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A Simplified Architecture of Type-2 TSK Fuzzy Logic Controller for Fuzzy Model of Double Inverted Pendulums

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Abstract

This paper proposes a novel inference mechanism for an interval type-2 Takagi-Sugeno-Kang fuzzy logic control system (IT2 TSK FLCS). This paper focuses on control applications for case both plant and controller use A2-C0 TSK models. The defuzzified output of the T2FLS is then obtained by averaging the defuzzified outputs of the resultant four embedded T1FLSs in order to reduce the computational burden of T2 TSK FS. A simplified T2 TSK FS based on a hybrid structure of four type-1 fuzzy systems (T1 TSK FS). A simulation example is presented to show the effectiveness of this method.

Keywords: Fuzzy control systems, simplified type-2 fuzzy logic system, double inverted pendulums.

1. Introduction

Fuzzy systems of Takagi-Sugeno (T-S) models (Takagi & Sugeno, 1985) have become an effective method to represent nonlinear system by fuzzy sets and fuzzy reasoning. In (Echanobe *et al.*, 2005) presented some important aspects concerning the analysis and implementation of a piecewise linear (PWL) fuzzy model with universal approximation capability. Reference (Sadighi & jong Kim, 2010) presented a combination of a Sugeno fuzzy model and neural networks. In (Guechi *et al.*, 2010) presented a new technique for tracking-error model-based Parallel Distributed Compensation (PDC) control and stabilizing controller by solved by LMI conditions for the tracking-error model.

A new stability analysis method for nonlinear processes with T-S fuzzy logic controllers (FLCs) without process linearization and without using the quadratic Lyapunov functions in the derivation and proof of the stability conditions was designed in (Tomescu *et al.*, 2007). In (Precup *et al.*, 2009) studied a new framework for the design of generic two-degree-of-freedom (2-DOF), linear and fuzzy, controllers dedicated to a class of integral processes specific to servo systems.

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Fuzzy systems first introduced by Zadeh. The membership degree of Type-1 fuzzy set is crisp value but it isT1FS in the Type-2 fuzzy sets (Mendel, 2001; Wu & Mendel, 2001; Mendel, 2007). Researchers have shown that T1FLS have difficulty in modeling and minimizing the effect of uncertainties (Zadeh, 1975).

In (Biglarbegian *et al.*, 2010), the WuMendel uncertainty bounds (WM UBs) to design stable interval type-2 TSK fuzzy logic control systems (IT2 TSK FLCS). Proposed Inference Methods for IT2 TSK FLCSs in (Mohammad, 2010). In (Ren *et al.*, 2011) showed IT2 TSK FLSs analyzes the sensibility of the outputs of a type-2 TSK fuzzy system, and discusses the approximation capacities of type-2 TSK FLS and its type-1 counterpart. In (Wu & Tan, 2004) the study is conducted by utilizing a type-2 FLC, evolved by a genetic algorithm(GA), to control a liquid-level process.The proposed algorithm of interval type-2 TSK FLS has been used in fuzzy modeling and uncertainty prediction in high precision manufacturing (Ren *et al.*, 2009).

In this paper, Proposed the new inference mechanisms. we reduced the computational burden of T2 TSK FS.A simplified T2 TSK FS have a hybrid structure of four type-1 fuzzy systems (T1 TSK FS).The final output of the T2 TSK FLS is then obtained by averaging the defuzzified outputs of each T1 TSK FLC. The rest of the chapter is organized as follows: Section II, we present an overview of dynamic Takagi Sugeno systems. In this section, deals with analytical design of Type-2 TSK fuzzy control and introduces the proposed simplified implementation of T2 TSK FLS using four embedded T1FSs. Some simulations are executed to verify the validity of the proposed approach in Section III. Section IV concludes the paper.

