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How many $SU(4)_L \otimes U(1)_Y$ Gauge models ?

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Abstract

We prove in this letter that the general method of solving gauge models with high symmetries proposed by Cotăescu several years ago can predict precisely two distinct classes of $SU(4)_L \otimes U(1)_Y$ electroweak models. Their fermion representations with respect to this gauge group are exactly obtained in rach case.

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1. Introduction

One of the most stringent topics in modern theoretical particle physics is to find the proper extension of the Standard Model (SM) able to accommodate (or even to predict) the new and richer observed phenomenology at colliders or in cosmology such as: (i) neutrino oscillation, (ii) 126 GeV Higgs signal at CERN-LHC, (iii) new Z'gauge boson. etc. More than a decade ago, Cotăescu (Cotăescu, 1997) proposed a general method for solving chiral gauge models of the type $SU(3)_c \otimes SU(N)_L \otimes U(1)_Y$ that undergo a spontaneous symmetry breaking (SSB) in its electroweak sector. Based on a particular parametrization of the scalar sector leading to an unusual Higgs mechanism to accomplish the SSB, the method established itself as a successful tool in investigating the phenomenology of interest at present facilities (CERN-LHC, Tevatron, LEP etc). Also, it can give some estimates of the expected processes.

We focus in this letter on the classification job the method supplies in the case of the $SU(3)_c \otimes SU(4)_L \otimes U(1)_Y$ gauge models, subject to a sustained research (R. Foot & Tran, 1994), (Pisano & Pleitez, 1995), (Doff & Pisano, 1999), (Doff & Pisano, 2001), (Fayyazuddin & Riazuddin, 2004), (W. A. Ponce & Sanchez, 2004), (L. A. Sanchez & Ponce, 2004), (Ponce & Sanchez, 2004), (L. A. Sanchez & Zuluaga, 2008), (Riazuddin & Fayyazuddin, 2008), (Palcu, 2009*c*), (Palcu,

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2009*a*), (Nisperuza & Sanchez, 2009), (Palcu, 2009*e*), (Palcu, 2009*d*), (Villada & Sanchez, 2009), (Palcu, 2009*b*), (Jaramillo & Sanchez, 2011), (Palcu, 2012) lately. More precisely, we obtain all the classes allowed by the general method when applied to this particular gauge group. Of course, in all these models the $SU(3)_c$ group is the color group of chromodynamics and it remains vector-like as usual, contributing in the case at hand only to the cancellation of the axial anomaly. Therefore it will be no longer mentioned, as the extension takes place only in the electroweak sector.

The paper is organized as follows: Sec. 2 briefly displays the main results of the general method with a special emphasise on the charge operators which are worked out in detail, while in Sec.3 our conclusions are sketched.

2. Charge operators in $SU(4)_L \otimes U(1)_Y$ models

To begin with, we present some general results of the method involved here with a particular focus on the charge operators and their concrete expressions.

2.1. Main results of the general method

2.1.1. Irreducible representations of $SU(N)_L \otimes U(1)_Y$

When constructing a gauge model, one must consider proper fermion representations of the $SU(N)_L \otimes U(1)_Y$ gauge group. usually, these are the fundamental irreducible unitary representations (irreps) $\mathbf{n}$ and $\mathbf{n}^*$ of the $SU(N)$group. They supply different classes of tensors of ranks (r, s) as direct products like $(\otimes\mathbf{n})^r \otimes (\otimes\mathbf{n}^*)^s$. These tensors exhibit r lower and s upper indices for which the notation $i, j, k, \cdots = 1, \cdots, n$. The irrep ρ of $SU(N)$ by indicating its dimension, $\mathbf{n}_\rho$. The $su(n)$ Lie algebra can have different parameterizations , but we prefer here a hybrid basis (see Ref. (Cotăescu, 1997)) consisting of $n-1$ diagonal generators of the Cartan sub-algebra, $D_{\hat{i}}$, labeled by indices $\hat{i}, \hat{j}, ...$ ranging from 1 to $n-1$, and the generators $E^i_j = H^i_j/\sqrt{2}$, $i \neq j$, related to the off-diagonal real generators H^i_j. We got thus the elements $\xi = D_{\hat{i}}\xi^{\hat{i}} + E^i_j\xi^j_i \in su(n)$ now parameterized by $n-1$ real parameters, $\xi^{\hat{i}}$, and by $n(n-1)/2$ c-number ones, $\xi^i_j = (\xi^j_i)^*$, for $i \neq j$. That is a suitable choice since the parameters ξ^i_j can be directly associated to the c-number gauge fields due to the factor $1/\sqrt{2}$ which gives their correct normalization. In addition, this basis ensures a convenient trace orthogonality relations:

$$Tr(D_{\hat{i}}D_{\hat{j}}) = \frac{1}{2}\delta_{\hat{i}\hat{j}}, \quad Tr(D_{\hat{i}}E^i_j) = 0\,, \quad Tr(E^i_jE^k_l) = \frac{1}{2}\delta^i_l\delta^k_j\,. \tag{2.1}$$

If one deals with different irreps, ρ of the $su(n)$ algebra one denotes $\xi^\rho = \rho(\xi)$ for each $\xi \in su(n)$ such that the corresponding basis-generators of the irrep ρ become $D^\rho_{\hat{i}} = \rho(D_{\hat{i}})$ and $E^{\rho\, i}_j = \rho(E^i_j)$.

2.1.2. Fermion sector

The $U(1)_Y$ transformations corresponding to the new hypercharge are simply phase factor multiplications. Therefore - once the coupling constants g for $SU(n)_L$ and g' for the $U(1)_Y$ are established - the transformation rule of the fermion tensor L^ρ with respect to the whole gauge group yields:

$$L^\rho \to U(\xi^0, \xi)L^\rho = e^{-i(g\xi^\rho + g' y_{ch}\xi^0)}L^\rho \tag{2.2}$$

where $\xi = \in su(n)$ and y_{ch} is the chiral hypercharge defining the irrep of the $U(1)_Y$ group parametrized by ξ^0. In order to simplify the notations, the general method used to deal with the character $y = y_{ch}g'/g$ instead of the chiral hypercharge y_{ch}. This small mathematical artifice does not alter at all the results. The irreps of the whole gauge group $SU(n)_L \otimes U(1)_Y$ are uniquely determined by identifying the dimension of the $SU(n)$ tensor and its character y for particular representations $\rho = (\mathbf{n}_\rho, y_\rho)$ of interest in each case.

2.1.3. Electric and neutral charges

In order to introduce specific interaction among fermions, a proper mechanism to conceive couplings must be set up. This goal is achieved by postulating the covariant derivatives in the manner: $D_\mu L^\rho = \partial_\mu L^\rho - ig(A^a_\mu T^\rho_a + y_\rho A^0_\mu)L^\rho$. Here T^ρ_a are generators (regardless they are diagonal or off-diagonal) defining the $su(n)$ algebra, expressed in the representation ρ. The gauge fields in our notation are $A^0_\mu = (A^0_\mu)^*$ and $A_\mu = A^+_\mu \in su(n)$ respectively.

The charge spectrum of the general method is essentially related to the problem of finding the basis of the physical neutral bosons after separating the electromagnetic massless A^{em}_μ. It corresponds to the residual $U(1)_{em}$ symmetry, that is to the one-dimensional subspace of the parameters ξ^{em} in the parameter space $\{\xi^0, \xi^{\hat{i}}\}$ of the whole Cartan sub-algebra. It is uniquely determined by the $n-1$ - dimensional unit vector ν and the angle θ giving the subspace equations $\xi^0 = \xi^{em}\cos\theta$ and $\xi^{\hat{i}} = \nu_{\hat{i}}\xi^{em}\sin\theta$.

