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Sensitivity Investigation of Fault Tree Analysis with Matrix-Algebraic Method

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Abstract

The Fault Tree Analysis (FTA) is a systematic, deductive (top-down type) and probabilistic risk assessment tool which shows the causal relations leading to a given undesired event, referred to as the ”Top Event” (TE). The events, which cannot be subdivided, are called the Basic Events. Fault Tree diagram displays the undesired state of the investigated system (top event) in terms of the states of its components (basic events). The Fault Tree Analysis is a graphical design technique main result of which is a tree, a dendritic graph. Probabilistic Fault Tree Analysis (PFTA) is a quantitative analysis method used to calculate the probability of Top Event from given failure probabilities of system components. The objective of the sensitivity analysis is to show how the change in any system parameter influences the resultant reliability value of the whole system.

The main aim of this study is to elaborate an easy-used algorithm for setting-up of Linear Fault Tree Sensitivity Model (LFTSM). This modular approach process uses matrix-algebraic method based upon the mathematical diagnostic modeling of aircraft systems and gas turbine engines. The paper shows the adaptation of linear mathematical diagnostic modeling methodology for setting-up of LFTSM and its possibility of use to investigate Fault Tree sensitivity by a demonstrative example.

Keywords: Reliability, Availability, Maintenance, Sensitivity Analysis.
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1. Introduction

Several analytical methods of dependability and reliability analysis are available, of which Fault Tree Analysis (FTA) is one. The purpose of each method and their individual or combined applicability in evaluating the reliability and availability of a given system or component should be examined by the analysts before starting the FTA. Consideration should also be given to the results available from each method. Data required performing the analysis, complexity of analysis, and other factors identified in the Standard lEC 1025 (IEC, 1990).

The Bell Telephone Laboratories developed the concept of FTA at the beginning of the 1960s for the U.S. Air Force for use with the Minuteman system. It was later adopted and extensively applied by the Boeing Company. Fault Tree Analysis is one of many symbolic ”analytical logic techniques” found in operations research and in system reliability.

Probabilistic Fault Tree Analysis (PFTA) is a quantitative analysis method used to calculate the probability of Top Event from given failure probabilities of system components. For PFTA we have to know probability data of component failures (basic events), which can be hunted by:

- technical books;
- data delivered by manufacturer;
- laboratory evidences

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- experts' opinions;

- statistical analysis of maintenance data.

It is easy to see, that these data have non-negligible uncertainties. Therefore, the sensitivity and uncertainty analysis of FTA is an important task. The objective of the sensitivity analysis is to show how the change in any system parameter influences the resultant reliability value of the whole system.

The paper of Tchorzewska-Cieslak and Boryczko contains the methodology of the FTA and an example of its application in order to analyze different failure scenarios in water distribution subsystem. They concluded that the FTA is particularly useful for the analysis of complex technical systems in which analysis of failure scenarios is a difficult process because it requires to examine a high number of cause-effect relationship. Undoubtedly the water distribution subsystem belongs to such systems. The FTA involves "thinking back", which allows the identification of failure events that cause the occurrence of the Top Event. In the case of very large fault trees it is advisable to use the computer methods (Tchorzewska-Cieslak & Boryczko, 2010).

Siontorou, Batzias, and Tsakiri investigated the causes of biosensors' malfunction by means of FTA and proposed a computer-aided method for diagnosing biosensor failure during operation through an algorithmic procedure that is based on a nested loop mechanism. The tree (dendritic graph) structure serves as the knowledge base decision mechanism is the inference engine for fault detection and isolation (Siontorou *et al.*, 2010).

In article of Ekaette et al., authors described the application of probabilistic Fault Tree Methods to assess the probability of radiation misadministration to patients at a large cancer treatment center (Ekaette *et al.*, 2007). To populate the FT they used subjective probabilities from experts and compared results with incident report data. Both the Fault Tree and the incident report analysis revealed simulation tasks to be most prone to incidents, and the treatment prescription task to be least prone to incidents. Ekaette et al. have demonstrated that the Fault Tree Method is useful in modeling the probability of incidents in complex medical systems. They were able to evaluate the reliability of the FTA using incident reports. The FTA helps them to understand the type of incidents that could occur and therefore supports proactive risk analysis. The discussions and analysis of possible incident pathways throughout the process of building the FT provided the medical staff better insight of the treatment system as a whole, how their individual areas of expertise and duties interrelate, the vulnerable aspects of the tasks for which they are responsible, and possible systematic interventions for better provision of care.

