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FORMALIZATION OF POINT OF VIEW BY THE FUZZY SET THEORY

ABSTRACT:

A point of view pv may bedefined as a couple pv =(D,) where D is a set whose elements are called determinants and is a mapping from D into[0,1]. The mathematical results in the theory of fuzzy sets permit us to have many constructions of different points of view. We can also compare two points of view by introducing the notion of imbrication and dissimilarity.

KEYWORDS:

point of view, determinant, taking into consideration, degree, fuzzy set.

1. INTRODUCTION

The purpose of this paper is to introduce the possibility to give a mathematical formalization of point of view. It permits us first to combine different points of view by mathematical operators and then to ratify the result of mathematical operation by comparison with our intuitions or experiences.

As our investigation object is imprecise or fuzzy, we choose the fuzzy set as domain of mathematical formalization. In effect, the theory of fuzzy set is actually developed and may be used in different domains, so that it seems useful to us to exploit the possibility of this theory in order to manipulate and control the points of view and their different combination beyond our intuition.

Thus mathematical formalization of point of view founded on the theory of fuzzy set shows that it is possible to manipulate the fuzzy sets and to applicate them to humain thoughts.

This paper is organized as follows. After a brief recalling of the fuzzy set theory in the section 2, we introduce in the section 3 the notion of point of view. The section 4 is devoted to different operations on the points of view. Then we describe, in the section 5, the mathematical formalization of the similarities of point of view. Finally we conclude our paper in the section 6.

2 FUZZY SET

The previous consideration permits us to explain the reason of using the theory of fuzzy set. Let X an universal set.

Definition 2.1 *A fuzzy set of support $A \subset X$ is the data of the couple (A, α) where α is a mapping from A to* [0,1]. *α is called membership function. For $x \in A$, $\alpha(x)$ represents the degree of membership of x to A.*

Remark 2.1 *The degree of membership of an element x doesn't belong only to the pair* $\{0,1\}$ *like in a classical theory of set but it belongs to* $[0,1]$

- *If* $\alpha(x)=0$, *x doesn't belong at all to A.*

- *If* $\alpha(x)=1$, *x belongs completely to A.*

- *If* $0<\alpha(x)<1$ *the membership of x to A is more or less complete.*

Remark 2.2 *The mathematical result given by the theory of fuzzy set permits to have many constructions and combination of different points of viev from the set operators: union, intersection, inclusion, which may be confronted with what we think or feel.*

3 POINT OF VIEW

We begin by giving a definition of point of view with some simple hypothesis.

Definition 3.1 *A point of view pv is determined by a couple pv =(D, α) where D is a set whose the elements are called determinants and α is a mapping from D into* $[0,1]$.
If $d \in D$, $\alpha(d)$ *represents the taking into consideration for the d by the point of view pv.*
We assume that the following hypothesis are satisfied:

- ***H1:*** *A point of view is specified by a set of determinants that it take into consideration.*

- ***H2:*** *Those determinants are of different natures: criterion, indicator, rule, constraint, aspect,... That can put in relation to a measure or some evaluation..*

• ***H3:*** *Those determinants are taken into consideration by point of view with varied degree.*

We denote by $D = \{d_i, i \in I\}$ the set of determinants of point of view in one given context, that we don't give some precision in this step. Therefore D is a set in the classical sense.
Twopoints of view $pv_1 = (D_1, \alpha_1)$ and $pv_2 = (D_2, \alpha_2)$ may bedistinguished oneself of two manners:

• The first distinction replace on the set difference between D_1 and D_2.

• The second distinction replace on the related importance of taking consideration of one determinant by the point of view pv_1 and pv_2.

Example 3.1 *Let* $pv_1 = (D_1, \alpha_1)$ *and* $pv_2 = (D_2, \alpha_2)$ *with* $D_1 = D_2 = D$ = *lowground, forest, grassland, fallowfield*
Suppose that one does a survey about the importance of these determinants. Let α_1 *the result of rural survey and* α_2 *the result of urban survey: Let be* d_1 = *lowground*, d_2 = *forest*, d_3=*grassland*, d_4 = *fallowfield*.