The remaining massive neutral gauge fields $A'^{\hat{i}}_\mu$ will exhibit non-diagonal mass matrix successively the SSB via a proper Higgs mechanism (whose details we will overpass here). The mass basis can be reached by resorting to a $SO(n-1)$ transformation, namely $A'^{\hat{i}}_\mu = \omega^{\hat{i}}_{\cdot\,\hat{j}} Z^{\hat{j}}_\mu$ where $Z^{\hat{i}}_\mu$ are the physical neutral bosons with well-defined masses. Explicitly, this $SO(n-1)$ transformation works in the manner:

$$\begin{aligned} A^0_\mu &= A^{em}_\mu \cos\theta - \nu_{\hat{i}}\omega^{\hat{i}}_{\cdot\,\hat{j}} Z^{\hat{j}}_\mu \sin\theta, \\ A^{\hat{k}}_\mu &= \nu^{\hat{k}} A^{em}_\mu \sin\theta + \left(\delta^{\hat{k}}_{\hat{i}} - \nu^{\hat{k}}\nu_{\hat{i}}(1-\cos\theta)\right)\omega^{\hat{i}}_{\cdot\,\hat{j}} Z^{\hat{j}}_\mu. \end{aligned} \tag{2.3}$$

It connects the gauge basis $(A^0_\mu, A^{\hat{i}}_\mu)$ to the physical one $(A^{em}_\mu, Z^{\hat{i}}_\mu)$. This transformation ω is called the generalized Weinberg transformation (gWt).

At this stage, one can easily identify the charges of the fermions involved with respect to the above determined physical bosons. The spinor multiplet L^ρ acquires the following electric charge matrix:

$$Q^\rho = g\left[(D^\rho \cdot \nu)\sin\theta + y_\rho \cos\theta\right], \tag{2.4}$$

and $n-1$ neutral charge matrices:

$$Q^\rho(Z^{\hat{i}}) = g\left[D^\rho_{\hat{k}} - \nu_{\hat{k}}(D^\rho \cdot \nu)(1-\cos\theta) - y_\rho \nu_{\hat{k}}\sin\theta\right]\omega^{\hat{k}}_{\cdot\,\hat{i}} \tag{2.5}$$

each corresponding to the $n-1$ neutral physical fields, $Z^{\hat{i}}_\mu$.

2.2. $SU(4)_L \otimes U(1)_Y$ gauge group

In the particular $SU(4)_L \otimes U(1)_Y$ gauge model one has to properly identify the diagonal generators and set up the possible options for the versor ν. For our purpose, the standard generators T_a of the $su(4)$ algebra are the Hermitian diagonal generators of the Cartan sub-algebra, namely $D_1 = T_3 = \frac{1}{2}\mathrm{diag}(1,-1,0,0)$, $D_2 = T_8 = \frac{1}{2\sqrt{3}}\mathrm{diag}(1,1,-2,0)$, and $D_3 = T_{15} = \frac{1}{2\sqrt{6}}\mathrm{diag}(1,1,1,-3)$.

We will prefer in the following to denote the irreps of the gauge group by $\rho = (\mathbf{n}_\rho, y^\rho_{ch})$ indicating the genuine chiral hypercharge y_{ch} instead of y. Hence, the multiplets of the 3-4-1 model under consideration here - subject to anomaly cancellation in order to keep renormalizable the whole theory - will be denoted by $(\mathbf{n}_{color}, \mathbf{n}_\rho, y^\rho_{ch})$. The condition $e = g\sin\theta_W$ established in the SM is valid throughout.

In principle, there will be three distinct cases in choosing the versors. They are:

- versors $\nu_1 = 1$, $\nu_2 = 0$, $\nu_3 = 0$,
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- versors $\nu_1 = 0$, $\nu_2 = 0$, $\nu_3 = 1$.

2.2.1. Class A ($\nu_1 = 1$, $\nu_2 = 0$, $\nu_3 = 0$)

The lepton quadruplet obeys the fundamental irrep of the gauge group $\rho = (\mathbf{4}, 0)$. Eq. (2.4) yields:

$$Q^{(4,0)} = eT_3^{(4)}\frac{\sin\theta}{\sin\theta_W}, \tag{2.6}$$

which denotes the lepton representation $\begin{pmatrix} e^c_\alpha, & e_\alpha, & \nu_\alpha, & N_\alpha \end{pmatrix}^T_L \sim (\mathbf{4},0)$ if and only if $\sin\theta = 2\sin\theta_W$ holds.