2. Takagi-sugeno fuzzy model

A dynamic T-S fuzzy model is described by a set of fuzzy IF THEN rules with fuzzy sets in the antecedents and dynamic linear time-invariant systems in the consequents. A generic T-S plant rule can be written as follows (Dorato *et al.*, 1995; Khaber *et al.*, 2006):

$$i^{th} Plant\ Rule:\ IF x_1(t)\ is\ M_{i1} \ldots, x_n(t)\ is\ M_{in}\ THEN\ \dot{x} = A_i x + B_i u,$$

where $x_{n\times1}$ is the state vector, r is the number of rules, M_{ij} are input fuzzy sets, $u_{m\times1}$ is the input and $A_{n\times n}$, $B_{n\times m}$ are state matrix and input matrix respectively. Using singleton fuzzifier, max-product inference and center average defuzzifier, we can write the aggregated fuzzy model as:

$$\dot{x} = \frac{\sum_{i=1}^{r} \omega_i(x)(A_i x + B_i u)}{\sum_{i=1}^{r} \omega_i(x)}, \tag{2.1}$$

with the term ω_i is defined by:

$$\omega_i(x) = \prod_{j=1}^{n} \mu_{ij(x_j)}, \tag{2.2}$$

where μ_{ij} is the membership function of the jth fuzzy set in the ith rule. Defining the coefficients α_i as:

$$\alpha_i = \frac{\omega_i}{\sum_{i=1}^{r} \omega_i} \tag{2.3}$$

we can write (2.1) as:

$$\dot{x} = \sum_{i=1}^{r} \alpha_i(x)(A_i x + B_i u) i = 1, \ldots, r, \tag{2.4}$$

where $\alpha_i > 0$ and $\sum_{i=1}^{r} \alpha_i (x) = 1$.

Using the same method for generating T-S fuzzy rules for the controller, we have:

$$i^{th} controllerRule:$$

$$IF x_1(t) \; is \; M_1^i \; and \; \ldots x_n(t) \; is \; M_n^i \; then \; u(t) = -K_i x(t), \; i = 1, \ldots, r,$$

The over all controllers would be

$$u = -\sum_{i=1}^{r} \alpha_i (x) K_i x. \tag{2.5}$$

Replacing (2.5) in (2.4), we obtain the following equation for the closed loop system:

$$\dot{x} = \sum_{i=1}^{r} \sum_{j=1}^{r} \alpha_i (x) \alpha_j (x) (A_i x + B_i u) x. \tag{2.6}$$

3. IT2 TSK FLSs

This chapter first presents the design of IT2 TSK FLSs for modeling and control applications. Second, WM UBs are introduced and third, a new inference engines for IT2 TSK FLSs are introduced. The general structure of an interval A2-C0 TSK model for a system is as follows (Mohammad, 2010):

$$If \; x_1 \; is \; \tilde{F}_1^i \; and \; x_2 \; is \; \tilde{F}_2^i \; and \; \ldots x_n \; is \; \tilde{F}_n^i, \; Then \; y_i = a_0^i x_1 + a_0^i x_2 + \ldots + a_n^i x_n \tag{3.1}$$

where $\tilde{F}_j^i$, $i = 1, \ldots, M$ represents the IT2 FS of input state j in rule i, $x_1, \ldots, x_n$ are states, $a_0^i, \ldots, a_n^i$ are the coefficients of the output function for rule i (and hence are crisp numbers, i.e., type-0 FSs), y_i is the output of the i^{th} rule, and M is the number of rules. The above rules allow us to model the uncertainties encountered in the antecedents.

In an IT2 TSK A2-C0 model, $\bar{f}^i(x)$ and $\underline{f}^i(x)$, lower and upper firing strengths of the i^{th} rule, respectively, are given by

$$\bar{f}^i(x^*) = \bar{\mu}_{\tilde{F}_1^i}(x_1) \star \ldots \star \bar{\mu}_{\tilde{F}_n^i}(x_n), \tag{3.2}$$

$$\underline{f}^i(x^*) = {}_{\tilde{F}_1^i}(x_1) \star \ldots \star {}_{\tilde{F}_n^i}(x_n), \tag{3.3}$$

where $\underline{\mu}_{\tilde{F}_j^i}$ and $\bar{\mu}_{\tilde{F}_j^i}$ represent the j^{th} ($j = 1 \ldots M$) lower and upper MFs of rule i, and "$\star$" is a t-norm operator. State vector is defined as