An algorithm of vague Fault Tree Analysis is proposed in Chang et al.'s paper to calculate fault interval of system components from integrating expert's knowledge and experience in terms of providing the possibility of failure of bottom events. This paper also modifies Tanaka's definition on fault-tree analysis and integrates vague set arithmetic for implementing fault-tree analysis on weapon system fault diagnosis (Chang *et al.*, 2006).

The Author has studied mathematical modeling of deterministic and stochastic technical systems, types of mathematical models' uncertainties and the parametrical uncertainties' investigation methods in mathematical modeling, engineering simulation and maintenance management's decision making (Pokorádi & Szabolcsi, 1999), (Pokorádi, 2008), (Pokorádi, 2009*a*), (Pokorádi, 2009*b*), (Pokorádi, 2010).

A method of FT sensitivity analysis is shown in paper of Csiba by an example of the complete railway vehicle. The determination of resultant reliability is indispensable for the investigation. The determination of the reliability and the fulfillment of sensitivity analysis will be carried out by using the failure tree method. In the investigation of Csiba, the reliability of a railway carriage has been determined. The reliability model of the railway carriage has been built by 16 main constructional units (Csiba, 2008).

One of the disadvantages of Csiba's work, that only sensitivity of Top Event's probability can be investigated in case of changing of basic events' probabilities. His method cannot analyze sensitivities of intermediate events, which can represent sensitivity of element groups' or subsystems' reliability. Another disadvantage is that calculation of sensitivity coefficient can be difficult in a real, complicate situation. (It will be shown in Chapter 3 by demonstrative example of this paper)

The main aim of this study is to elaborate an easy-used algorithm for setting-up of Linear Fault Tree Sensitivity Model (LFTSM). This modular approach process uses matrix-algebraic method that is based upon the mathematical diagnostics methodology of aircraft systems and gas turbine engines by the Author (Pokorádi, 2008). This paper will show adaptation of linear mathematical diagnostic modeling methodology for setting-up of LFTSM and its possibility of use to investigate Fault Tree sensitivity by a demonstrative example.

The outline of the paper is as follows: Section 2 shows the Fault Tree Analysis shortly. Section 3 presents a modular approach algorithm for setting-up of Linear Fault Tree Sensitivity Model theoretically and by a practical demonstration. Section 4 interprets a simply application of the LFTSM. Section 5 summaries the paper, outlines the prospective scientific work of the Author.

2. The Fault Tree Analysis

The Fault Tree Analysis (FTA) is a systematic, deductive (top-down type) and probabilistic risk assessment tool which shows the causal relations leading to a given undesired event, referred to as the ”Top Event” (TE). The events, which cannot be subdivided, are called the Basic Events. Fault Tree diagram displays the undesired state of the investigated system (top event) in terms of the states of its components (basic events). The Fault Tree Analysis is a graphical design technique. The main result of FTA is a graph that has a dendritic structure.

The FTA can be used:

- to determine faults, fault-combinations that can occur the TE and their causes;
- to detect especially critical events and/or event-chains;
- to perform reliability and dependability investigations;
- to demonstrate failure-mechanisms illustratively.

The first step in a Fault Tree Analysis is the selection of the Top Event that is a specific undesirable system's state or failure. Then the experts should analyze the system or process to discover logical dependencies between TE and all Basic Events. To represent logical dependencies, basically the AND or OR logical gates and so-called intermediate events can be used. The intermediate events can denote subsystem's faults.

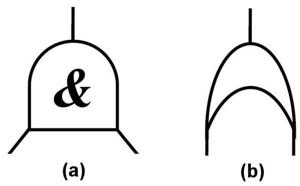

Figure 1. Logical Gates of Fault Tree.

