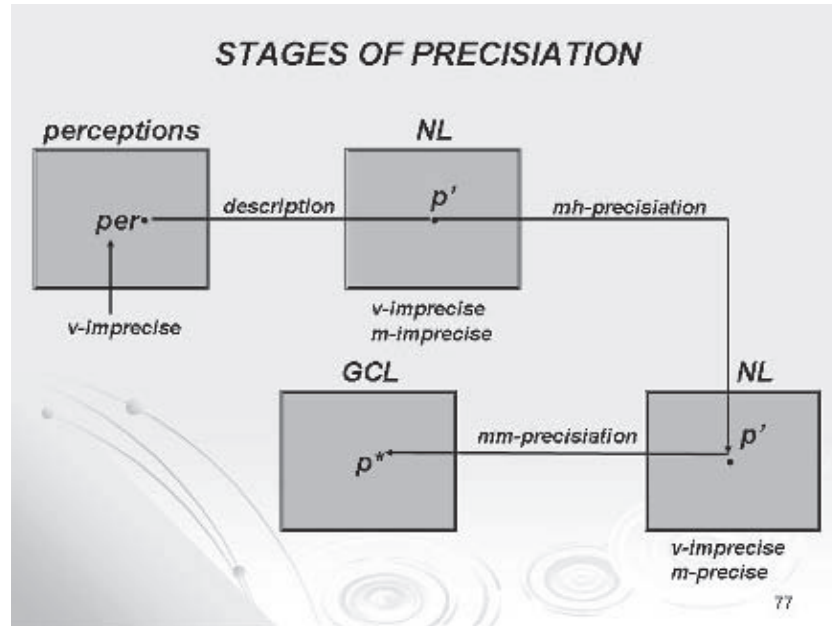
$p \rightarrow X/A \text{ isr } R/B \leftarrow \text{GC-form of } p$

example

p : Carol lives in a small city near San Francisco

$X/\text{Location}(\text{Residence}(\text{Carol})) \text{ is } R/\text{NEAR}[\text{City}] \wedge \text{SMALL}[\text{City}]$

76

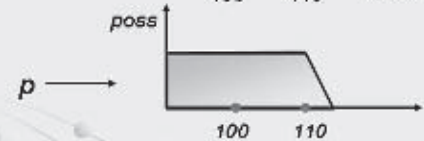


EXAMPLE

- p : Speed limit is 100 km/h



cg-precisiation
 $r = \text{blank (possibilistic)}$



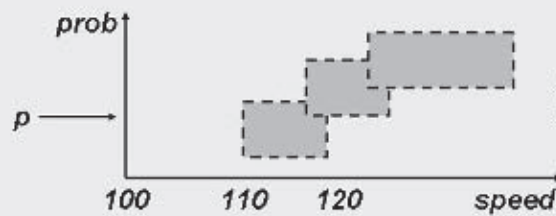
g-precisiation
 $r = \text{blank (possibilistic)}$



g-precisiation
 $r = p \text{ (probabilistic)}$

81

CONTINUED



g-precisiation
 $r = \text{bm (bimodal)}$

If Speed is less than *110, Prob(Ticket) is low

If Speed is between *110 and *120, Prob(Ticket) is medium

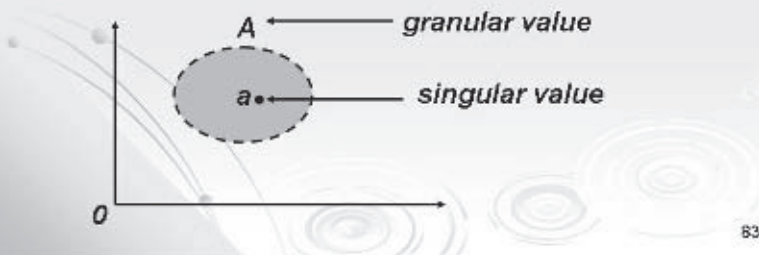
If Speed is greater than *120, Prob(Ticket) is high

82

THE CONCEPT OF GRANULAR VALUE

X is a ← **singular value**
↳ *singleton*

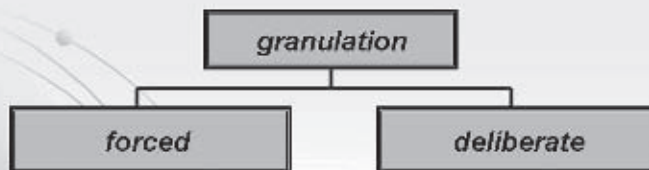
X is A ← **granular value**
↳ *granule*



CONTINUED

➤ A is defined as a generalized constraint
example

X is *small* ← **granular value**
↳ *fuzzy set*



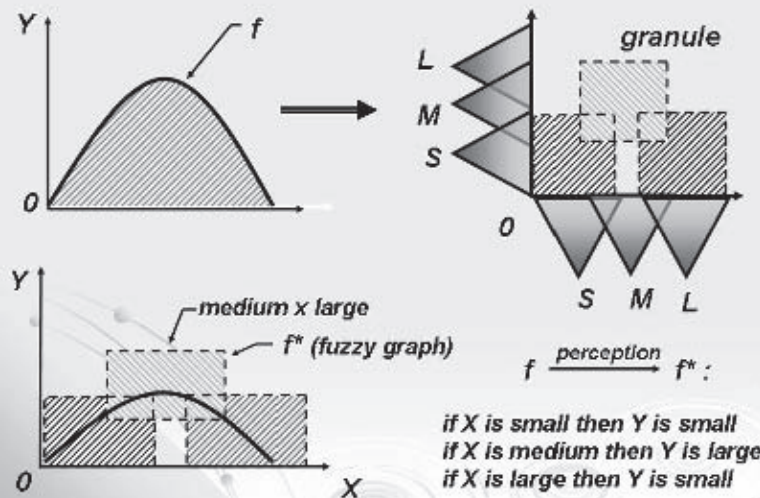
84

GRANULAR COMPUTING (GrC)

- *The objects of computation in granular computing are granular values of variables and parameters*
- *Granular computing has a position of centrality in fuzzy logic*
- *Granular computing plays a key role in precisiation and deduction*

85

GRANULAR DEFINITION OF A FUNCTION



86

PRECISIATION AND DEDUCTION

- *p*: most Swedes are tall
- p**: $\Sigma \text{Count}(\text{tall.Swedes}/\text{Swedes})$ is most

further precisiation

h(u): height density function
h(u)du: fraction of Swedes whose height is in $[u, u+du]$, $a \leq u \leq b$

$$\int_a^b h(u)du = 1$$

87

CONTINUED

- $\Sigma \text{Count}(\text{tall.Swedes}/\text{Swedes}) = \int_a^b h(u)\mu_{\text{tall}}(u)du$

