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Stability Analysis Results Concerning the Fuzzy Control of a Class of Nonlinear Time-Varying Systems

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Abstract

The paper offers new stability results concerning the Takagi-Sugeno (T-S) fuzzy control systems dedicated to a class of Single Input-Single Output (SISO) nonlinear time-varying systems. Lyapunov's approach based on quadratic positive definite Lyapunov function candidates is emplyed to derive a sufficient uniform asymptotic stability condition. An illustrative example validates the stability analysis results by the design of a T-S fuzzy control system for a SISO nonlinear process.

Keywords: Lyapunov function candidates, nonlinear time-varying systems, Takagi-Sugeno fuzzy control systems, uniform asymptotic stability.
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1. Introduction

Fuzzy controller (FCs) prove to be useful for processes which are subjected to difficulties in deriving mathematical models or coping with performance limitations. These nonlinear controllers are easily understandable version of other complex nonlinear controllers used in such situations (Johanyák & Kovács, 2006), (Johanyák *et al.*, 2006), (Škrjanc *et al.*, 2004), (Vaščák, 2008), (Vaščák, 2009).

One of the major concerns of dynamical systems theory is the necessity to guarantee their stability therefore thorough stability analyses must be conducted. The importance of this problem increases when a control system is involved. Although fuzzy controllers have been proposed for a long time and applied successfully in many applications (Zhao *et al.*, 2010), (Kruszewski *et al.*, 2008), (Chin-Tzong & Yung-Yih, 2008), (Precup *et al.*, 2009), (Haber *et al.*, 2010), (Kurnaz *et al.*, 2010), (Precup *et al.*, 2010) a comprehensive work on the proof of stability for fuzzy control systems has begun only recently. These systems have resulted in convenient and relatively easily understandable approaches to nonlinear control of complex and even ill-defined processes. A proof of stability in the conditions of all possible automatic control system operating conditions is necessary before the fuzzy control system is put into real practice.

In principle, for the stability analysis of fuzzy control systems controlling nonlinear processes any method can be utilized to be suitable for the analysis of nonlinear dynamical systems. Which method is the best one to use depends only on the prerequisites. There, the structure of the system, the type of information describing the process and the

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type of sufficient conditions for the stability are usually the key points.

Many stability analyses become difficult to apply when there are developed fuzzy controllers to cope with complex processes including the nonlinear time-varying (LTV) ones. LTV systems are used in practice because most real-world systems are time-varying as a result of their parametric modifications over time as it is reported in (Neerhoff & van der Kloet, 2001), (van der Kloet & Neerhoff, 2002), (Yoneyama, 2007). LTV systems may also be a result of linearizing nonlinear systems in the vicinity of a set of operating points or of a trajectory. Several techniques are employed in the analysis and development of control systems meant for LTV systems. These techniques deal mainly with the popular eigenstructure assignment (Choi *et al.*, 2001), (Lee & Jiang, 2005).

Several methods for the stable design of FCs employing the stability analysis in the framework of considering the fuzzy control systems as nonlinear systems have been proposed recently. Some critical points of view on fuzzy controllers and on the stability analysis of fuzzy control systems are presented in (Michels *et al.*, 2006), (Arino & Sala, 2007), (Mozelli *et al.*, 2009), (Feng, 2010), (Lendek *et al.*, 2010).

The first objective of the paper is to give a mathematical characterization of a class of Takagi-Sugeno (T-S) models for Single Input-Single Output (SISO) nonlinear time-varying (NTV) systems viewed as controlled processes. On the basis of these models the second objective is to propose a stability analysis for the accepted class of control systems with T-S fuzzy controllers controlling the SISO NTV processes. The contribution of this paper with respect to (Tomescu *et al.*, 2008) concerns the new details inserted in the theoretical part while an extended set of digital simulation results is included.

The paper is organized as follows. Section 2 expresses the T-S fuzzy models for SISO NTV systems. Section 3 presents the new stability analysis results expressed as an original theorem based on Lyapunov's theorem for time-varying systems. Details on the Lorenz chaotic systems expressed in terms of the well known and accepted three equations are presented as case study in Section 4. Section 5 designs a stable fuzzy control system for the Lorenz chaotic system considered as illustrative example of NTV systems. The conclusions are outlined in Section 6.