The AND logical gate (figure 1.a) should be used if output event occurs only if all input events occur simultaneously. If output event occurs if any of the input events occur, either alone or in any combination, the OR logical gate (figure 1.b) should be used.

The Figure 2. shows a demonstrative Fault Tree. In the figure events 1; 2; 11 and 22 are intermediate events. The events 12; 21; 111; 112; 221 and 222 are Basic Events.

Probabilistic Fault Tree Analysis (PFTA) is a quantitative analysis method used to calculate the probability of TE from given failure probabilities of system components. The probability of occurrence of a (non-basic) event can be determined by probabilities of input events and the knowledge of logical gate describing connection.

In case of **AND** logical gate (Figure 1.a):

$$P = \prod_{i=1}^{n} P_i \tag{2.1}$$

where:

$P_i, \quad P_i \in [0,1] \subset \Re$ - probability of occurrence of i-th input event;

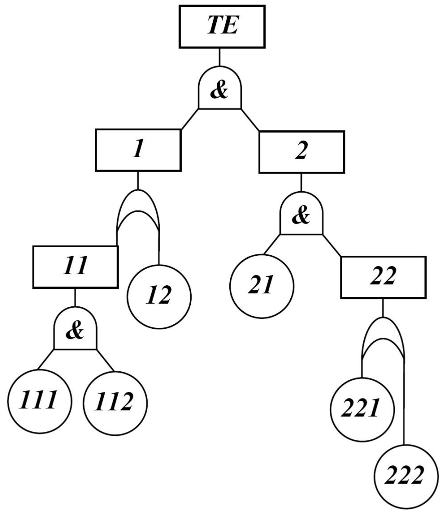

Figure 2. Fault Tree (Example).

$n, \quad n \in \aleph$ - number of input events.

In case of OR logical gate (Figure 1.b):

$$P = 1 - \prod_{i=1}^{n} (1 - P_i), \tag{2.2}$$

but for two-input **OR** gate logical gate:

$$P = 1 - ((1 - P_C)(1 - P_D)) = P_C + P_D - P_C P_D. \tag{2.3}$$

Using equations (2.1), (2.2) and (2.3) the model of investigated fault tree (Figure 2):

$$P_{TE} = P_1 P_2 \tag{2.4}$$

$$P_1 = P_{11} + P_{12} - P_{11} P_{12} \tag{2.5}$$

$$P_2 = P_{21} P_{22} \tag{2.6}$$

$$P_{11} = P_{111} P_{112} \tag{2.7}$$

$$P_{22} = P_{221} + P_{222} - P_{221} P_{222}. \tag{2.8}$$

For further investigation, firstly the nominal (average, typical) probabilities of occurrence of Basic Events should be recorded. The Table 1. shows nominal values of basic events' probabilities of occurrence. Then - using equations (2.4) - (2.8) - the probabilities of intermediate events and at last probabilities of the Top Event should be determined. The Table 2. shows the results of result of the used model.

Table 1. Primary Data.

$P_{12} = 0,01$	$P_{21} = 0,02$	$P_{111} = 0,05$
$P_{112} = 0,06$	$P_{221} = 0,04$	$P_{222} = 0,03$

Table 2. Probabilities Calculated by Equations (2.4) - (2.8).

$P_{22} = 0,0688$	$P_{11} = 0,003$
$P_2 = 0,001376$	$P_1 = 0,01297$
$P_{TE} = 1,1784610^{-5}$	

For demonstration of main essence of shown modular approach methodology, the probability of TE can be determined by equation

$$P_{TE} = (P_{111}P_{112} + P_{12} - P_{111}P_{112}P_{12})(P_{21}(P_{221} + P_{222} - P_{221}P_{222})). \tag{2.9}$$

3. Setting-up of Sensitivity Model

For sensitivity investigation, the sensitivity model of the discussed Fault Tree should be set-up. In this chapter the method of modular approach sensitivity model setting up will be depicted theoretically and demonstrated practically by the example of the sample FT mentioned above (Figure 2).