- constraint on *h*

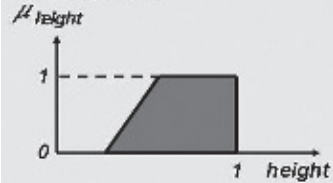
- $\int_a^b h(u)\mu_{\text{tall}}(u)du$ is most

- $\pi(h) = \mu_{\text{most}}(\int_a^b h(u)\mu_{\text{tall}}(u)du)$

88

CALIBRATION / PRECISIATION

- calibration



- precision

most Swedes are tall $\longrightarrow \pi(h) = \mu_{most}(\int_a^b h(u)\mu_{tall}(u)du)$

h : count density function

- Frege principle of compositionality—precisiated version
- precision of a proposition requires precisiations (calibrations) of its constituents

89

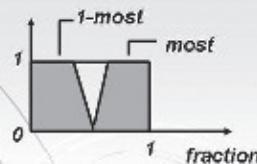
DEDUCTION

q : How many Swedes are not tall

q^+ : $\int_a^b h(u)\mu_{not_tall}(u)du$ is ? Q

solution: $\int_a^b h(u)(1 - \mu_{tall}(u))du =$

$$\int_a^b h(u)du - \int_a^b h(u)\mu_{tall}(u)du = 1 - most$$



90

DEDUCTION

q: How many Swedes are short

q:* $\int_a^b h(u)\mu_{short}(u)du$ is ? Q

solution: $\int_a^b h(u)\mu_{tall}(u)du$ is most

$\int_a^b h(u)\mu_{short}(u)du$ is ? Q

extension principle

$$\mu_q(v) = \sup_u (\mu_{most}(\int_a^b h(u)\mu_{tall}(u)du))$$

subject to

$$v = \int_a^b h(u)\mu_{short}(u)du$$

91

CONTINUED

q: What is the average height of Swedes?

q:* $\int_a^b h(u)udu$ is ? Q

solution: $\int_a^b h(u)\mu_{tall}(u)du$ is most

$\int_a^b h(u)udu$ is ? Q

extension principle

$$\mu_q(v) = \sup_h (\mu_{most}(\int_a^b h(u)\mu_{tall}(u)du))$$

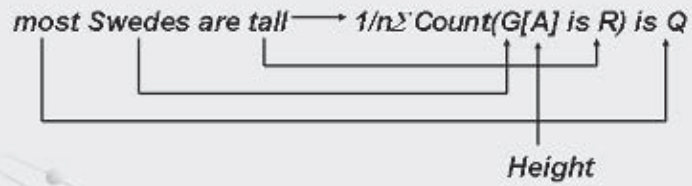
subject to

$$v = \int_a^b h(u)udu$$

92

PROTOFORMAL DEDUCTION

Example:



93

PROTOFORMAL DEDUCTION RULE

$$\frac{1/n \sum \text{Count}(G[A] \text{ is } R) \text{ is } Q}{1/n \sum \text{Count}(G[A] \text{ is } S) \text{ is } T}$$

$$\frac{\sum \mu_R(A) \text{ is } Q}{\sum \mu_S(A) \text{ is } T}$$

$$\mu_T(v) = \sup_{A_1, \dots, A_n} (\mu_Q(\sum_i \mu_R(A_i)))$$

subject to

$$v = \sum \mu_S(A_i)$$

94

PROTOFORM LANGUAGE AND PROTOFORMAL DEDUCTION

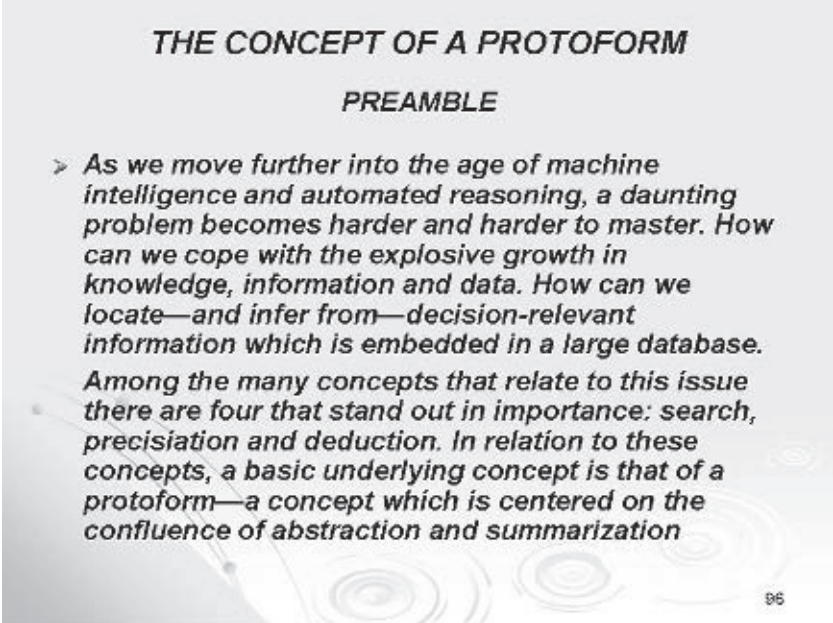
PFL



95

THE CONCEPT OF A PROTOFORM

PREAMBLE

- *As we move further into the age of machine intelligence and automated reasoning, a daunting problem becomes harder and harder to master. How can we cope with the explosive growth in knowledge, information and data. How can we locate—and infer from—decision-relevant information which is embedded in a large database. Among the many concepts that relate to this issue there are four that stand out in importance: search, precision and deduction. In relation to these concepts, a basic underlying concept is that of a protoform—a concept which is centered on the confluence of abstraction and summarization*
- 

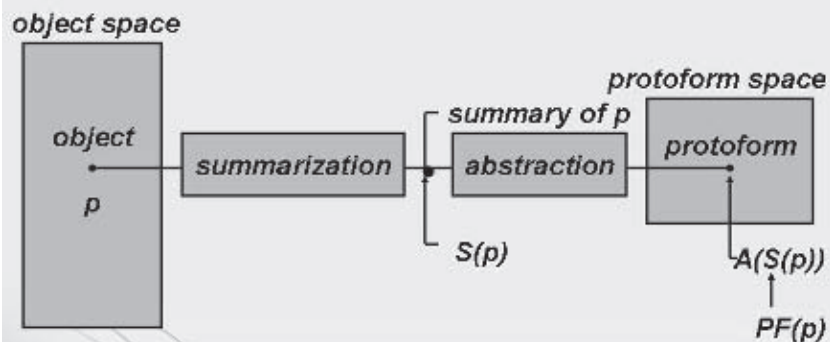
96

WHAT IS A PROTOFORM?