2. Definition of a class of fuzzy control systems

In this paper the fuzzy control system is accepted to consist of a process and a T-S FC as shown in Fig. 1. The process of extracting the knowledge from human operators in the form of fuzzy control rules is by no means trivial, nor is the process of deriving the rules based on heuristics and good understanding of the process and control systems theory is needed.

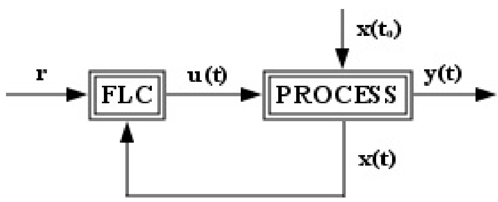


Figure 1. Fuzzy control system structure.

Let $D \subset \mathbb{R}^n$ be a universe of discourse. Consider the nonlinear autonomous system of the following form representing the state-space equations of the controlled process:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t) + \mathbf{b}(\mathbf{x}, t)u \tag{2.1}$$

$$\mathbf{x}(t_0) = \mathbf{x}_0$$

where: $\mathbf{x} \in D$, $\mathbf{x} = [\ x_1 \quad x_2 \quad ... \quad x_n\ ]^T$, is the state vector, $n \in \mathbb{N}^*$, $\dot{\mathbf{x}} = [\ \dot{x}_1 \quad \dot{x}_2 \quad ... \quad \dot{x}_n\ ]^T$ is the derivative of $\mathbf{x}$ with respect to the time variable t, $f, b : [0, \infty) \times D \to IR^n$ are continuous in t, and:

$$\mathbf{f}(\mathbf{x}, t) = [f_1 \ (\mathbf{x}, t) \quad f_2(\mathbf{x}, t) \quad ... \quad f_n(\mathbf{x}, t)\ ]^T \tag{2.2}$$

and

$$\mathbf{b}(\mathbf{x}, t) = [\ b_1(\mathbf{x}, t) \quad b_2(\mathbf{x}, t) \quad ... \quad b_n(\mathbf{x}, t)\ ]^T \tag{2.3}$$

are functions describing the dynamics of the process, u is the control signal fed to the process, obtained by the weighted-sum defuzzification method for T-S FCs.

The FC consists of r fuzzy rules. The *i-th* IF-THEN rule in the fuzzy rule base of the FC, referred to as Takagi-Sugeno fuzzy rule, is expressed in terms of the following form:

$$\begin{aligned} &Rule\ i : \text{IF } x_1 \text{ IS } X_{i,1} \text{ AND } x_2 \text{ IS } X_{i,2} \text{ AND ... AND } x_n \text{ IS } X_{i,n} \\ &\qquad \text{THEN } u = u_i\,(x)\,,\ i = \overline{1, r}, r \in \mathbb{N}^*, \end{aligned} \tag{2.4}$$

where r is the total number of rules, $X_{i,1}, X_{i,2}, .., X_{i,n}$ are fuzzy sets that describe the linguistics terms (LTs) of input variables, $u = u_i\,(x)$ is the control signal of rule i, similar to the case of parallel distributed compensation, and the function AND is a t-norm. u_i can be a single value or a function of the state vector, $\mathbf{x}(t)$.

Each fuzzy rule generates an activation degree referred to also as firing strength and defined in (2.5):

$$\alpha_i \in [0, 1]\,,\ i = \overline{1, r},$$

$$\alpha_i(\mathbf{x}) = AND(\mu_{\tilde{X}_{i,1}}(x_1), \mu_{\tilde{X}_{i,2}}(x_2), ..., \mu_{\tilde{X}_{i,n}}(x_n)). \tag{2.5}$$

It is assumed that for any $\mathbf{x}$ belonging to the input universe of discourse, , there exists at least one α_i among all rules that is not equal to zero.