$$x = [x_1, x_2, \ldots, x_n]^T \tag{3.4}$$

The final output of the IT2 TSK A2-C0 is given as:

$$Y_{TSK/A2-C0} = [y_l, y_r] = \int_{f^1\in\left[\underline{f}^1,\bar{f}^1\right]} \ldots \int_{f^M\in\left[\underline{f}^M,\bar{f}^M\right]} 1\Bigg/\frac{\sum_{k=1}^{M} f^i(x)y_i}{\sum_{k=1}^{M} f^i(x)}, \tag{3.5}$$

where y_i is given by the consequent part of (3.1). $Y_{TSK/A2-C0}$ is an interval T1 set and only depends on its left and right end-points y_l, y_r, which can be computed using the iterative KM algorithms. Therefore, the final output is given as The final output of the IT2 TSK A2-C0 is given as:

$$Y_{output}(x) = \frac{y_r(x) + y_l(x)}{2}. \tag{3.6}$$

KM Algorithm (Mohammad, 2010):

The KM algorithm presents iterative procedures to compute y_l, y_r in as follows:

Set $y^i = y_l^i$ (or y_r^i) for $i = 1, \ldots, N$;

Arrange y^i in ascending order;

Set $f^i = \frac{\underline{f}^i+\bar{f}^i}{2}$ for $i = 1, \ldots, N$;

$y' = \frac{\sum_{i=1}^{N} y^i f^i}{\sum_{i=1}^{N} f^i}$;

Do

$y'' = y'$;

Find $k \in [1, N-1]$ such that $y^k \leq y' \leq y^{k+1}$;

Set $f^i = \bar{f}^i$ (or ^{i}) for $i \leq k$;

Set $f^i = \underline{f}^i$ (or $\bar{f}^i$) for $i \geq k+1$;

$y' = \frac{\sum_{i=1}^{N} y^i f^i}{\sum_{i=1}^{N} f^i}$;

While $y' \neq y''$

y_l (or y_r) $= y'$.

It has been proven that this iterative procedure can converge in at most N iterations (Mohammad, 2010).

4. A simplified implementation of T2 TSK FS

As shown in the Figure 1, each T2MF can represents by two T1MFs, upper MF and lower MF. Therefore, each one of two neighbor T2MFs intersects each other in four points and object to get four MFs, upper MF, lower MF, left MF and right MF showing in Figure 2 (Hameed *et al.*, 2011). Thus four T1 TSK Fuzzy controller supplanted are used discretely. The MFs in each controller supplanted by upper MF, lower MF, left MF and right MF, and will create upper fuzzy controller (UFC), lower fuzzy controller (LFC), left fuzzy controller (LEFTFC) and right fuzzy controller (RFC) respectively.

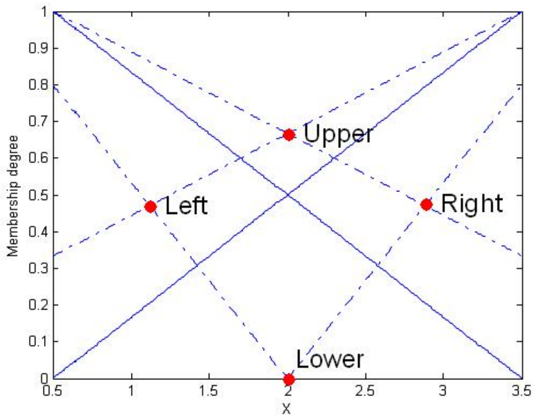

Figure 1. Illustration of decomposing T2MFs into 4 T1MFs.