- *protoform = abbreviation of prototypical form*
- *informally, a protoform, A, of an object, B, written as $A=PF(B)$, is an abstracted summary of B*
- *usually, B is lexical entity such as proposition, question, command, scenario, decision problem, etc*
- *more generally, B may be a relation, system, geometrical form or an object of arbitrary complexity*
- *usually, A is a symbolic expression, but, like B, it may be a complex object*
- *the primary function of $PF(B)$ is to place in evidence the deep semantic structure of B*

97

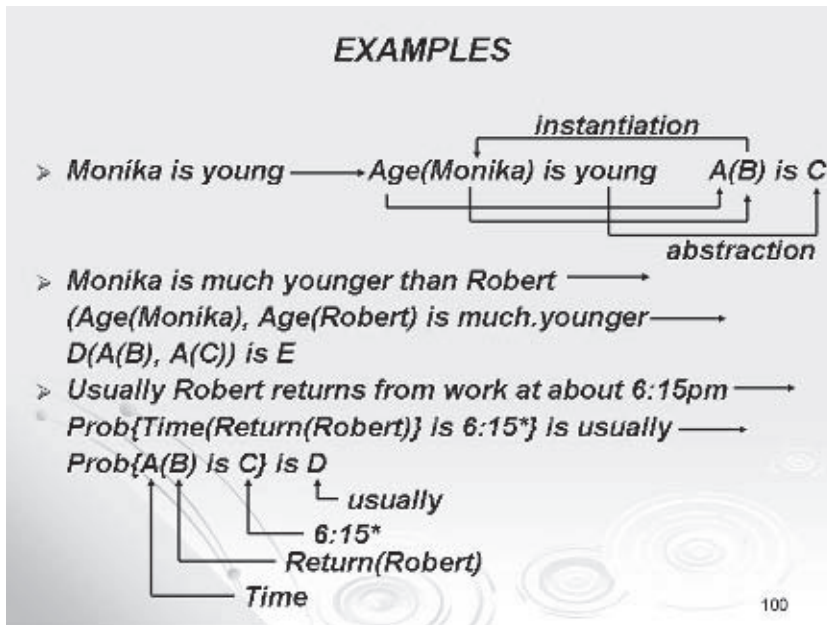
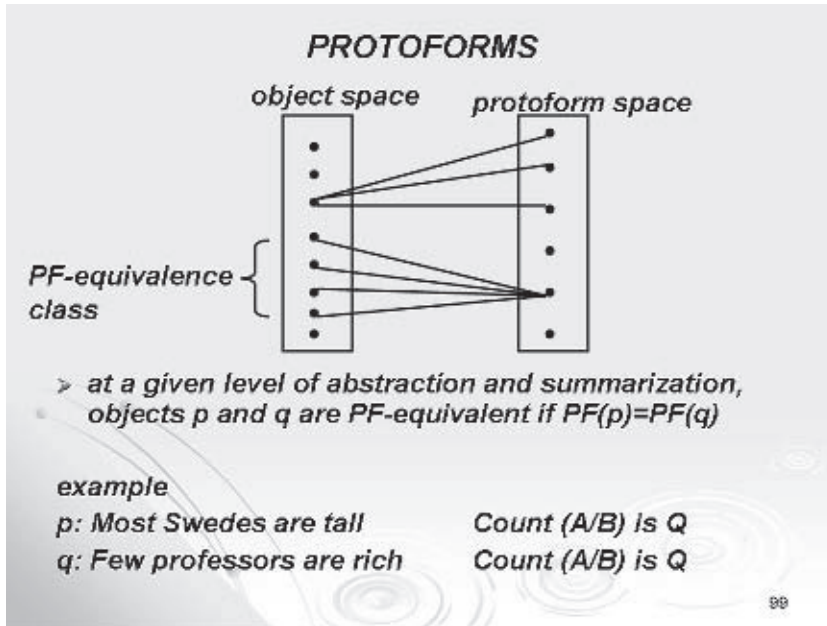
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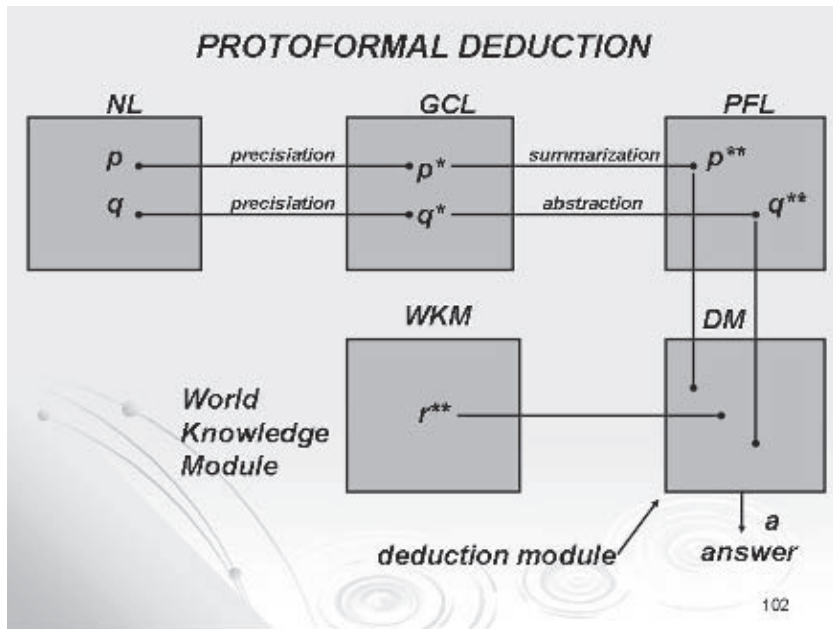
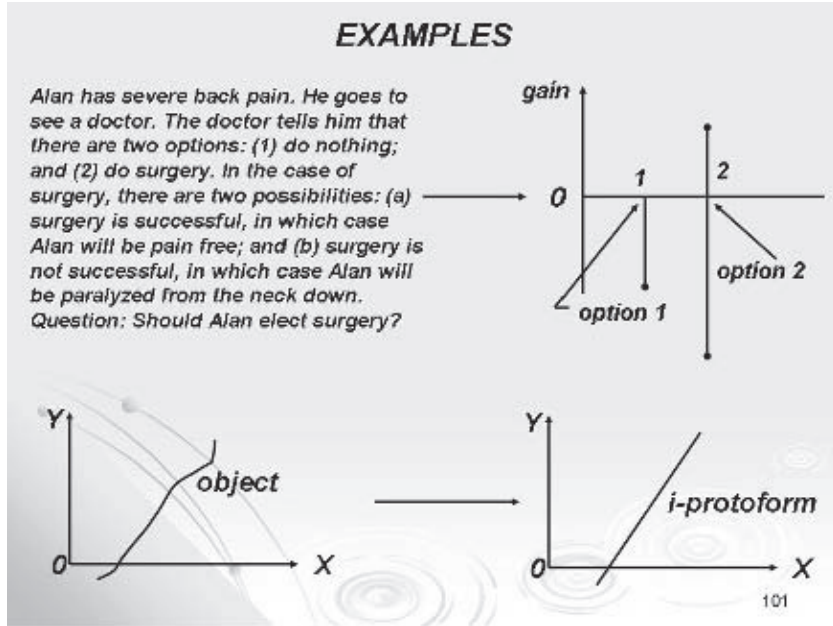