$\alpha_1(d_1) = 0.7$; $\alpha_1(d_2) = 0.4$; $\alpha_1(d_3) = 0.5$; $\alpha_1(d_4) = 0.4$;

$\alpha_2(d_1) = 0.5$; $\alpha_2(d_2) = 0.8$; $\alpha_2(d_3) = 0.4$; $\alpha_2(d_4) = 0.3$;

In this example, we have $\alpha_1(d_1) \geq \alpha_2(d_1)$; *that means that determinant* d_1 *is more taken into consideration by* pv_1 *than* pv_2, *or* $d1$ *is more important for rural people than for town-dweller.*

Notation:

	d_1	d_2	d_3	d_4
α_1	0.7	0.4	0.5	0.1
α_2	0.5	0.8	0.4	0.3

4. OPERATION OF POINTS OF VIEW

Let $pv_1 = (D_1, \alpha_1)$ and $pv_2 = (D_2, \alpha_2)$, two points of view, the operations between these two points of view concern both D_i and α_i.

4.1 Inclusion

The inclusion of two points of view $pv_1 = (D_1, \alpha_1)$ and $pv_2 = (D_2, \alpha_2)$ is denoted by $pv_1 \subset pv_2$, if $D_1 \subset D_2$ and for all $d_1 \in D_1$, $\alpha_1(d_1) \leq \alpha_2(d_2)$

4.2 Intersection and T-norm

4.2.1 Intersection

The Intersection of two points of view $pv_1 = (D_1, \alpha_1)$ and $pv_2 = (D_2, \alpha_2)$ is denoted by $pv_1 \wedge pv_2 = (D_1 \cap D_2, \alpha_1 \wedge \alpha_2)$, where $\alpha_1 \wedge \alpha_2$ is the infinimum of α_1 and α_2, (ie) for all determinant d in $D_1 \cap D_2$, $\alpha_1 \wedge \alpha_2(d)= \inf(\alpha_1(d), \alpha_2(d))$.

4.2.2 T-norm

The taking into consideration of determinants at once by pv_1 and pv_2 which gives us the point of view $pv_1 \cap pv_2$ can be proceeded by different manners beyond using the infinimum. TheT-norms are tools that combine pv_1 and pv_2 otherwise in order to have an other type of intersection that we denote always by $pv_1 \wedge pv_2$

Here are some examples, that are more used in the theory of fuzzy set to express the intersection of α_1 and α_2 that we denote always by $pv_1 \wedge pv_2$

• min operator: *min*(1(*d*),2 (*d*))

• algebraic product $\alpha_1 \cdot \alpha_2(d) = \alpha_1(d) \cdot \alpha_2(d)$

• limited product $min(1,(\alpha_1(d) + \alpha_2(d)))$

• drastic product $\begin{cases} \alpha_2(d)\_if\ \_\alpha_1(d)=1 \\ \alpha_1(d)\_if\ \_\alpha_2(d)=1 \\ 0\_\_\_\_if\ \_\alpha_1(d)\_and\ \_\alpha_2(d)>0 \end{cases}$

• Einsten product: $\dfrac{\alpha_1(d)\cdot\alpha_2(d)}{2-\alpha_1(d)-\alpha_2(d)+\alpha_1(d)\cdot\alpha_2(d)}$

• Hamasher product: $\dfrac{\alpha_1(d)\cdot\alpha_2(d)}{\alpha_1(d)+\alpha_2(d)-\alpha_1(d)\cdot\alpha_2(d)}$

These T-norm considered have no parameters by they could have one.

4.3 Union and T-co-norm

4.3.1 Union

The union of two points of view $pv_1 = (D_1, \alpha_1)$ and $pv_2 = (D_2, \alpha_2)$ is denoted by $pv_1 \vee pv_2 = (D_1 \cup D_2, \alpha_1 \vee \alpha_2)$, where $\alpha_1 \vee \alpha_2$ is the supremum of α_1 and α_2, (ie) for all determinant *d* in $D_1 \cup D_2$, $\alpha_1 \vee \alpha_2(d) = sup(\alpha_1(d), \alpha_2(d))$.

4.3.2 T-co-norm

The taking into consideration of determinant by pv_1 and pv_2 which gives the point of view $pv_1 \vee pv_2$ can be processed by different manners otherwise on using supremum.