The defuzzified output of the T2FLS is then obtained by averaging the defuzzified outputs of the resultant four embedded T1FLSs, as shown in Figure 3.

$$Y(x) = \frac{1}{4}y_{upper}(x) + \frac{1}{4}y_{lower}(x) + \frac{1}{4}y_{left}(x) + \frac{1}{4}y_{right}(x)\,. \tag{4.1}$$

5. Simulation

A two-inverted pendulum system is shown in Figure 4. It consists of two cart-pole inverted pendulums. The inverted pendulums are linked by a spring in the middle. The carts will move to and from during the operation. The control objective is to balance the inverted pendulums vertically despite the movings of the spring and carts by applying forces to the tips of the pendulums. Referring to Figure 4, M and m are the masses of the carts and the pendulums, respectively, m=10 kg and M =100 kg. L =1 m is the length of the pendulums. The spring has a stiffness constant $k = 1N/m$. $y_1(t) = sin(2t)$ and $y_2(t) = L + sin(3t)$ are the trajectories of the moving carts. $u_1(t)$ and $u_2(t)$ are the forces applied to the pendulums. $\theta_1(t)$ and $\theta_2(t)$ are the angular displacements of the pendulums measured from the vertical. The dynamic equation of the two-inverted pendulum system can be written as follows (Lam *et al.*, 2000):

$$\dot{X} = A(x(t))\,x(t) + Bu(t) \tag{5.1}$$

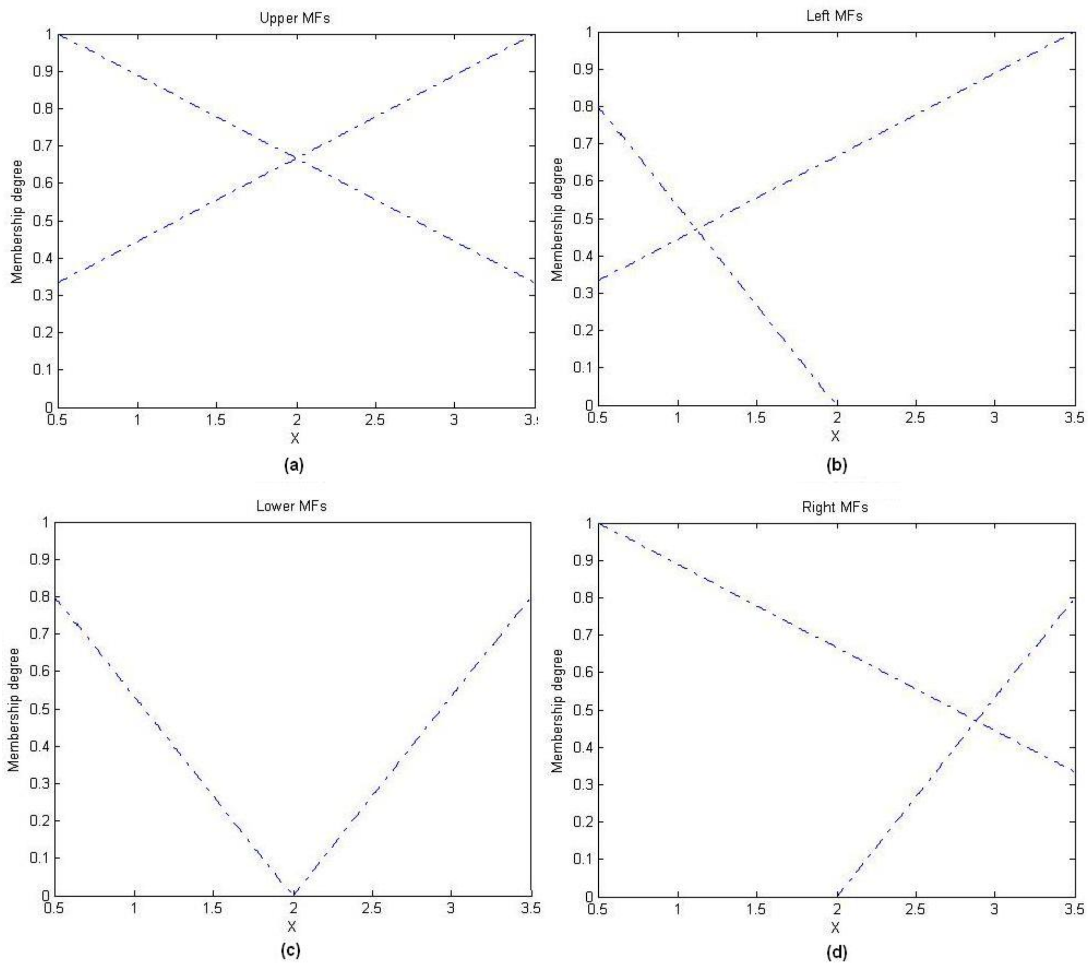

Figure 2: (a) Membership functions of upper intersection points. (b) Membership functions of left intersection points. (c) Membership functions of lower intersection points, and (d) Membership functions of right intersection points.