$PF(p)$: abstracted summary of p
deep structure of p

- protoform equivalence
- protoform similarity

98





PROTOFORMAL DEDUCTION

▸ Rules of deduction in the Deduction Database (DDB) are protoformal
 examples: (a) compositional rule of inference

$$\frac{X \text{ is } A}{(X, Y) \text{ is } B}$$

$$Y \text{ is } A \cdot B$$

$$\mu_B(v) = \sup(\mu_A(u) \wedge \mu_B(u, v))$$

symbolic
↑
computational

(b) extension principle

$$\frac{X \text{ is } A}{Y = f(X)}$$

$$Y = f(A)$$

$$\mu_Y(v) = \sup_u(\mu_A(u))$$

Subject to: $v = f(u)$

symbolic
↑
computational
103

RULES OF DEDUCTION

▸ Rules of deduction are basically rules governing generalized constraint propagation
 ▸ The principal rule of deduction is the extension principle

$$\frac{X \text{ is } A}{f(X) \text{ is } B}$$

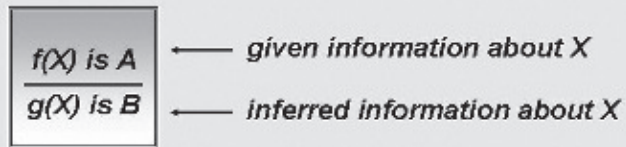
$$\mu_B(v) = \sup_u(\mu_A(u))$$

Subject to: $v = f(u)$

symbolic
↑
computational
104

GENERALIZATIONS OF THE EXTENSION PRINCIPLE

information = constraint on a variable



$$\mu_B(\mathbf{v}) = \sup_u (\mu_A(f(u)))$$

subject to: $\mathbf{v} = g(u)$

105

CONTINUED

$$\frac{f(X_1, \dots, X_n) \text{ is } A}{g(X_1, \dots, X_n) \text{ is } B}$$

$$\mu_B(\mathbf{v}) = \sup_u (\mu_A(f(u)))$$

Subject to: $\mathbf{v} = g(u)$

$$\frac{\begin{array}{l} (X_1, \dots, X_n) \text{ is } A \\ g_j(X_1, \dots, X_n) \text{ is } Y_j, \quad j=1, \dots, n \\ (Y_1, \dots, Y_n) \text{ is } B \end{array}}$$

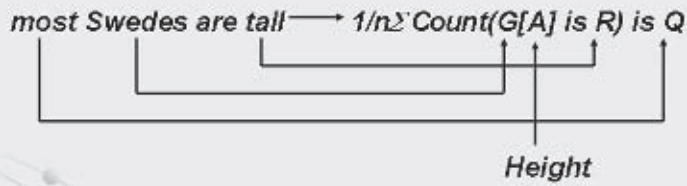
$$\mu_B(\mathbf{v}) = \sup_u (\mu_A(f(u)))$$

Subject to: $\mathbf{v} = g(u)$
 $j = 1, \dots, n$

106

PROTOFORMAL DEDUCTION

Example:



107

PROTOFORMAL DEDUCTION RULE

$$\frac{1/n \sum \text{Count}(G[A] \text{ is } R) \text{ is } Q}{1/n \sum \text{Count}(G[A] \text{ is } S) \text{ is } T}$$

$$\frac{\sum \mu_R(A_j) \text{ is } Q}{\sum \mu_S(A_j) \text{ is } T}$$

$$\mu_T(v) = \sup_{A_1, \dots, A_n} (\mu_Q(\sum_i \mu_R(A_i)))$$

subject to

$$v = \sum \mu_S(A_j)$$

108

EXAMPLE OF DEDUCTION

p: Most Swedes are much taller than most Italians

q: What is the difference in the average height of Swedes and Italians?

TPM-based solution

Step 1. *precisiation: translation of p into GCL*

$S = \{S_1, \dots, S_m\}$: population of Swedes

$I = \{I_1, \dots, I_n\}$: population of Italians

$g_i = \text{height of } S_i$, $g = (g_1, \dots, g_m)$

$h_j = \text{height of } I_j$, $h = (h_1, \dots, h_n)$

$\mu_{ij} = \mu_{\text{much.taller}}(g_i, h_j) = \text{degree to which } S_i \text{ is much taller than } I_j$

109

CONTINUED

$r_i = \frac{1}{n} \sum_j \mu_{ij}$ = Relative Σ Count of Italians in relation to whom S_i is much taller

$t_i = \mu_{\text{most}}(r_i)$ = degree to which S_i is much taller than most Italians

$v = \frac{1}{m} \sum_i t_i$ = Relative Σ Count of Swedes who are much taller than most Italians