The control signal u, which must be applied to the process, is a function of α_i and u_i. By applying the weighted-sum defuzzification method, the output of the FC is given by (2.6):

$$u = \frac{\sum_{i=1}^{r} \alpha_i u_i}{\sum_{i=1}^{r} \alpha_i}. \tag{2.6}$$

3. Stability analysis of Takagi-Sugeno fuzzy control systems

The stability analysis theorem presented here is based on the well acknowledged Lyapunov's theorem for time-varying systems referred in (Slotine & Li, 1991), (Khalil, 2001). The theorem ensures sufficient stability conditions for the fuzzy control systems with the structure described in the previous section. This section is focused on Theorem 3.1 that can be expressed in terms of a stability analysis algorithm.

Let $V : D{\times}[0, \infty) \to \mathbb{R}$, $V(\mathbf{x}, t) = \mathbf{x}^T P\mathbf{x}{\cdot}g(t)$, where P is an $n{\times}n$ constant positive definite matrix and $g \geq 0, \forall t \geq 0$, a continuously-differentiable function. The time derivative of $V(\mathbf{x}, t)$ along the open-loop trajectory (3.6) is given by:

$$\dot{V}(\mathbf{x}, t) = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x}\dot{\mathbf{x}} = \mathbf{x}^T P\mathbf{x} \cdot \dot{g}(t) + (\dot{\mathbf{x}}^T P\mathbf{x} + \mathbf{x}^T P\dot{\mathbf{x}})g(t) =$$

$$\mathbf{x}^T P\mathbf{x} \cdot \dot{g}(t) + g(t)(f(\mathbf{x}, t) + b(\mathbf{x}, t)u)^T Px + g(t)x^T P(f(\mathbf{x}, t) + b(\mathbf{x}, t)u) =$$

$$F(\mathbf{x}, t) + B(\mathbf{x}, t)u, \tag{3.1}$$

where:

$$F(\mathbf{x}, t) = g(t)f(\mathbf{x}, t)^T P\mathbf{x} + g(t)\mathbf{x}^T Pf(\mathbf{x}, t) + \mathbf{x}^T P\mathbf{x} \cdot \dot{g}(t) \tag{3.2}$$

and

$$B(\mathbf{x},t) = g(t)b(\mathbf{x},t)^T P\mathbf{x} + g(t)\mathbf{x}^T Pb(\mathbf{x},t). \tag{3.3}$$

The time derivative of $V(\mathbf{x})$ along the trajectory (2.1) for $\mathbf{u} = \mathbf{u}_k(\mathbf{x})$ is

$$\dot{V}_k(\mathbf{x},t) = F(\mathbf{x},t) + B(\mathbf{x},t)\mathbf{u}_k(\mathbf{x}) \tag{3.4}$$

The next theorem is based on the Lyapunov's theorem for time-varying systems (Khalil, 2001) and (Slotine & Li, 1991). Its proof is presented in (Precup *et al.*, 2009).

Theorem 3.1. *Consider the fuzzy control system consisting of the T-S FC defined in Section 2 and the nonlinear time-varying process with the state-space equations (2.1). Let* $\mathbf{x} = \mathbf{0}$ *be an equilibrium point for (2.1) and* $D \subset \mathbb{R}^n$ *be a domain containing the origin* $\mathbf{x} = \mathbf{0}$*. Let the Lyapunov function candidate* $V : D \times [0,\infty) \to \mathbb{R}$, $V(\mathbf{x},t) = \mathbf{x}^T P\mathbf{x} \cdot g(t)$ *where* P *is an* $n \times n$ *constant positive definite matrix and* $g \geq 0, \forall t \geq 0$ *a continuously-differentiable function, such that:*

$$V(\mathbf{x},t) \leq W^1(\mathbf{x}) \tag{3.5}$$

$$\dot{V}_k(\mathbf{x},t) = F(\mathbf{x},t) + B(\mathbf{x},t)\mathbf{u}_k(\mathbf{x}) \leq -W_k^2(\mathbf{x}), \forall k = \overline{1,r}, \forall t \geq 0, \forall \mathbf{x} \in D, \tag{3.6}$$

where $W^1(\mathbf{x})$ *and* $W_k^2(\mathbf{x})$ *are continuous positive definite functions on* D*. Then* $\mathbf{x} = 0$ *is uniformly asymptotically stable.*

The above stability theorem ensures sufficient stability conditions concerning the considered class of T-S fuzzy control systems described briefly in Section 2.