Where

$$X(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} = \begin{bmatrix} \theta_1(t) \\ \dot{\theta}_1(t) \\ \theta_2(t) \\ \dot{\theta}_2(t) \end{bmatrix}, \; x_1 \in [x_{1min} x_{1max}] = \left[-\tfrac{\pi}{2} \tfrac{\pi}{2}\right], \; x_3 \in [x_{3min} x_{3max}] = \left[-\tfrac{\pi}{2} \tfrac{\pi}{2}\right],$$

$$A(x(t)) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ f_1(x_1(t)) & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & f_2(x_3(t)) & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ \lambda & 0 \\ 0 & 0 \\ 0 & \lambda \end{bmatrix} \text{ and } u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix},$$

$$f_1(t) = \frac{2}{L} - \frac{m}{M}\sin(x_1(t))\, x_1(t), \; f_2(t) = \frac{2}{L} - \frac{m}{M}\sin(x_3(t))\, x_3(t) \text{ and } \lambda = \frac{2}{mL^2}.$$

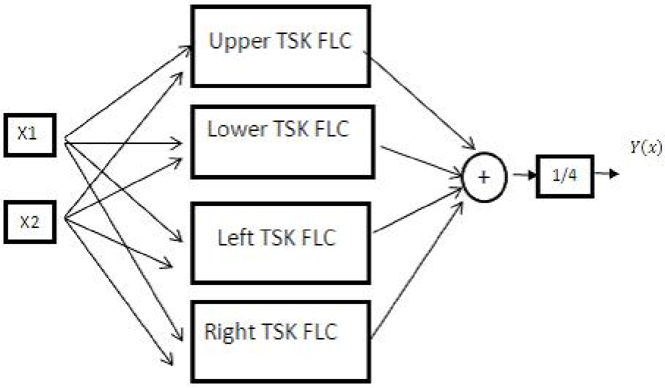

Figure 3. Simplified type-2 TSK fuzzy Logic Controller.

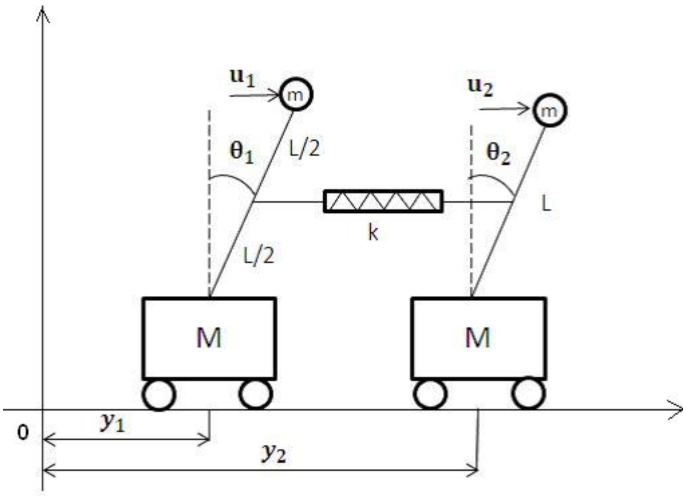

Figure 4. Two-inverted pendulum system.