3.1. Theoretical Solution

To determine the sensitivity coefficient as a first step, the total differential of both sides of the initial equation

$$y = f(x_1, x_2, ..., x_n), f : \Re^n \to \Re, \tag{3.1}$$

should be formed:

$$dy = \frac{\partial f(x_1; x_2; \dots x_n)}{\partial x_1} dx_1 + \dots + \frac{\partial f(x_1; x_2; \dots x_n)}{\partial x_n} dx_n. \tag{3.2}$$

Then both sides of the last equation should be multiplied by same sides of the general equation and all elements should be multiplied by $\frac{x_i}{x_i}$

$$\frac{dy}{y} = \frac{\partial f(x_1; x_2; \dots x_n)}{\partial x_1} \frac{x_1}{f(x_1; x_2; \dots x_n) x_1} dx_1 + \dots + \frac{\partial f(x_1; x_2; \dots x_n)}{\partial x_n} \frac{x_n}{f(x_1; x_2; \dots x_n) x_n} dx_n \tag{3.3}$$

Introducing the sensitivity coefficients:

$$K_{y;x_i} = \frac{\partial f(x_1; x_2; \dots x_n)}{\partial x_i} \frac{x_i}{f(x_1; x_2; \dots x_n)} = \frac{\partial y}{\partial x_i} \frac{x_i}{y}, \tag{3.4}$$

a short sign is K_i, and

$$\frac{d\eta}{\eta} \approx \frac{\Delta\eta}{\eta} = \delta\eta \tag{3.5}$$

equation, the following linear system can be achieved:

$$\delta y = K_{y;x_1} \delta x_{y;x_2} + \dots + K_{y;x_n} \delta x_n. \tag{3.6}$$

Using equation, mentioned above, how sensitive dependent system output parameters will be to uncertainties of input ones.

The sensitivity coefficients of Fault Tree gates can be determined by the following way:

Using equation (2.1), in case of **AND** logical gate:

$$K_i = 1, \forall i \in \{1, 2, ..., n\} \tag{3.7}$$

where n is the number of causer events

Using equations (2.2) and (2.3) in case of **OR** logical gate:

$$K_j = \frac{P_j}{P} \prod_{\substack{i=1 \\ i \neq j}}^{n} (1 - P_i). \tag{3.8}$$

or

$$K_C = \frac{\partial (1 - ((1 - P_C)(1 - P_D)))}{\partial P_C} \frac{P_C}{P} = (1 - P_D) \frac{P_C}{P}. \tag{3.9}$$

Next task is to separate events of Fault Tree into basic events and non-basic (top and intermediate) ones. The probabilities of basic and non-basic events should be arranged into vectors **x** and **y**. Then, the connection between probabilities of basic and non-basic events can be described by

$$\mathbf{A}\delta\mathbf{y} = \mathbf{B}\delta\mathbf{x}, \tag{3.10}$$

where $\mathbf{A} \in \Re^{n\times n}$ and $\mathbf{B} \in \Re^{n\times m}$ are coefficient matrices of basic and non-basic events of the investigated Fault Tree, and $\delta\mathbf{y} \in \Re^{n\times 1}$, $\delta\mathbf{x} \in \Re^{m\times 1}$, **n** is the number of non-basic events, **m** is the number of basic events.

Using the

$$\mathbf{D} = \mathbf{A}^{-1}\mathbf{B} \in \Re^{n\times m} \tag{3.11}$$

relative sensitivity coefficient matrix of investigated Fault Tree, the equation

$$\delta\mathbf{y} = \mathbf{D}\delta\mathbf{x} \tag{3.12}$$

can be used for sensitivity investigations.