In the quark sector there are two families ($i = 1,2$) transforming similarly under the gauge group $\begin{pmatrix} J_i, & u_i, & d_i, & D_i \end{pmatrix}^T_L \sim (\mathbf{4}^*, -1/3)$ and a third one transforming as $\begin{pmatrix} J_3, & d_3, & u_3, & U_3 \end{pmatrix}^T_L \sim (\mathbf{4}, +2/3)$. Their electric charge operators will take, respectively, the forms

$$Q^{(4^*,-\frac{1}{3})} = e\left[T_3^{(4^*)}\frac{\sin\theta}{\sin\theta_W} - \frac{1}{3}\left(\frac{g'}{g}\right)\frac{\cos\theta}{\sin\theta_W}\right], \tag{2.7}$$

$$Q^{(4,+\frac{2}{3})} = e\left[T_3^{(4)}\frac{\sin\theta}{\sin\theta_W} + \frac{2}{3}\left(\frac{g'}{g}\right)\frac{\cos\theta}{\sin\theta_W}\right], \tag{2.8}$$

Now, in order to get the known electric charges of the quarks one must enforce the coupling match:

$$\frac{g'}{g} = \frac{\sin\theta_W}{\sqrt{1-4\sin^2\theta_W}}. \tag{2.9}$$

The anomaly-free content in the fermion sector of this class of 3-4-1 models stands:

Lepton families

$$f_{\alpha L}=\begin{pmatrix} e^c_\alpha \\ e_\alpha \\ \nu_\alpha \\ N_\alpha \end{pmatrix}_L \sim (\mathbf{1},\mathbf{4},0) \tag{2.10}$$

Quark families

$$Q_{iL}=\begin{pmatrix} J_i \\ u_i \\ d_i \\ D_i \end{pmatrix}_L \sim (\mathbf{3},\mathbf{4}^*,-1/3) \qquad Q_{3L}=\begin{pmatrix} J_3 \\ -d_3 \\ u_3 \\ U_3 \end{pmatrix}_L \sim (\mathbf{3},\mathbf{4},+2/3) \tag{2.11}$$

$$(d_{3L})^c, (d_{iL})^c, (D_{iL})^c, \sim (\mathbf{3},\mathbf{1},+1/3) \tag{2.12}$$

$$(u_{3L})^c, (u_{iL})^c, (U_{3L})^c \sim (\mathbf{3},\mathbf{1},-2/3) \tag{2.13}$$

$$(J_{3L})^c \sim (\mathbf{3},\mathbf{1},-5/3) \qquad (J_{iL})^c \sim (\mathbf{3},\mathbf{1},+4/3) \tag{2.14}$$

with $\alpha = 1,2,3$ and $i = 1,2$.

The capital letters J label some exotic quarks included in each family. They exhibit exotic electric charges $\pm 4/3$ and $\pm 5/3$.

Based on Eq.(2.5) one can compute the neutral charges for the above model A. They are presented in Table 1. Obviously, the SM fermions exhibit the same neutral charges as they do in the SM framework with respect to the Z boson.

2.2.2. Class B ($\nu_1 = 0$, $\nu_2 = 1$, $\nu_3 = 0$)

Due to $T_8 = \frac{1}{2\sqrt{3}}\,\mathrm{diag}(1,1,-2,0)$ there is no room for a plausible electric charge operator, since there is only 0 and $\pm e$ allowed in the lepton quadruplets. Therefore, this case must be ruled out from phenomenological reasons.

2.2.3. Class C ($\nu_1 = 0$, $\nu_2 = 0$, $\nu_3 = -1$)

In this case, one can assign two different chiral hypercharges $-\frac{1}{4}$ and $-\frac{3}{4}$ respectively for the lepton quadruplet. Hence, we get two sub-cases leading to two different versions of this class. The coupling matching yields the same relation in both sub-cases.