A four-rule TS-fuzzy plant model is used to represent the two inverted pendulum system. The *i-th* rule of the TS-fuzzy plant model is given by

$$Rule\ i = IF f_1\left(x_1\left(t\right)\right)\ is\ M_{i1}\ and\ f_2\left(x_3\left(t\right)\right)\ is\ M_{i2}\ then\ \dot{X} = A_i x\left(t\right) + Bu\left(t\right),\ i = 1,2,3,4 \quad (5.2)$$

where M_i is a fuzzy term of rule i, $i = 1,2,3,4$. Then, the system dynamics is described by

$$\dot{X} = \sum_{i=1}^{4} w_i \left[A_i x\left(t\right) + Bu\left(t\right)\right], \quad (5.3)$$

where

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ f_{1min} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & f_{2min} & 0 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ f_{1min} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & f_{2max} & 0 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ f_{1max} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & f_{2min} & 0 \end{bmatrix}, A_4 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ f_{1max} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & f_{2max} & 0 \end{bmatrix},$$

$$w_i = \frac{\mu_{M_1^i}(f_1(x_1(t))) \times \mu_{M_2^i}(f_2(x_3(t)))}{\sum_{i=1}^{4}\left(\mu_{M_1^i}(f_1(x_1(t))) \times \mu_{M_2^i}(f_2(x_3(t)))\right)},$$

$\mu_{M_1^\beta}(f_1(x_1(t))) = \frac{-f_1(x_1(t))+f_{1max}}{f_{1max}-f_{1min}}$ for $\beta = 1,2$ and $\mu_{M_1^\delta}(f_1(x_1(t))) = 1-\mu_{M_1^1}(f_1(x_1(t)))$ for $\delta = 3,4$
$\mu_{M_1^\varepsilon}(f_2(x_3(t))) = \frac{-f_2(x_3(t))+f_{2max}}{f_{2max}-f_{2min}}$ for $\varepsilon = 1,3$ and $\mu_{M_1^\phi}(f_2(x_3(t))) = 1-\mu_{M_2^1}(f_2(x_3(t)))$ for $\delta = 2,4$
$f_{1max} = \frac{2}{L} + x_{1max}$ and $f_{1min} = \frac{2}{L} + x_{1min}$, $f_{2max} = f_{1max}$ and $f_{2min} = f_{1min}$.

Figure 5 shows a controller in which the inputs are the states *x(k)* and the output is *u(k)*. For this system, the general *i-th* rule has the following form:

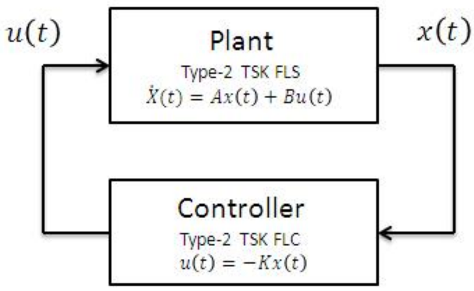

Figure 5. Closed-loop T2 TSK fuzzy control system.

To compare the performance of the IT2 TS FLC with the T1 controller, the model of the plant is kept as a T1 TS and only the controller is replaced with an IT2 TS model. To make a fair comparison, the parameters of the plants and controllers are kept unchanged for four control systems, and only the MFs for the IT2 controller are designed. MFs for this Example showed Figures 1 and 2. In this paper, the simplified Type 2 TSK Fuzzy controller of scaling factors are tuned by trial-and-error approach. A four-rule fuzzy controller is designed as following equation (Lam *et al.*, 2000).

$$Rule\ i = If\ x_1(t)\ is\ \tilde{M}_1^i\ and\ x_3(t)\ is\ \tilde{M}_2^i\ then\ u(t) = G_j x(t)\ j = 1,2,3,4.$$

The feedback gains for each fuzzy controller are then chosen as:

$$G_1 = \begin{bmatrix} -116.6410 & -119.7827 & -95.0589 & -39.6463 \\ -79.0293 & -40.024 & -260.5216 & -180.2173 \end{bmatrix},$$

$$G_2 = \begin{bmatrix} -116.6410 & -119.7827 & -95.0589 & -39.6463 \\ -97.0293 & -40.024 & -260.5216 & -180.2173 \end{bmatrix},$$

$$G_3 = \begin{bmatrix} -179.4729 & -119.7827 & -95.0589 & -39.6463 \\ -97.0293 & -40.024 & -260.5216 & -180.2173 \end{bmatrix}$$

and

$$G_4 = \begin{bmatrix} -179.4729 & -119.7827 & -95.0589 & -39.6463 \\ -97.0293 & -40.024 & -323.3534 & -180.2173 \end{bmatrix}$$

The zero-input responses of the system under the initial conditions:

$$x(0)\,(rad) = [\frac{88\pi}{180}0 - \frac{88\pi}{180}0].$$

The responses for T1 TSK Fuzzy and simplified T2 TSK Fuzzy controllers are shown in Figures 6-7 comparison between the two types of TSK FLCs have done. The reciprocal of the Root squared error (RMSE) of the response showed in Table I.