4. Case study

This section presents preliminary information about the Lorenz chaotic system known also as the Lorenz attractor (Lorenz, 1963), (Lorenz, 1993). In the first scientific papers about concept of deterministic chaos, chaotic behavior has been regarded as an exotic phenomenon that might be of interest only as a mathematical speculation and would never be encountered in practice. The Lorenz equation is commonly defined as three coupled ordinary differential equations expressed in (4.1) to model the convective motion of fluid cell, which is warmed from below and cooled from above:

$$\begin{aligned} \tfrac{dx}{dt} &= \sigma(y-x) \\ \tfrac{dy}{dt} &= x(\rho - z) - y \\ \tfrac{dz}{dt} &= xy - \beta z \end{aligned} \tag{4.1}$$

where the three parameters $\sigma, \rho, \beta > 0$ are called the Prandtl number, the Rayleigh number, and a physical proportion, respectively. These constants determine the behavior of the system and these three equations exhibit chaotic behavior i.e. they are extremely sensitive to initial conditions. A small change of the initial conditions leads quickly to large modifications of the corresponding solutions. The classical values used to demonstrate chaos are $\sigma = 10, \beta = \frac{8}{3}$ and ρ is variable in time.

5. Stable design of fuzzy control systems

The design of the fuzzy control system with T-S FC starts with rewriting the ordinary differential equation (4.1) in the following form representing the state-space equations of the nonlinear time-varying controlled process:

$$\dot{\mathbf{x}} = \begin{pmatrix} \sigma(x_2 - x_1) \\ x_1(\rho(t) - x_3) - x_2 \\ x_1x_2 - \beta x_3 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} u, \quad x(t_0) = x_0 \tag{5.1}$$

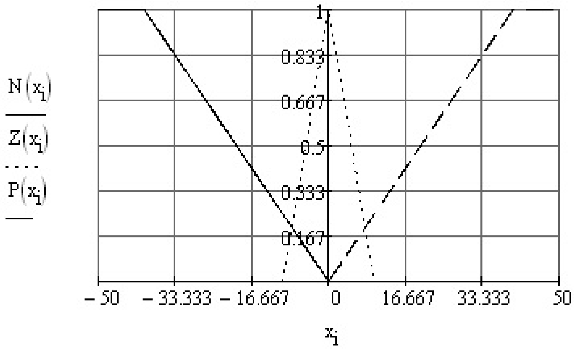


Figure 2. Membership functions of x_1 and x_2 .

where $\sigma = 10, \beta = \frac{8}{3}$ and $\rho(t) \in (0, 100]$. Next, the fuzzification module of T-S FC is set according to Fig. 2 which illustrates the membership functions that describe the LTs of the linguistic variables of x_1 and x_2. The linguistics terms representing Positive, Zero and Negative values are highlighted by P, Z and N, respectively.

The inference engine employs the fuzzy logic operators AND and OR implemented by the MIN and MAX functions, respectively, as pointed out in Section 2. The inference engine is assisted by the complete set of fuzzy control rules illustrated in Table 1, and the weighted sum defuzzification method is utilized. Summarizing, the only parameters to be calculated are the consequents u_i, $i = \overline{1, 9}$, in the nine fuzzy control rules.

In order to find the values of u_i for which the system (5.1) can be stabilized with the above described T-S FC, we will apply Theorem 3.1.

Table 1
Fuzzy Control Rule Base

Rule	Antecedent		Consequent
	x_1	x_2	u
1	P	P	u_1
2	N	N	u_2
3	P	N	u_3
4	N	P	u_4
5	P	Z	u_5
6	N	Z	u_6
7	Z	P	u_7
8	Z	N	u_8
9	Z	Z	u_9

Let the universe of discourse be $D = (-100, 100] \times (-100, 100]$. Let us consider the next Lyapunov function:

$$V(\mathbf{x}, t) = (x_1^2 + x_2^2 + x_3^2)(1 + e^{-t}) \leq 2(x_1^2 + x_2^2 + x_3^2) = W^1(\mathbf{x}) \tag{5.2}$$

which is a continuously differentiable positive function on domain D. The total derivative of V with respect to time using (5.1) is:

$$\dot{V}(\mathbf{x}, t) = [-2(\sigma x_1^2 + x_2^2 + \beta x_3^2) + 2x_1 x_2(\sigma + \rho(t)) + 2x_1 u](1 + e^{-t}) -$$

$$-e^{-t}(x_1^2 + x_2^2 + x_3^2). \tag{5.3}$$

After analyzing each fuzzy control rule, in respect to Theorem1, results:

For rule 1: $u_1 = -100(\sigma + \rho(t))$.