Using equation (3.5), vector of relative changing of basic events' probabilities of occurrences can be determined by

$$\delta\mathbf{x} = \mathbf{X}_{nom}^{-1}\Delta\mathbf{x}, \tag{3.13}$$

where:

$\mathbf{X}_{nom} \in \Re^{m\times m}$ - nominal values matrix of basis events' probabilities of occurrence:

$$\mathbf{X_{nom}} = \begin{bmatrix} P_{1nom} & 0 & \dots & 0 \\ 0 & P_{2nom} & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & \dots & 0 & P_{nnom} \end{bmatrix} \tag{3.14}$$

$\Delta\mathbf{x} \in \Re^{m\times 1}$ - vector of measured changing of basis events' probabilities. Knowing the vector measured changing of basis events' probabilities, by relative sensitivity coefficient matrix - equation (3.12) - the vector of relative changing of non-basic event' probabilities

$$\delta\mathbf{y} = \mathbf{D}\delta\mathbf{x} = \mathbf{D}\mathbf{X}_{nom}^{-1}\Delta\mathbf{x} \tag{3.15}$$

can be determined. Applying the nominal values matrix of non-basis events' probabilities of occurrence $\mathbf{Y}_{nom} \in \Re^{n\times n}$, the vector of measured changing of non-basis events' probabilities:

$$\Delta\mathbf{y} = \mathbf{Y}_{nom}\delta\mathbf{y} = \mathbf{Y}_{nom}\mathbf{D}\mathbf{X}_{nom}^{-1}\Delta\mathbf{x} \tag{3.16}$$

Introducing measured sensitivity coefficient matrix of investigated Fault Tree

$$\mathbf{S} = \mathbf{Y}_{nom}\mathbf{D}\mathbf{X}_{nom}^{-1}, \tag{3.17}$$

the equation (3.16) can be simplified

$$\Delta\mathbf{y} = \mathbf{S}\Delta\mathbf{x}, \tag{3.18}$$

where:

$\mathbf{Y}_{nom} \in \Re^{n\times n}$ - nominal values matrix of basis events' probabilities of occurrence:

$$\mathbf{Y}_{nom} = \begin{bmatrix} P_{1nom} & 0 & \dots & 0 \\ 0 & P_{2nom} & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & \dots & 0 & P_{mnom} \end{bmatrix}. \tag{3.19}$$

The vector of measured changing of basic events' probabilities can be determined by

$$\Delta \mathbf{x} = \mathbf{x} - \mathbf{x}_{nom} \tag{3.20}$$

where: $\mathbf{x}_{nom}$ - nominal values vector of basis events' probabilities of occurrence. The vector of measured changing of non-basic events' probabilities can be determined by

$$\Delta \mathbf{y} = \mathbf{y} - \mathbf{y}_{nom} \tag{3.21}$$

where: $\mathbf{y}_{nom}$ - nominal values vector of non-basic events' probabilities of occurrence. Using equations (3.18); (3.20) and (3.21) vector of measured values of non-basic events' probabilities can be determined by

$$\mathbf{y} = \mathbf{y}_{nom} + \mathbf{S}\left(\mathbf{x} - \mathbf{x}_{nom}\right). \tag{3.22}$$

If during a Fault Tree Analysis only the probability of TE and its sensitivities are investigated only the first row of matrix $\mathbf{S}$ will be used as vector $\mathbf{s} \in \Re^{m\times 1}$. So the changing of TE probability of occurrence can be calculated by

$$\Delta P_{TE} = \mathbf{s}^T \Delta \mathbf{x}, \tag{3.23}$$

and

$$P_{TE} = P_{TEnom} + \mathbf{s}^T \left(\mathbf{x} - \mathbf{x}_{nom}\right). \tag{3.24}$$

3.2. Practical Demonstration

To demonstrate setting-up methodology mentioned above, lets' study the Fault Tree shown by Figure 2. The sensitivity coefficients of the investigated Fault Tree - using equations (2.4) - (2.8) are:

$$\delta P_{TE} = K_1 \delta P_1 + K_2 \delta P_2 \tag{3.25}$$

$$K_1 = 1, K_2 = 1$$

$$\delta P_1 = K_{11} \delta P_{11} + K_{12} \delta P_{12} \tag{3.26}$$

$$K_{11} = \left(1 - P_{12}\right) \frac{P_{11}}{P_1}, K_{12} = \left(1 - P_{11}\right) \frac{P_{12}}{P_1}$$