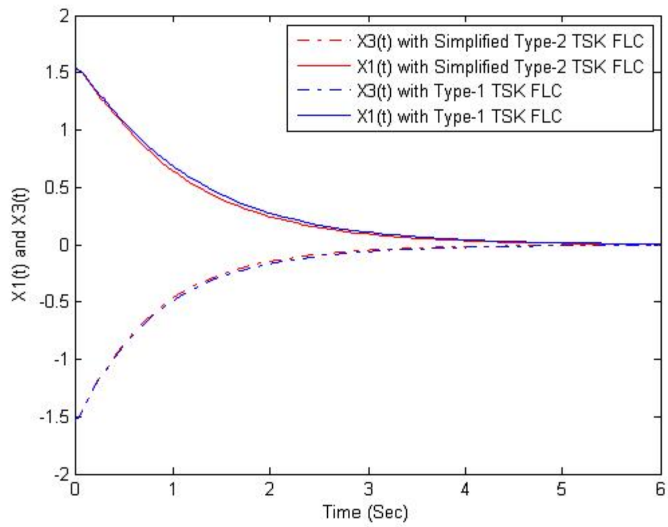

Figure 6: Responses of (solid line) and (dotted line) of the two-inverted pendulum system under T1 TSK FLC and Simplified T2 TSK FLC with M = 100 kg.

Table 1
RMSE of the responses

RMSE	X1(t)	X2(t)	X3(t)	X4(t)
T1 TSK FLC	6.8299	3.8769	6.3953	5.1933
T2 TSK FLC	6.7483	4.0474	6.3542	5.3271

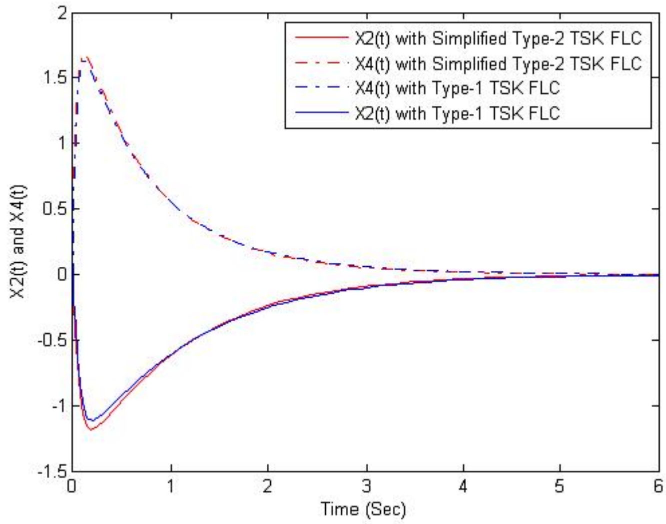

Figure 7: Responses of (solid line) and (dotted line) of the two-inverted pendulum system under T1 TSK FLC and Simplified T2 TSK FLC with M = 100 kg.

6. Conclusion

The nonlinear, T1, and IT2 controllers are capable of stabilizing the system. With attention to table. 1 in before section, value RMSE reduced in the X1(t) and X3(t) in the T2 TSK FLC with respect to T1 TSK FLC. Therefore output system is robustness. In this case study, it is shown that the proposed IT2 TSK FLC is capable of stabilizing the coupled two inverted pendulum while achieving a better performance compared to its T1 TSK FLC

References

Biglarbegian, M., W. W. Melek and J. M. Mendel (2010). On the stability of interval type-2 TSK fuzzy logic control systems. *Systems, Man, and Cybernetics, Part B: Cybernetics, IEEE Transactions on* **40**(3), 798 –818.