For rule 2: $u_2 = 100(\sigma + \rho(t))$.

For rule 3: $u_3 = 0$.

For rule 4: $u_4 = 0$.

For rule 5: $u_5 = -10(\sigma + \rho(t))$.

For rule 6: $u_6 = 10(\sigma + \rho(t))$.

For rule 7: $u_7 = -x_2(\sigma + \rho(t))$.

For rule 8: $u_8 = -x_2(\sigma + \rho(t))$.

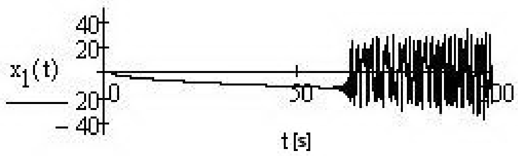
For rule 9: $u_9 = -x_2(\sigma + \rho(t))$.

We choose

$$W^2(\mathbf{x}) = W_k^2(\mathbf{x}) = (\sigma x_1^2 + x_2^2 + \beta x_3^2) \tag{5.4}$$

for any u_k.

Considering the values of process parameters $\sigma = 10, \beta = \frac{8}{3}, \rho(t) \in (0, 100]$, the initial state $x_1(0) = 1$, $x_2(0) = -1$ and $x_3(0) = 1$, the responses of x_1, x_2 and x_3 versus time in the closed-loop system are shown in Figs. 3-6. But, as already mentioned, the simulation of the behavior of chaotic system is extremely important due to the initial conditions of the system of ordinary differential equations (3.6). Therefore the Adams method has been employed here in the numerical solving (integration) of the ordinary differential equations afferent to the mathematical model of the fuzzy control system (4.1). The mathematical model is obtained by coupling the equations (2.1)-(2.4) and (5.3).



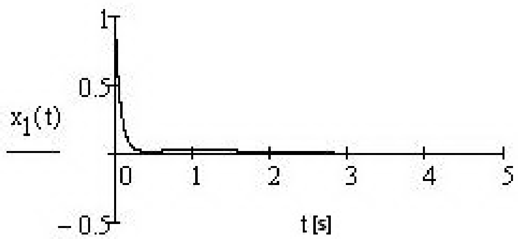


Figure 3. State variable x_1 versus time of Lorenz chaotic without FC (a) and with FC (b).

6. Conclusion

New results concerning the uniform asymptotic stability analysis of T-S fuzzy control systems dedicated to a class of nonlinear processes characterized by SISO nonlinear time-varying systems have been introduced. The theoretical results have been applied resulting in a simple and effective Takagi-Sugeno FC viewed as an alternative to the stabilization of the Lorenz chaotic system.

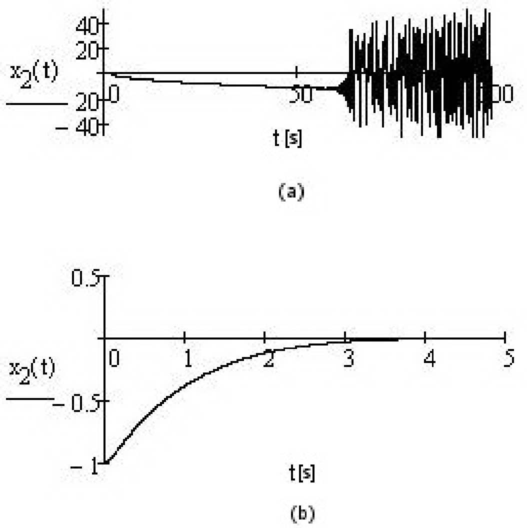


Figure 4. State variable x_2 versus time of Lorenz chaotic without FC (a) and with FC (b).