The T-co-norm are tools which combine $pv_1 \vee pv_2$ in order to have an other type of union. Here are some examples :

• max operator: :$max(\alpha_1(d), \alpha_2(d))$

• algebraic sum: $\alpha 1(d) + \alpha_2(d)$

• limited sum: $max(0, \alpha_1(\mathrm{d}) + \alpha_2(d) - 1)$

• drastic sum: : $\begin{cases} \alpha_2(d)\_if\ \_\alpha_1(d) = 0 \\ \alpha_1(d)\_if\ \_\alpha_2(d) = 0 \\ 1\_\_\_\_if\ \_\alpha_1(d)\_and\ \_\alpha_2(d) > 0 \end{cases}$

• Einstein sum: $\dfrac{\alpha_1(d) + \alpha_2(d)}{1 + \alpha_1(d)\alpha_2(d)}$

• Hamasher sum: $\dfrac{\alpha_1(d) + \alpha_2(d) - 2\alpha_1(d)\alpha_2(d)}{1 + \alpha_1(d)\alpha_2(d)}$

• disjunctive sum $max(min(\alpha_1(d), 1-\alpha_2(d)), min(1-\alpha_1(d), \alpha_2(d)))$
The operators of T-co-norm considered here have no parameters but they could have one.

4.4 Complementarity

The complementarity of pv_1 is denoted by $pv_1^c = (D_1, 1-\alpha_1)$

Remark 4.1 *Let* $pv_1 = (D_1, \alpha 1)$ *a point of view.*

• $i - pv_1 \wedge pv_1^c \neq \phi$

• *ii – The complementarity concept here means that if pv_1 takes account one determinant with x percent so then pv_1^c takes account with* (1 - *x*) *percent*.

• *iii - The remarks 1 and 2 induce us to say that there is a kind of imbrication between the point of view pv_1 and his complementarity pv_1^c which we define later in* ***4.5***

• *iv – The concept of complementarity with regard to determinant considered or not is taken into account by the previous definition with the following manner:*

One determinant, which is not taken into account, has a degree of taking into account equal to zero. For example, if one determinant $d_1$is not taken into consideration by one point of view pv_2, his degree of taking into account is zero; in other hand, if one determinant d_2 is not taken into account by the point of view pv_1, then it has a degree of taking into account zero, but if it is taken into consideration with a degree upper by pv_2, then the point of view pv_1 and pv_2 are complementary with regard to determinants d_1,d_2.

4.5 Cuts

The concept of cuts for a point of view $pv = (D, \alpha)$ permits to show the set of determinants d in D which are taken into account by pv beyond a certain choosen degree. Let be $r \in [0,1]$. We denote that r-cuts of point of view pv the point of view $pv_r = (D_r, \alpha)$ such that $D_r = \{d, d \in D/(d) \geq r\}$, D_r is a set included in D. A r-strict cuts is r defined by $pv^r = (D^r, \alpha)$ such that $D^r = \{d,d \in D/\alpha(d)>r\}$,

Property 4.1 *Let be* $r \in [0,1]$
$i - (pv_1 \cap pv_2)^r = (pv_1)^r \cap pv_2)^r$
$ii - (pv_1 \cup pv_1)^r = (pv_1)^r \cup (pv_2)^r$.

$iii - (pv^c)_r = pv^c_{1-r} \neq pv_r$ *if* $r \neq ½$ *and* $r \neq 1$.

4.6 Size of points of view

The size of point of view *pv* is defined by $|pv| = \sum_{d \in D} (\alpha(d))$

If *D* is finite the relative size of point of view is $\|pv\| = \dfrac{|pv|}{CardD}$

If the set of determinant is continuous that we denote by X, the relative size is defined by $|pv| = \int_D \alpha(x)dx$.

4.7 Imbrication between two points of view

In this, section, we can say about the imbrication between two points of view . In fact, in the classical theory of sets, we have one of two relations inclusion $A \subset B$ or $B \subset A$.
The two relations are mutually exclusive. But in the theory of fuzzy set if $A \subset B$, B can be embedded partially in A. The imbrication of B in A is graduate. If A is in B then the degree of imbrication of A in B is equal to 1. If $A \cap B = \emptyset$; the degree of imbrication is equal to zero.