$$\delta P_2 = K_{21} \delta P_{21} + K_{22} \delta P_{22} \tag{3.27}$$

$$K_{22} = 1, K_{21} = 1$$

$$\delta P_{11} = K_{111} \delta P_{111} + K_{112} \delta P_{112} \tag{3.28}$$

$$K_{111} = 1, K_{112} = 1$$

$$\delta P_{22} = K_{221} \delta P_{221} + K_{222} \delta P_{222} \tag{3.29}$$

$$K_{221} = \left(1 - P_{222}\right) \frac{P_{221}}{P_{22}}, K_{222} = \left(1 - P_{221}\right) \frac{P_{222}}{P_{22}}$$

The vectors of probabilities of basic, and non- basic events are:

$$\mathbf{x}^T = \left[P_{12}; P_{21}; P_{111}; P_{112}; P_{221}; P_{222}\right], \tag{3.30}$$

$$\mathbf{y}^T = \left[P_{TE}; P_1; P_2; P_{11}; P_{22}\right]. \tag{3.31}$$

The coefficient matrices of basic, and non-basic events are:

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & -1 & 0 & 0 \\ 0 & 1 & 0 & -0.229 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}; \tag{3.32}$$

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0,767 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0,564 & 0,419 \end{bmatrix}. \tag{3.33}$$

The relative sensitivity matrix:

$$\mathbf{D} = \begin{bmatrix} 0,769 & 1 & 0,229 & 0,229 & 0,564 & 0,419 \\ 0,769 & 0 & 0,229 & 0,229 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0,564 & 0,419 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0,564 & 0,419 \end{bmatrix}. \tag{3.34}$$

By tables 1. and 2. the matrices of nominal values of basic, and non-basic events are:

$$\mathbf{X}_{nom} = \begin{bmatrix} 0,01 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0,02 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0,05 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0,06 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0,04 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0,03 \end{bmatrix}; \tag{3.35}$$

$$\mathbf{Y}_{nom} = \begin{bmatrix} 1,178 \cdot 10^{-5} & 0 & 0 & 0 & 0 \\ 0 & 1,297 \cdot 10^{-2} & 0 & 0 & 0 \\ 0 & 0 & 1,376 \cdot 10^{-3} & 0 & 0 \\ 0 & 0 & 0 & 3 \cdot 10^{-3} & 0 \\ 0 & 0 & 0 & 0 & 6,88 \cdot 10^{-2} \end{bmatrix}. \tag{3.36}$$

The vectors of nominal values of basic, and non- basic events are:

$$\mathbf{x}_{nom}^T = \begin{bmatrix} 0,01 & 0,02 & 0,05 & 0,06 & 0,04 & 0,03 \end{bmatrix}, \tag{3.37}$$

$$\mathbf{y}_{nom}^T = \begin{bmatrix} 1,178 \cdot 10^{-5} & 1,297 \cdot 10^{-2} & 1,376 \cdot 10^{-3} & 3 \cdot 10^{-3} & 6,88 \cdot 10^{-2} \end{bmatrix}. \tag{3.38}$$

The measured sensitivity matrix of the demonstrative FT (Figure 2):

$$\mathbf{S} = \begin{bmatrix} 1,372 \cdot 10^{-3} & 8,923 \cdot 10^{-4} & 8,173 \cdot 10^{-5} & 6,811 \cdot 10^{-5} & 2,516 \cdot 10^{-4} & 2.490 \cdot 10^{-4} \\ 0,997 & 0 & 5,940 \cdot 10^{-2} & 4,950 \cdot 10^{-2} & 0 & 0 \\ 0 & 6,880 \cdot 10^{-2} & 0 & 0 & 1,940 \cdot 10^{-2} & 1,940 \cdot 10^{-2} \\ 0 & 0 & 5,999 \cdot 10^{-2} & 0,500 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0,970 & 0,960 \end{bmatrix}, \tag{3.39}$$

and measured sensitivity vector of TE:

$$\mathbf{s}^T = \begin{bmatrix} 1,372 \cdot 10^{-3} & 8,923 \cdot 10^{-4} & 8,173 \cdot 10^{-5} & 6,811 \cdot 10^{-5} & 2,516 \cdot 10^{-4} & 2.490 \cdot 10^{-4} \end{bmatrix}. \tag{3.40}$$