$ts(g, h) = \mu_{\text{most}}(v)$

$p \rightarrow$ generalized constraint on S and I

$q: d = \frac{1}{m} \sum_i g_i - \frac{1}{n} \sum_j h_j$

110

CONTINUED

Step 2. Deduction via extension principle

$$\mu_q(d) = \sup_{g,h} ts(g,h)$$

subject to

$$d = \frac{1}{m} \sum_i g_i - \frac{1}{n} \sum_j h_j$$

111

DEDUCTION PRINCIPLE

- Point of departure: question, q
- Data: $D = (X_r/u_r, \dots, X_r/u_r)$

u_i is a generic value of X_i

- $Ans(q)$: answer to q
- If we knew the values of the X_r, u_r, \dots, u_r , we could express $Ans(q)$ as a function of the u_i

$$Ans(q) = g(u_1, \dots, u_n) \quad u = (u_1, \dots, u_n)$$

- Our information about the $u_i, i(u_1, \dots, u_n)$ is a generalized constraint on the u_i . The constraint is defined by its test-score function

$$f(u) = f(u_1, \dots, u_n)$$

112

CONTINUED

➤ Use the extension principle

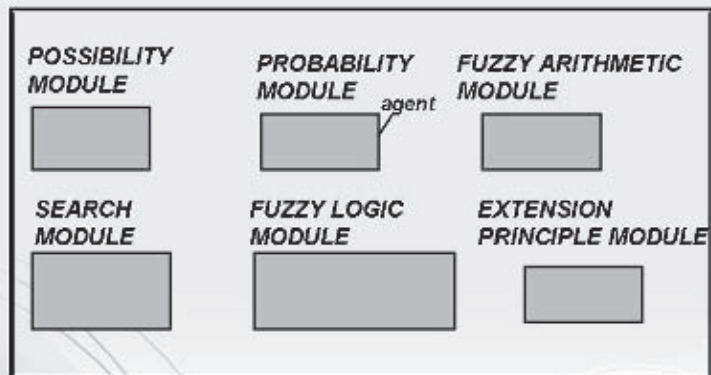
$$\mu_{Ans(q)}(v) = \sup_u (ts(u))$$

subject to

$$v = g(u)$$

113

MODULAR DEDUCTION DATABASE



114

PROBABILITY MODULE

115

THE CONCEPT OF BIMODAL DISTRIBUTION (ZADEH 1979)

X isbm R

└─ bimodal distribution
└─ random variable

- A bimodal distribution is a collection of ordered pairs of the form

$$R: \{(P_1, A_1), \dots, (P_n, A_n)\}$$

- or equivalently

$$\sum_i (P_i | A_i), \quad i=1, \dots, n$$

where the P_i are fuzzy probabilities and the A_i are fuzzy sets

116

CONTINUED

Special cases:

1. *The P_i are crisp; the A_i are fuzzy*
2. *The P_i are fuzzy; the A_i are crisp*
3. *The P_i are crisp; the A_i are crisp*

➤ *The Dempster-Shafer theory of evidence is basically a theory of crisp bimodal distributions*

117

EXAMPLE: FORD STOCK

➤ *I am considering buying Ford stock. I ask my stockbroker, "What is your perception of the near-term prospects for Ford stock?" He tells me, "A moderate decline is very likely; a steep decline is unlikely; and a moderate gain is not likely." My question is: What is the probability of a large gain?*

118

CONTINUED

- *Information provided by my stockbroker may be represented as a collection of ordered pairs:*

*Price: ((unlikely, steep.decline),
(very.likely, moderate.decline),
(not.likely, moderate.gain))*

In this collection, the second element of an ordered pair is a fuzzy event or, equivalently, a possibility distribution, and the first element is a fuzzy probability.

- *The importance of the concept of a bimodal distribution derives from the fact that in the context of human-centric systems, most probability distributions are bimodal*

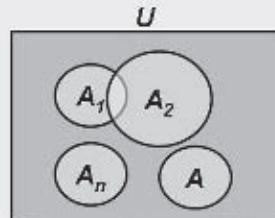
119

BIMODAL DISTRIBUTIONS

- *Bimodal distributions can assume a variety of forms. The principal types are Type 1, Type 2 and Type 3. Type 1, 2 and 3 bimodal distributions have a common framework but differ in important detail*

120

BIMODAL DISTRIBUTIONS (Type 1, 2, 3)



- **Type 1 (default):** X is a random variable taking values in U
 A_1, \dots, A_n, A are events (fuzzy sets)
 $p_i = \text{Prob}(X \text{ is } A_i)$, $i = 1, \dots, n$
 $\sum_i p_i$ is unconstrained
 p_i is P_i (granular probability)

BMD: bimodal distribution: $((P_1, A_1), \dots, (P_n, A_n))$
 $X \text{ isbm } (P_1 \setminus A_1 + \dots + P_n \setminus A_n)$

Problem: What is the probability, p , of A ? In general, this probability is fuzzy-set-valued, that is, granular

121

CONTINUED

- **Type 2 (fuzzy random set):** X is a fuzzy-set-valued random variable with values A_1, \dots, A_n (fuzzy sets)
 $p_i = \text{Prob}(X = A_i)$, $i = 1, \dots, n$

BMD: $X \text{ isrs } (p_1 \setminus A_1 + \dots + p_n \setminus A_n)$
 $\sum_i p_i = 1$

Problem: What is the probability, p , of A ? p is not definable. What are definable are (a) the expected value of the conditional possibility of A given BMD, and (b) the expected value of the conditional necessity of A given BMD

122

CONTINUED

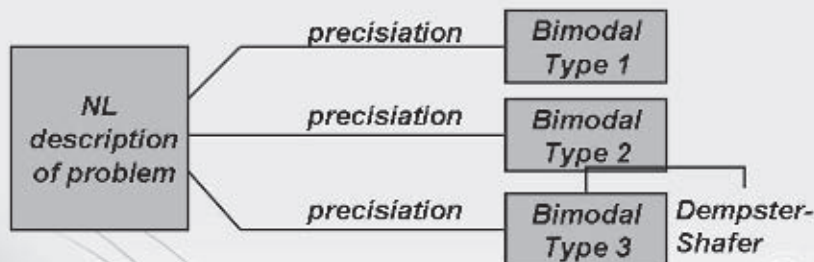
- **Type 3 (augmented random set; Dempster-Shafer):**
 X is a set-valued random variable taking the values X_1, \dots, X_n with respective probabilities p_1, \dots, p_n
 - Y_i is a random variable taking values in $A_i, i = 1, \dots, n$
 - Probability distribution of Y_i in $A_i, i = 1, \dots, n$, is not specified
 - X is $p(\{p_1 \setminus X_1 + \dots + p_n \setminus X_n\})$

Problem: What is the probability, p , that Y_1 or $Y_2 \dots$ or Y_n is in A ? Because probability distributions of the Y_i in the A_i are not specified, p is interval-valued. What is important to note is that the concepts of upper and lower probabilities break down when the A_i are fuzzy sets

123

IS DEMPSTER SHAFER COINTENSIVE?

