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MODELLING THE LONGITUDINAL MOTION OF AN AIRCRAFT

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ABSTRACT:

The paper is making a short introduction into the field of the aircraft modeling. A basic aircraft model is build, aiming to obtain a simulation platform for different related control algorithms and for further studies on the switching controllers effects in avionics. A Matlab-Simulink implementation is provided.

KEYWORDS:

switching controllers, mathematical model, state space.

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INTRODUCTION

The Stability and the Control of the airplanes was a key issue from the very beginning of the aviation. A great step forward in the field was the introduction of the automatic pilot (robot pilot, autopilot). The initial purpose of an autopilot was to replace the human pilot during cruise modes. They are expected to perform more rapidly and with greater precision than the human pilot and to make the aircraft fly in the same manner as a highly trained pilot: smoothly and with no sudden and erratic maneuvers. The modern autopilots are implemented by complex digital computers and they are able to stabilize the aircraft, protect the aircraft from undesirable maneuvers, and realize automatic landings.

Although at the first glance the reliability of the digital computers seem to be indubitable, in particular circumstances, the perturbations produced when switching between automate pilot and manual pilot may cause sudden and erratic instabilities that can cause fatal airplane crashes. Official and reliable reports on such accidents are not easy to find, but it is unanimously accepted that the on-line switching of two different controllers may produce uncontrollable transient regimes and even destabilizations.

In some previous papers we investigated the *Switching Controllers Instability* (SCI) for the case of some second order plants. The objective of this paper is to choose an appropriate mathematical model of an aircraft, that could stand for a simulation support, in further studies on control algorithms and on the switching controllers effects in avionics.

MATHEMATICAL MODELS FOR AIRCRAFTS

The first mathematical model of an aircraft was proposed by G.H. Bryan, in a fundamental early book: *Stability in Aviation*, 1911. Bryan's model is a system of 6-degrees-of-freedom equations that are still in use for the computer simulation of the most advanced of today's aircrafts, with some supplementary developments needed for the airplane control [1]. An interesting fact about the Brian's model is that his simplifying assumptions, which are affecting the model's accuracy for the subsonic aircrafts, are more suitable for the supersonic aircraft models.

$$
\begin{aligned}
W\frac{du}{gdt} &= W\epsilon\cos\theta_0 + \delta H - uX_u - vX_v - rX_r \\
W\left(\frac{dv}{gdt} + \frac{rU}{g}\right) &= -W\epsilon\sin\theta_0 - uY_u - vY_v - rY_r \\
C\frac{dr}{gdt} &= -h\delta H - uN_u - vN_v - rN_r
\end{aligned}
$$

$$
\begin{aligned}
W\left(\frac{dw}{gdt} - \frac{qU}{g}\right) &= -W\phi\cos\theta_0 - wZ_w - pZ_p - qZ_q \\
A\frac{dp}{gdt} - F\frac{dq}{gdt} &= -wL_w - pL_p - qL_q \\
B\frac{dq}{gdt} - F\frac{dp}{gdt} &= -wM_w - pM_p - qM_q
\end{aligned}
$$

Figure 1. The Brian's aircraft parametrical model

Starting from this model, that offers a structural view of the aircraft's dynamics, off-line or on-line accurate experimental models can be obtained.

An on-line identification was communicated in Ref. [2]. The identified aircraft is an L-410 Turbolet, manufactured by the Czech aircraft manufacturer LET. L-410 is a twin engine short-range transport aircraft (see **Fig. 2**).

The state vector that was used for the aircraft longitudinal motion modeling contains four state variables: aircraft velocity v, angle of attack α, pitch angle φ and derivative pitch angle φ'. The control vector contains only one input variable: the elevator angle δ.

Figure 2. The L-410 Turbolet

It was used a linear model:

$$X' = \Phi(X) + \Gamma(U) \tag{1}$$

where Φ is the plant matrix (n x n), Γ the control matrix (n x r), X the state vector (n x 1) and U the control vector (r x 1).

The state vector and the control vector are the following:

$$\mathbf{X} = [\ v\ \ \alpha\ \ \varphi\ \ \varphi'\ ]^{\mathbf{T}} \tag{2}$$
$$U = [\ \delta\ ]^{\mathbf{T}} \tag{3}$$

The final result of the identification, after n = 21 data measurements (t = 2.0 s), using a Matlab implemented version of the classic least squares method [3] is the following:

$$[\Phi_{21}, \Gamma_{21}] = \begin{bmatrix} 9.5079e\text{-}001 & 3.4779e{+}001 & -1.7931e{+}001 & -1.4134e{+}001 & -1.3064e{+}001 \\ 3.8000e\text{-}004 & 8.2254e\text{-}001 & 3.3844e\text{-}002 & 9.5337e\text{-}002 & -1.4376e\text{-}002 \\ -1.5513e\text{-}004 & 9.8492e\text{-}002 & 9.3600e\text{-}001 & 6.8752e\text{-}002 & -2.7441e\text{-}002 \\ 3.3045e\text{-}004 & 1.5789e\text{-}001 & -2.5812e\text{-}001 & 6.4224e\text{-}001 & -5.3852e\text{-}001 \end{bmatrix} \quad (4)$$

A SIMULINK IMPLEMENTATION

The previous mathematical model is implemented in Simulink-Matlab as shown in **Fig. 3**. This deployed version is more complicate that the state-space model, but has the advantage of a transparent and complete control of the initial values of the state variables.