3.3. Discussion

Using method shown by reference (Csiba, 2008), the sensitivity coefficients can be determined by derivations - see equations (3.1) - (3.4)) - of equation (2.9). It is easy to see that this task can be difficult or error-risky for engineers or technical expert (who is not mathematician) in a real, complicate Fault Tree Analysis. Because in cases of real system reliability analysis, the Fault Tree can has more then 5 ~ 6 levels of intermediate events. In addition the result of sensitivity investigation shows only the Top Event's sensitivity. The analyzers cannot gather information about the sensitivities of intermediate events that is reliability and dependability of element groups or subsystems. This result can be got using shown matrix algebraic method and equation (3.24).

On the other hand, advantages of the modular approach method shown above are followings:

The connections between input and output event of all logical gates can be written easily - using equations (2.1), (2.2) or (2.3). Using these connections, the elements of vectors and matrices of nominal values of basic ($\mathbf{x}_{nom}$ and $\mathbf{X}_{nom}$) and non-basic ($\mathbf{y}_{nom}$ and $\mathbf{Y}_{nom}$) events' probabilities can be determined.

Therefore, employing equations (3.6) or (3.7), the sensitivity coefficients of all logical gates as modules of investigated Fault Tree that is elements of coefficient matrices of basic (**A**) and non-basic (**B**) events can be determined easily.

After determination of required vectors and matrices, the experts can use matrix-algebraic method to analyze sensitivity characters of given Fault Tree from given investigational point of view. (The next Chapter will show a simply example of possibility of use Linear Fault Tree Sensitivity Model.)

4. Application of Linear Fault Tree Sensitivity Model

The Linear Fault Tree Sensitivity Model (LFTSM) elaborated and set up above can be used to investigate reliability and dependability of a technical system analyzed by Fault Tree.

Knowing the sensitivity model of Fault Tree, sensitivity of the Top Event and intermediate events can be investigated by modification of vector of basic events' probabilities (as independent variables)$\Delta\mathbf{x}$. Results of sensitivity analysis can be used for conclusions to come about features of the given system and its behavior in case of simulated basic events' probability changing. This changing of probability can be produced by constructional modification of given element that occur the modeled probability changing, of investigated technical system.

The essence of the sensitivity analysis is that the anomalies and variations of dependent system parameters are simulated by changing of its independent (input and inner) variables. On the basis of the mathematical model of the investigated system can be determined how sensitive dependent system variables will be for simulated changes. If only one of independent variables is changed, the investigation will be called one-parameter sensitivity analysis. If the number of the changed independent variables is more than one, the several-parameter sensitivity analysis is used.

It is important to mention that changes of independent variables cannot be more than about 1 or 5 %, depending on the intensity of the original model's nonlinearity. Depending on the nonlinearity of the original model, results of the sensitivity analysis can have difference from real influences of simulated changes. But these results show the direction and order of the magnitude of real simulated changes.

Six one-parameter sensitivity investigations of the Fault Tree shown by figure 2. - using equations (3.24) and (3.40) - were performed. During investigations probabilities of every basic events were decreased (in other words reliability of all components were improved) by 0,005.

The Figure 3. and Table 3. show the results. It is seeable, that effects of basic events' probabilities 12 and 21 are the largest to the probability of Top Event reliability that is reliability of the system.

Table 3. Results of one-parameter sensitivity investigations of Top Events.

Event	12	21	111	112	221	222
Nominal	$1{,}17846\ 10^{-5}$					
Modeled	$4{,}925\ 10^{-06}$	$7{,}323\ 10^{-06}$	$1{,}138\ 10^{-05}$	$1{,}144\ 10^{-05}$	$1{,}053\ 10^{-05}$	$1{,}054\ 10^{-05}$

The Figure 4 and Table 4 show the results of increasing by 0,005 of basic event 12 probability (from 0,01 to 0,015). In the diagram can be seen that only probability of Top Event and intermediate event 1 will change on account of increasing of probability of basic event 12. It is important to mention that axe calibration of probability is a logarithmic one.