- In applying Dempster Shafer theory, it is important to check on whether the data fit Type 3 model.

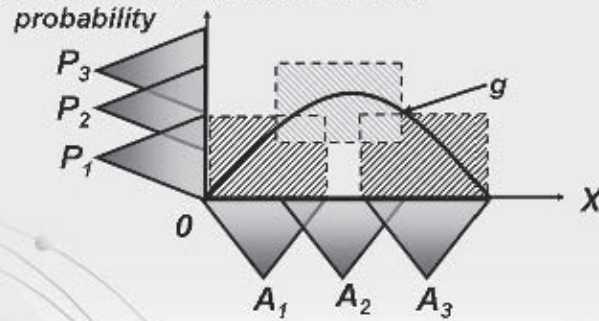


- **Caveat:** In many cases the cointensive (well-fitting) precisian (model) of a problem statement is bimodal distribution of Type 1 rather than Type 3 (Dempster-Shafer)

124

**BASIC BIMODAL DISTRIBUTION (BMD) (Type 1)
(PERCEPTION-BASED PROBABILITY DISTRIBUTION)**

X is a real-valued random variable

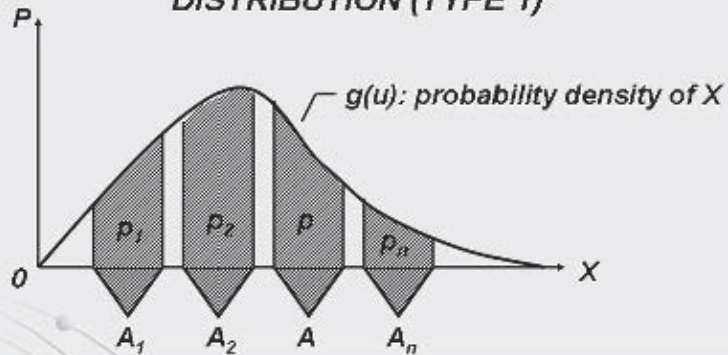


$$\text{BMD: } P(X) = P_{i(1)} \setminus A_1 + P_{i(2)} \setminus A_2 + P_{i(3)} \setminus A_3$$

Prob {X is A_i } is $P_{i(0)}$

$P(X) = \text{low} \setminus \text{small} + \text{high} \setminus \text{medium} + \text{low} \setminus \text{large}$ 125

INTERPOLATION OF A BASIC BIMODAL DISTRIBUTION (TYPE 1)



p_i is P_i : granular value of $p_i, i=1, \dots, n$

$(P_i, A_i), i=1, \dots, n$ are given

A is given

$(?P, A)$

126

INTERPOLATION MODULE AND PROBABILITY MODULE

$$\frac{\text{Prob } \{X \text{ is } A_i\} \text{ is } P_i, \quad i = 1, \dots, n}{\text{Prob } \{X \text{ is } A\} \text{ is } Q}$$

$$\mu_Q(v) = \sup_g (\mu_{P_1}(\int_U \mu_{A_1}(u)g(u)du) \wedge \dots \wedge$$

$$\mu_{P_n}(\int_U \mu_{A_n}(u)g(u)du))$$

subject to

$$U = \int_U \mu_A(u)g(u)du$$

127

OPERATIONS ON BIMODAL DISTRIBUTIONS

$P(X)$ defines possibility distribution of g

$$\pi(g) = \mu_{P_1}(\int_U \mu_{A_1}(u)g(u)du) \wedge \dots \wedge \mu_{P_n}(\int_U \mu_{A_n}(u)g(u)du)$$

problem

a) what is the expected value of X

128

EXPECTED VALUE OF A BIMODAL DISTRIBUTION

$$E(P^*) = \int_U ug(u)du = f(g)$$

Extension Principle

$$\mu_{E(P^*)}(v) = \sup_g (\mu_{P_1}(\int_U \mu_{A_1}(u)g(u)du) \wedge \dots \wedge \mu_{P_n}(\int_U \mu_{A_n}(u)g(u)du))$$

subject to: $v = \int_U ug(u)du$

129

SUMMATION

- *addition of significant question-answering capability to search engines is a complex, open-ended problem*
- *incremental progress, but not much more, is achievable through the use of bivalent-logic-based methods*
- *to achieve significant progress, it is imperative to develop and employ new methods based on computing with words, protoform theory, precisiated natural language and computational theory of precisiation of meaning*
- *The centerpiece of new methods is the concept of a generalized constraint*

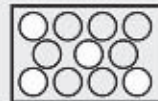
130

APPENDIX

131

DEDUCTION THE BALLS-IN-BOX PROBLEM

Version 1. Measurement-based



A flat box contains a layer of black and white balls. You can see the balls and are allowed as much time as you need to count them

- q_1 : *What is the number of white balls?*
- q_2 : *What is the probability that a ball drawn at random is white?*
- q_1 and q_2 *remain the same in the next version*

132

DEDUCTION

Version 2. Perception-based

You are allowed n seconds to look at the box. n seconds is not enough to allow you to count the balls

You describe your perceptions in a natural language

p_1 : there are about 20 balls

p_2 : most are black

p_3 : there are several times as many black balls as white balls

PT's solution?

133

MEASUREMENT-BASED

version 1

- *a box contains 20 black and white balls*
- *over seventy percent are black*
- *there are three times as many black balls as white balls*
- *what is the number of white balls?*
- *what is the probability that a ball picked at random is white?*

PERCEPTION-BASED

version 2

- *a box contains about 20 black and white balls*
- *most are black*
- *there are several times as many black balls as white balls*
- *what is the number of white balls?*
- *what is the probability that a ball drawn at random is white?*

134

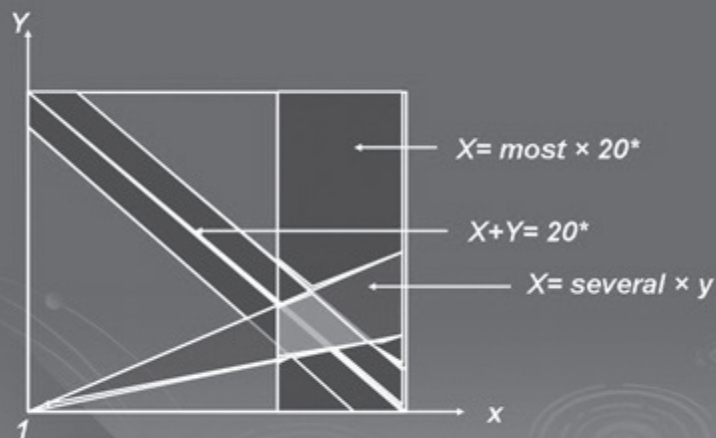
COMPUTATION (version 2)