Thus, we denote that the degree of imbrication of pv_1 in pv_2 by:

$$I(pv_1, pv_2) = \frac{|pv_1 \cap pv_2|}{|pv|}$$

Definition 4.1 *Let* $pv = (D, \alpha)$, *we call the support of pv the subset supp(pv)* $= \{d \in D / \alpha(d) \neq 0\}$

4.8 Entropy of point of view

Let us remark that: $supp(pv) \cup supp(pv)^c \neq D$ and $supp(pv) \cap supp(pv)^c$ Ø;. This shows the fact that there is an overlap between *pv* and pv^c or in other word, there is a certain disorderin *pv*. The measure of this disorder is expressed by the entropy concept and for all *pv* that we denote by:

$$E(pv) = \frac{|pv \cap pv^c|}{|pv \cup pv^c|}$$

4.9 Distance between two points of view

Here, we consider two points of view which have the same set of determinants *D*, even if some determinants are not taken into consideration by one or other point of view . Let $pv_1 = (D, \alpha_1)$ and $pv_2 = (D, \alpha_2)$, the distance between two points of view is a tool which permits to evaluate the difference between them. It exists many types of distance but here, we are interested by the Hamming distance and Euclidean distance.
In the finite case, we define:

• Hamming distance : we define the Hamming distance between $pv_1 = (\mathrm{D}, \alpha_1)$ and $pv_2 = (D, \alpha_2)$ by:

$$d(pv_1, pv_2) = \sum_{d \in D} |\alpha_1(d) - \alpha_2(d)|$$

• Euclidean distance: The euclidean distance between $pv_1 = (D, \alpha_1)$ and $pv_2 =(D, \alpha_2)$ is defined by:

$$d(pv_1, pv_2) = \sqrt{\sum_{d \in D} (\alpha_1 d - \alpha_2 d))^2}$$

5. DISSIMILARITY AND SIMILARITY

Let D a set of determinants, let us denote by PV the set of points of view $pv_i = (D, \alpha_i)$, $i \in I$. In order to compare the points of view which have the same set of determinants, we introduce the notion of dissimilarity between two points of view

Definition 5.1 *A mapping* $ds : PV \times PV \to [0,1]$ *is a dissimilarity if and only if:*

- *For all* $pv \in PV$, $ds(pv, pv)=0$

- *For all couple* $(pv_1,pv_2) \in PV \times PV$, $ds(pv_1, pv_2) = ds(pv_2, pv_1)$

Such mapping permits to measure the degree of dissimilarity between two points of view .
In dual manner,we can measure the degree of similarity between two points of view by a mapping: $s : PV \times PV \to [0,1]$ which satisfies the following conditions:

- For all couple $(pv_1, pv_2) \in PV \times PV$, $s(pv_1, pv_2) = s(pv_2, pv_1)$

- For all $pv_1, pv_2, pv_3 \in PV$, $s(pv_1, pv_2) \leq s(pv_3, pv3) = 1$

Thus, we can associate to all dissimilarity ds a similarity s defined by: $s = 1- ds$

Example 5.1 *Let us consider*

$$ds(pv_2, pv_1) = \frac{\sum_{d \in D} |\alpha_1(d) - \alpha_2(d)|}{\sum_{d \in D} \max(\alpha_1(d), \alpha_2(d))} \Rightarrow s(pv_2, pv_1) =$$

$$= 1 - \frac{\sum_{d \in D} |\alpha_1(d) - \alpha_2(d)|}{\sum_{d \in D} \max(\alpha_1(d), \alpha_2(d))}$$

We can verify easily that ds is a dissimilarity and s is a similarity.
Let us consider the previous example in **3.1.**

	d_1	d_2	d_3	d_4
α_1	0.7	0.4	0.5	0.1
α_2	0.5	0.8	0.4	0.3

We can calculate the dissimilarity between the two points of view, we get pv_1 and pv_2.

$ds(pv_2, pv_1) = 0.39 \Rightarrow s(pv_2, pv_1) = 0.61$.

Therefore we can conclude that the point of view of the town-dweller and the rural people, about the different varieties of earth are not similar for $s(pv_2, pv_1)$ is strictly inferior to 1.

CONCLUSION

The present work permits us to enlarge into an interval $[0,r] \subset [0,1]$ the values of taking into account criterions by a point of view instead of $r \in [0,1]$. That amounts to replacing the value point r of [0,1] by an interval, i.e by a set of infinite cardinal but measurable $[0, r]$, included in [0,1]. Thus, it consists to hang up the points of view to the theory of fuzzy sets which permits to graduate the equality in addition to the graduation of membership.
In the next time, we project to link up the points of view with the category theory, in particular with the ”topos theory”, via the theory of fuzzy sets. The points of view will be manipulated by the procedure of category construction, under existence

conditon. This second step tries to answer to our preoccupation for connecting the point of view and the transcendence to the process of human rational understanding in order that thepoint of view and the transcendence would notbedifficultfor the reason to understand them.

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