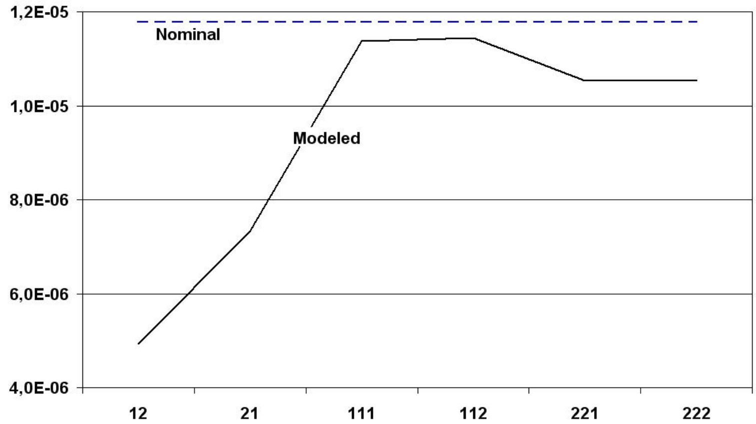

Figure 3. Results of one-parameter sensitivity investigation of Top Events.

Table 4. Results of one-parameter sensitivity investigation ($\Delta P_{12} = 0,005$).

Event	TE	1	2	11	22
Nominal	$1,17846\ 10^{-5}$	0,01297	0,001376	0,003	0,0688
Modeled	$8,64403\ 10^{-05}$	0,017955	0,001376	0,003	0,0688

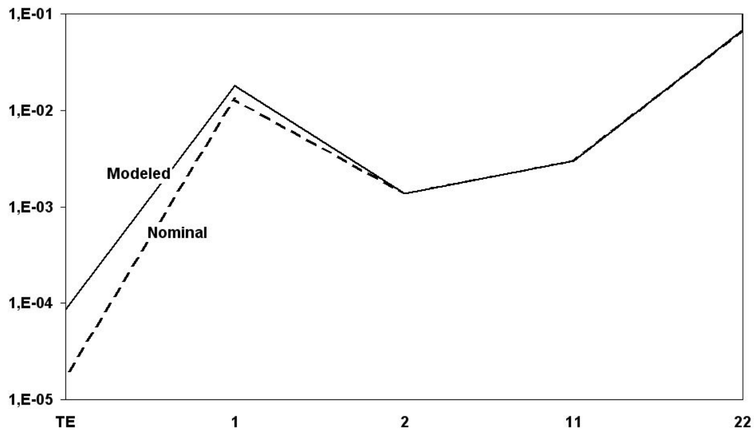

Figure 4. Results of one-parameter sensitivity investigation ($\Delta P_{12} = 0,005$).

5. Conclusions

This paper discussed a new sensitivity investigation method of Fault Tree Analysis elaborated by the Author. The paper showed the adaptation of linear mathematical diagnostic modeling methodology for setting-up of Linear Fault Tree Sensitivity Model (LFTSM) The LFTSM is a modular approach tool that uses matrix-algebraic method based upon the mathematical diagnostics methodology of aircraft systems and gas turbine engines. In this paper the possibility of use of LFTSM was demonstrated to investigate Fault Tree sensitivity by a simply example. Using demonstrated mathematical connections and procedure, the technical experts can get a easy-used methodology to build up sensitivity model of the given FT. This Linear Fault Tree Sensitivity Model (LFTSM) can be used to investigate system reliability and dependability from required point of view.

During prospective scientific research related to this field of applied mathematics and maintenance management decision making, the Author would like to work out methodologies of Fault Tree uncertainty investigation using other mathematical tools, for example linear interval equations, Monte-Carlo simulation and fuzzy set theory, on basis of Linear Fault Tree Sensitivity Model (LFTSM).

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