➤ *measurement-based*
 $X = \text{number of black balls}$
 $Y = \text{number of white balls}$
 $X \geq 0.7 \cdot 20 = 14$
 $X + Y = 20$
 $X = 3Y$
 $X = 15 ; Y = 5$
 $p = 5/20 = .25$

➤ *perception-based*
 $X = \text{number of black balls}$
 $Y = \text{number of white balls}$
 $X = \text{most} \times 20^*$
 $X = \text{several} \times Y$
 $X + Y = 20^*$
 $P = Y/N$

135

FUZZY INTEGER PROGRAMMING



136

RELEVANCE, REDUNDANCE AND DELETABILITY

DECISION TABLE

Name	A_1	A_j	A_n	D
$Name_1$	a_{11}	a_{1j}	a_{1n}	d_1
.
$Name_k$	a_{k1}	a_{kj}	a_{kn}	d_1
$Name_{k+1}$	$a_{k+1, 1}$	$a_{k+1, j}$	$a_{k+1, n}$	d_2
.
$Name_l$	a_{l1}	a_{lj}	a_{ln}	d_l
.
$Name_n$	a_{n1}	a_{nj}	a_{nn}	d_r

A_j : j th symptom

a_y : value of j th symptom of Name

D : diagnosis

137

REDUNDANCE \longrightarrow DELETABILITY

Name	A_1	A_j	A_n	D
.
$Name_r$	a_{r1}	*	a_{rn}	d_2
.

A_j is conditionally redundant for $Name_r$, A_1 is a_{r1} , A_n is a_{rn}
if D is d_2 for all possible values of A_j in *

A_j is redundant if it is conditionally redundant for all values of Name

• compactification algorithm (Zadeh, 1976); Quine-McCluskey algorithm

138

RELEVANCE

D is ? d if A_j is a_{rj}

constraint on A_j induces a constraint on D
 example: (blood pressure is high) constrains D
 $(A_j \text{ is } a_{rj})$ is uninformative if D is unconstrained

A_j is irrelevant if it A_j is uninformative for all a_{rj}

irrelevance \rightarrow deletability

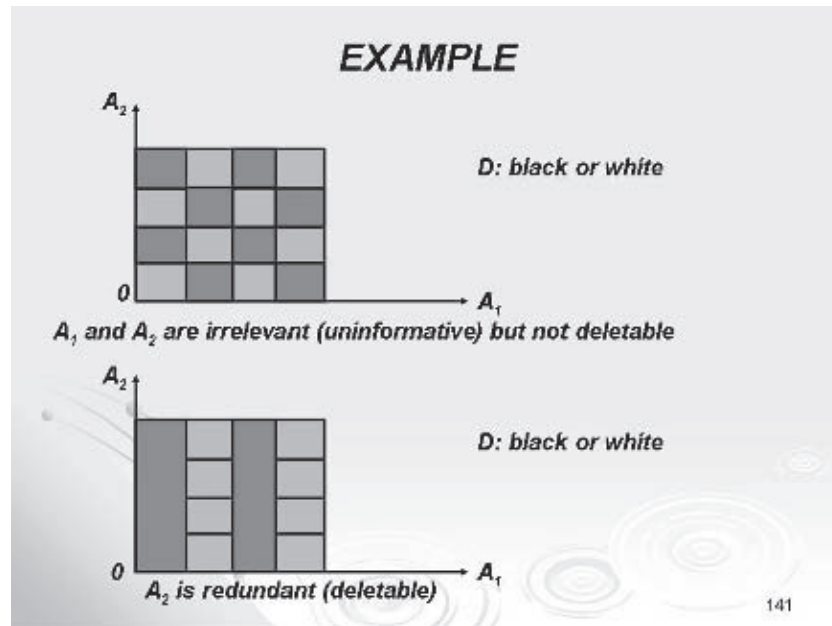
139

IRRELEVANCE (UNINFORMATIVENESS)

Name	A_1	A_j	A_n	D
Name	.	a_{ij}	.	d_1
r	.	a_{ij}	.	d_1
Name	.	a_{ij}	.	d_2
$i+s$.	a_{ij}	.	d_2

$(A_j \text{ is } a_{ij})$ is irrelevant
(uninformative)

140



KEY POINT—THE ROLE OF FUZZY LOGIC

- Existing approaches to the enhancement of web intelligence are based on classical, Aristotelian, bivalent logic and bivalent-logic-based probability theory. In our approach, bivalence is abandoned. What is employed instead is fuzzy logic—a logical system which subsumes bivalent logic as a special case.
- Fuzzy logic is not fuzzy
- Fuzzy logic is a precise logic of fuzziness and imprecision
- The centerpiece of fuzzy logic is the concept of a generalized constraint.

142

- *In bivalent logic, BL, truth is bivalent, implying that every proposition, p , is either true or false, with no degrees of truth allowed*
- *In multivalent logic, ML, truth is a matter of degree*
- *In fuzzy logic, FL:*
 - *everything is, or is allowed to be, to be partial, i.e., a matter of degree*
 - *everything is, or is allowed to be, imprecise (approximate)*
 - *everything is, or is allowed to be, granular (linguistic)*
 - *everything is, or is allowed to be, perception based*

143

CONTINUED

- *The generality of fuzzy logic is needed to cope with the great complexity of problems related to search and question-answering in the context of world knowledge; to deal computationally with perception-based information and natural languages; and to provide a foundation for management of uncertainty and decision analysis in realistic settings*

144

January 26, 2005

Factual Information About the Impact of Fuzzy Logic

PATENTS

- ③ **Number of fuzzy-logic-related patents applied for in Japan: 17,740**
- ③ **Number of fuzzy-logic-related patents issued in Japan: 4,801**
- ③ **Number of fuzzy-logic-related patents issued in the US: around 1,700**

145

PUBLICATIONS

Count of papers containing the word "fuzzy" in title, as cited in INSPEC and MATH.SCI.NET databases.