Dorato, P., C.T . Abdallah and V. Cerone (1995). *Linear Quadratic Control: An Introduction*. PrenticeHall, Englewood Cliffs, NJ.

Echanobe, J., I. del Campo and J.M. Tarela (2005). Issues concerning the analysis and implementation of a class of fuzzy controllers. *Fuzzy Sets and Systems* **155**(2), 252 – 271.

Guechi, El-Hadi, Jimmy Lauber, Michel Dambrine, Gregor Klančar and Saso Blažič (2010). PDC control design for non-holonomic wheeled mobile robots with delayed outputs. *J. Intell. Robotics Syst.* **60**(3-4), 395–414.

Hameed, Ibrahim A., Claus G. Sorensen and Ole Gree (2011). Building an intelligent controller using simple genetic type-2 fuzzy logic system. In: *Fuzzy Controllers, Theory and Applications, Dr. Lucian Grigorie (Ed.)*. pp. 147–162.

Khaber, F., K. Zehar and A. Hamzaoui (2006). Optimal fuzzy control system using the cross-entropy method. a case study of a drilling process. *International Journal of Computational Intelligence* **2**(3), 148.

Lam, H.K., F.H.F. Leung and P.K.S. Tam (2000). Stable and robust fuzzy control for uncertain nonlinear systems. *Systems, Man and Cybernetics, Part A: Systems and Humans, IEEE Transactions on* **30**(6), 825 –840.

Mendel, J. M. (2001). *Rule-Based Fuzzy Logic Systems: Introduction and New Directions*. NJ: Prentice-Hall.

Mendel, Jerry M. (2007). Advances in type-2 fuzzy sets and systems. *Information Sciences* **177**(1), 84 – 110.

Mohammad, Biglar Begian (2010). Systematic Design of Type-2 Fuzzy Logic Systems for Modeling and Control with Applications to Modular and Reconfigurable Robots. PhD thesis. University of Waterloo.

Precup, Radu-Emil, Stefan Preitl, Emil M. Petriu, Jzsef K. Tar, Marius L. Tomescu and Claudiu Pozna (2009). Generic two-degree-of-freedom linear and fuzzy controllers for integral processes. *Journal of the Franklin Institute* **346**(10), 980 – 1003.

Ren, Qun, M. Balazinski L. and Baron (2009). Uncertainty prediction for tool wear condition using type-2 TSK fuzzy approach. In: *Systems, Man and Cybernetics, 2009. SMC 2009. IEEE International Conference on*. pp. 660 –665.

Ren, Qun, Marek Balazinski and Luc Baron (2011). Type-2 TSK fuzzy logic system and its type-1 counterpart. *International Journal of Computer Applications* **20**(6), 8–13. Published by Foundation of Computer Science.

Sadighi, A. and Won jong Kim (2010). Intelligent sensorless control of a smart-material actuator. In: *Advanced Intelligent Mechatronics (AIM), 2010 IEEE/ASME International Conference on*. pp. 248 –253.

Takagi, T. and M. Sugeno (1985). Fuzzy identification of systems and its applications to modeling and control. *Systems, Man and Cybernetics, IEEE Transactions on* **SMC-15**(1), 116 –132.

Tomescu, M.-L., St. Preitl, R.-E. Precup and J.K. Tar (2007). Stability analysis method for fuzzy control systems dedicated controlling nonlinear processes. *Acta Polytechnica Hungarica* **4**(3), 127–141.

Wu, D. and W.W. Tan (2004). A type-2 fuzzy logic controller for the liquid-level process. In: *Fuzzy Systems, 2004. Proceedings. 2004 IEEE International Conference on*. Vol. 2. pp. 953 – 958 vol.2.

Wu, Hongwei and J.M. Mendel (2001). Introduction to uncertainty bounds and their use in the design of interval type-2 fuzzy logic systems. In: *Fuzzy Systems, 2001. The 10th IEEE International Conference on*. Vol. 2. pp. 662 – 665 vol.3.

Zadeh, L.A. (1975). The concept of a linguistic variable and its application to approximate reasoning - I. *Information Sciences* **8**(3), 199 – 249.