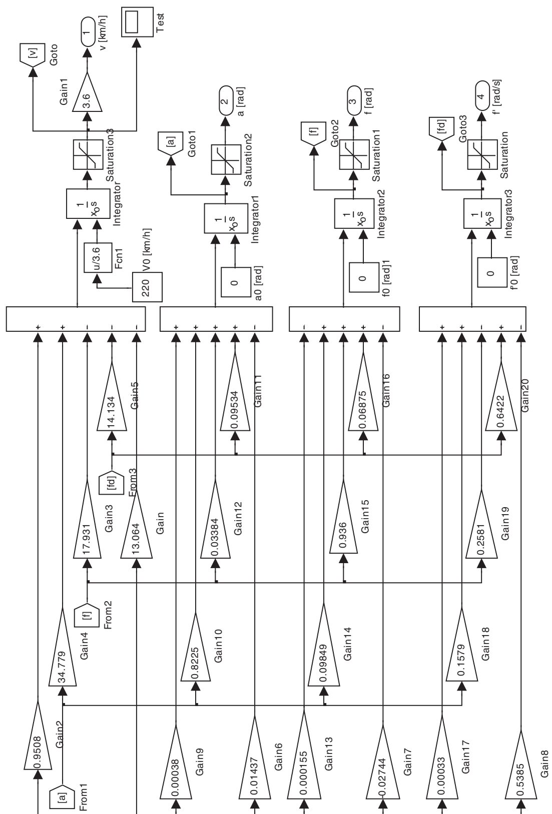

Figure 3. The aircraft longitudinal motion Simulink model

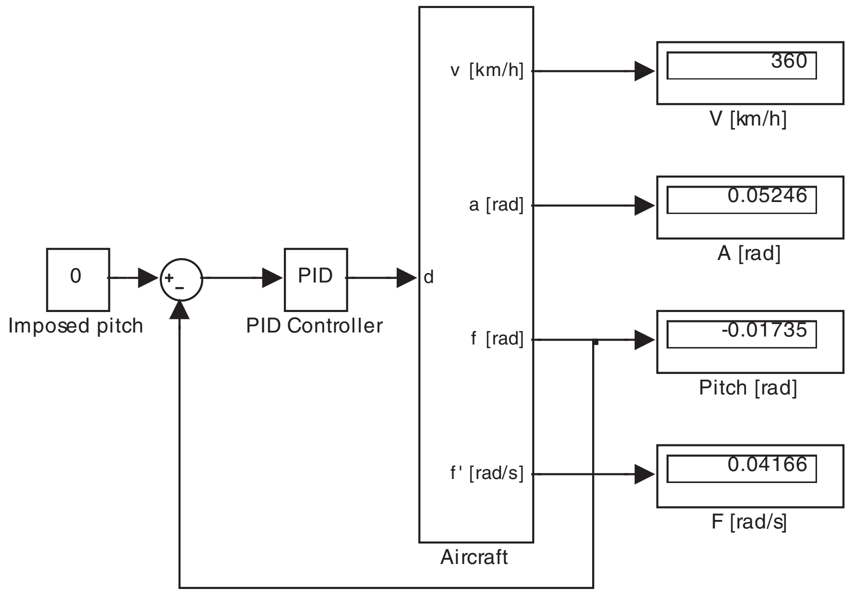

Figure 4. The control of the pitch by the elevator angle

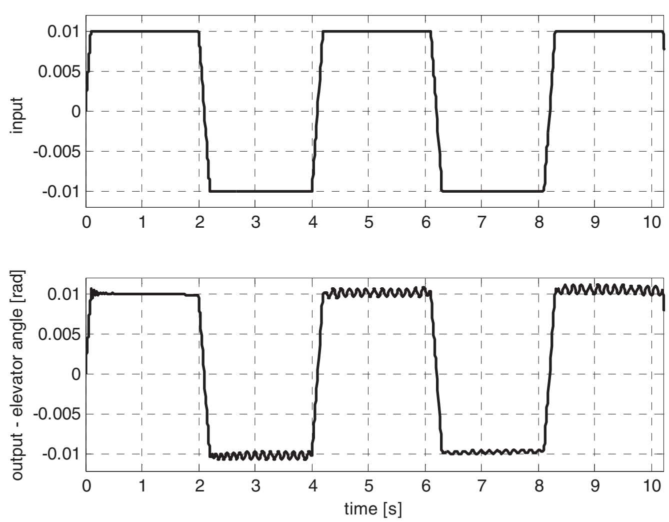

Figure 5. A simulation result

This model can be used now for testing different control algorithms. A simulation result is shown in **Fig. 5**.

The main advantages of this model are the simplicity, the linearity and the accuracy for the given identification conditions. It is adapted for the real-time identification of the specific steady flight regimes.

On the other side its nature is synthetic: no information about the physical structure of the airplane system is included. Although the state variables are physical parameters, they are not able to catch the nonlinear functionality of the system. That is why this model can be hardly used outside of its context.

CONCLUSIONS

The paper is presenting a deployed continuous time version of a state-space mathematical model of the longitudinal motion of an L-410 airplane.

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