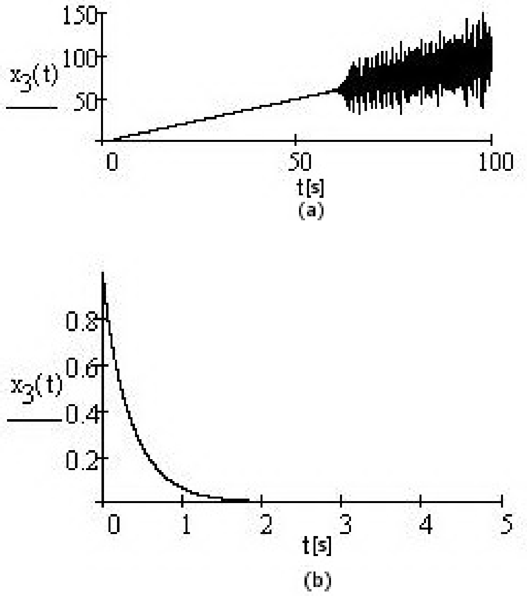


Figure 5. State variable x_3 versus time of Lorenz chaotic without FC (a) and with FC (b).

The stability analysis results proposes in this paper can be extended and applied in situations when the system has an equilibrium point different to the origin and / or the reference input r in Figure 1 is nonzero by an appropriately defined state transform. The new stability analysis results are different to the original Lyapunov's theorem and it is also different to the variety of more or less conservative LMI-based results. It allows more applications and it is well suited for controlling processes where the Lyapunov function candidates are not quadratic and the derivatives of the Lyapunov function candidates are negative definite. Therefore the applications of Theorem 3.1 to nonlinear processes controlled by T-S FCs will be successful for wide area of nonlinear time-varying systems. However the transparency of the proposed approach is not so good in comparison with that of LMI-based approaches.

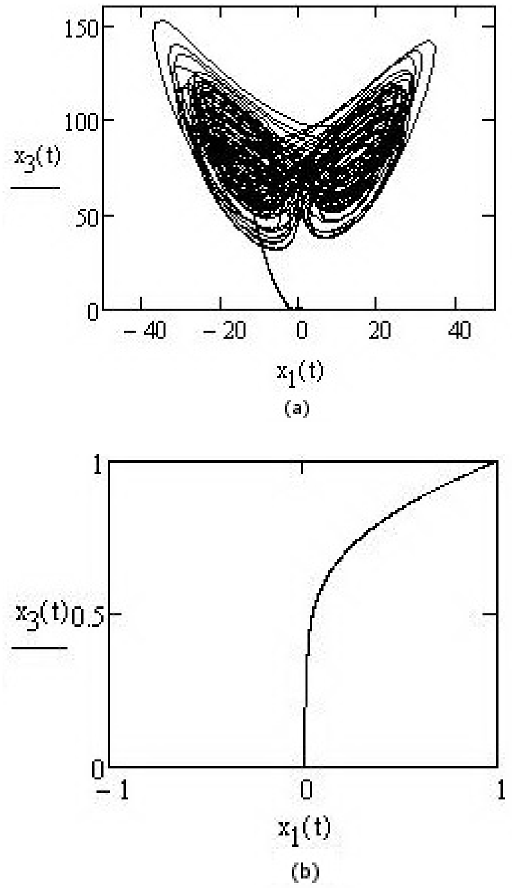


Figure 6. Phase portrait of Lorenz system without control (a) and with FC (b).

The digital simulation results presented in Section 5 prove that the proposed stability analysis approach is simpler than the nonfeedback control solutions proposed by Lima and Pettini (Lima & Pettini, 1990), the OGY method proposed by Ott, Grebogi and Yorke (Ott *et al.*, 1990) and Pyragas's method (Pyragas, 1992). The suggested T-S FC structure can be implemented as low cost automation solution.

Further research will be dedicated to offering other low cost fuzzy solutions for chaotic systems based on similar approaches and applications [(Horváth & Rudas, 2004), (Škrjanc *et al.*, 2005), (Johanyák & Kovács, 2007), (Precup, 2007), (Precup *et al.*, 2008)]. The careful stability analysis is necessary in all applications aiming the analysis of the stablity analysis algorithms by giving correct estimates of their complexity.

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