Compiled by Camille Wanat, Head, Engineering Library, UC Berkeley, December 22, 2004

Number of papers in INSPEC and MathSciNet which have "fuzzy" in their titles:

INSPEC - "fuzzy" in the title

1970-1979: 569
1980-1989: 2,404
1990-1999: 23,207
2000-present: 14,172
Total: 40,352

MathSciNet - "fuzzy" in the title

1970-1979: 443
1980-1989: 2,465
1990-1999: 5,483
2000-present: 3,960
Total: 12,351

146

JOURNALS (“fuzzy” or “soft computing” in title)

1. *Fuzzy Sets and Systems*
2. *IEEE Transactions on Fuzzy Systems*
3. *Fuzzy Optimization and Decision Making*
4. *Journal of Intelligent & Fuzzy Systems*
5. *Fuzzy Economic Review*
6. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*
7. *Journal of Japan Society for Fuzzy Theory and Systems*
8. *International Journal of Fuzzy Systems*
9. *Soft Computing*
10. *International Journal of Approximate Reasoning--Soft Computing in Recognition and Search*
11. *Intelligent Automation and Soft Computing*
12. *Journal of Multiple-Valued Logic and Soft Computing*
13. *Mathware and Soft Computing*
14. *Biomedical Soft Computing and Human Sciences*
15. *Applied Soft Computing*

147

APPLICATIONS

The range of application-areas of fuzzy logic is too wide for exhaustive listing. Following is a partial list of existing application-areas in which there is a record of substantial activity.

- | | |
|---|--|
| 1. <i>Industrial control</i> | 19. <i>Financial engineering</i> |
| 2. <i>Quality control</i> | 20. <i>Assessment of credit-worthiness</i> |
| 3. <i>Elevator control and scheduling</i> | 21. <i>Fraud detection</i> |
| 4. <i>Train control</i> | 22. <i>Mine detection</i> |
| 5. <i>Traffic control</i> | 23. <i>Pattern classification</i> |
| 6. <i>Loading crane control</i> | 24. <i>Oil exploration</i> |
| 7. <i>Reactor control</i> | 25. <i>Geology</i> |
| 8. <i>Automobile transmissions</i> | 26. <i>Civil Engineering</i> |
| 9. <i>Automobile climate control</i> | 27. <i>Chemistry</i> |
| 10. <i>Automobile body painting control</i> | 28. <i>Mathematics</i> |
| 11. <i>Automobile engine control</i> | 29. <i>Medicine</i> |
| 12. <i>Paper manufacturing</i> | 30. <i>Biomedical instrumentation</i> |
| 13. <i>Steel manufacturing</i> | 31. <i>Health-care products</i> |
| 14. <i>Power distribution control</i> | 32. <i>Economics</i> |
| 15. <i>Software engineering</i> | 33. <i>Social Sciences</i> |
| 16. <i>Expert systems</i> | 34. <i>Internet</i> |
| 17. <i>Operation research</i> | 35. <i>Library and Information Science</i> |
| 18. <i>Decision analysis</i> | |

148

Product Information Addendum 1

This addendum relates to information about products which employ fuzzy logic singly or in combination. The information which is presented came from SIEMENS and OMRON. It is fragmentary and far from complete. Such addenda will be sent to the Group from time to time.

SIEMENS:

- * washing machines, 2 million units sold
- * fuzzy guidance for navigation systems (Opel, Porsche)
- * OCS: Occupant Classification System (to determine, if a place in a car is occupied by a person or something else; to control the airbag as well as the intensity of the airbag). Here FL is used in the product as well as in the design process (optimization of parameters).
- * fuzzy automobile transmission (Porsche, Peugeot, Hyundai)

OMRON:

- * fuzzy logic blood pressure meter, 7.4 million units sold, approximate retail value \$740 million dollars

Note: If you have any information about products and or manufacturing which may be of relevance please communicate it to Dr. Vesa Niskanen vesa.niskanen@helsinki.fi and Masoud Nikravesh nikravesh@cs.berkeley.edu.

149

Product Information Addendum 2

This addendum relates to information about products which employ fuzzy logic singly or in combination. The information which is presented came from Professor Hideyuki Takagi, Kyushu University, Fukuoka, Japan. Professor Takagi is the co-inventor of neurofuzzy systems. Such addenda will be sent to the Group from time to time.

Facts on FL-based systems in Japan (as of 2/06/2004)

1. Sony's FL camcorders

Total amount of camcorder production of all companies in 1995-1998 times Sony's market share is the following. Fuzzy logic is used in all Sony's camcorders at least in these four years, i.e. total production of Sony's FL-based camcorders is 2.4 millions products in these four years.

1,228K units X 49% in 1995
1,315K units X 52% in 1996
1,381K units X 50% in 1997
1,416K units X 51% in 1998

2. FL control at Idemitsu oil factories

Fuzzy logic control is running at more than 10 places at 4 oil factories of Idemitsu Kosan Co. Ltd including not only pure FL control but also the combination of FL and conventional control.

They estimate that the effect of their FL control is more than 200 million YEN per year and it saves more than 4,000 hours per year.

150

3. Canon

Canon used (uses) FL in their cameras, camcorders, copy machine, and stepper alignment equipment for semiconductor production. But, they have a rule not to announce their production and sales data to public.

Canon holds 31 and 31 established FL patents in Japan and US, respectively.

4. Minolta cameras

Minolta has a rule not to announce their production and sales data to public, too.

whose name in US market was Maxxum 7xi. It used six FL systems in a camera and was put on the market in 1991 with 98,000 YEN (body price without lenses). It was produced 30,000 per month in 1991. Its sister cameras, alpha-9xi, alpha-5xi, and their successors used FL systems, too. But, total number of production is confidential.

151

5. FL plant controllers of Yamatake Corporation

Yamatake-Honeywell (Yamatake's former name) put FUZZICS, fuzzy software package for plant operation, on the market in 1992. It has been used at the plants of oil, oil chemical, chemical, pulp, and other industries where it is hard for conventional PID controllers to describe the plan process for these more than 10 years.

They planned to sell the FUZZICS 20 - 30 per year and total 200 million YEN.

As this software runs on Yamatake's own control systems, the software package itself is not expensive comparative to the hardware control systems.

6. Others

Names of 225 FL systems and products picked up from news articles in 1987 - 1996 are listed at http://www.adwin.com/elec/fuzzy/note_10.html in Japanese.)

Note: If you have any information about products and or manufacturing which may be of relevance please communicate it to Dr. Vesa Niskanen vesa.a.niskanen@helsinki.fi and Masoud Nikravesh Nikravesh@cs.berkeley.edu, with cc to me.

152