The lepton sector's electric charge operator for the first choice stands as

$$Q^{(4^*,-\frac{1}{4})} = e\left[-T^{(4^*)}_{15}\frac{\sin\theta}{\sin\theta_W} - \frac{1}{4}\left(\frac{g'}{g}\right)\frac{\cos\theta}{\sin\theta_W}\right], \tag{2.15}$$

for the first sub-case. This leads to the lepton representation $\begin{pmatrix} e_\alpha, & \nu_\alpha, & N_\alpha, & N'_\alpha \end{pmatrix}^T_L \sim (\mathbf{4}^*,-\frac{1}{4})$ including two new kinds of neutral leptons (N_α, N'_α) possibly right-handed neutrinos.

Table 1. Coupling coefficients of the neutral currents in 3-4-1 in Model A

Particle\Coupling($e/\sin 2\theta_W$)	$Z\bar{f}f$	$Z'\bar{f}f$	$Z''\bar{f}f$
$\nu_{eL}, \nu_{\mu L}, \nu_{\tau L}$	1	$-\frac{\sqrt{1-4\sin^2\theta_W}}{\sqrt{3}}$	$\frac{\cos\theta_W}{\sqrt{6}}$
e_L, μ_L, τ_L	$2\sin^2\theta_W - 1$	$-\frac{\sqrt{1-4\sin^2\theta_W}}{\sqrt{3}}$	$\frac{\cos\theta_W}{\sqrt{6}}$
e_R, μ_R, τ_R	$-2\sin^2\theta_W$	$\frac{2\sqrt{1-4\sin^2\theta_W}}{\sqrt{3}}$	$\frac{\cos\theta_W}{\sqrt{6}}$
$N_{eL}, N_{\mu L}, N_{\tau L}$	0	0	$-\sqrt{\frac{3}{2}}\cos\theta_W$
u_L, c_L	$1 - \frac{4}{3}\sin^2\theta_W$	$\frac{1}{\sqrt{3}}\left(\frac{1-2\sin^2\theta_W}{\sqrt{1-4\sin^2\theta_W}}\right)$	$-\frac{\cos\theta_W}{\sqrt{6}}$
d_L, s_L	$-1 + \frac{2}{3}\sin^2\theta_W$	$\frac{1}{\sqrt{3}}\left(\frac{1-2\sin^2\theta_W}{\sqrt{1-4\sin^2\theta_W}}\right)$	$-\frac{\cos\theta_W}{\sqrt{6}}$
t_L	$1 - \frac{4}{3}\sin^2\theta_W$	$\text{-}\left(\frac{1}{\sqrt{3}}\right)\frac{1}{\sqrt{1-4\sin^2\theta_W}}$	$\frac{\cos\theta_W}{\sqrt{6}}$
b_L	$-1 + \frac{2}{3}\sin^2\theta_W$	$-\left(\frac{1}{\sqrt{3}}\right)\frac{1}{\sqrt{1-4\sin^2\theta_W}}$	$\frac{\cos\theta_W}{\sqrt{6}}$
T_L	$-\frac{4}{3}\sin^2\theta_W$	$-\left(\frac{4}{\sqrt{3}}\right)\frac{\sin^2\theta_W}{\sqrt{1-4\sin^2\theta_W}}$	$-\sqrt{\frac{3}{2}}\cos\theta_W$
D_{1L}, D_{2L}	$\frac{2}{3}\sin^2\theta_W$	$\left(\frac{2}{\sqrt{3}}\right)\frac{\sin^2\theta_W}{\sqrt{1-4\sin^2\theta_W}}$	$\sqrt{\frac{3}{2}}\cos\theta_W$
u_R, c_R, t_R, T_R	$-\frac{4}{3}\sin^2\theta_W$	$\left(\frac{4}{\sqrt{3}}\right)\frac{\sin^2\theta_W}{\sqrt{1-4\sin^2\theta_W}}$	0
d_R, s_R, b_R, D_{iR}	$+\frac{2}{3}\sin^2\theta_W$	$-\left(\frac{2}{\sqrt{3}}\right)\frac{\sin^2\theta_W}{\sqrt{1-4\sin^2\theta_W}}$	0
J_{1L}, J_{2L}	$\frac{8}{3}\sin^2\theta_W$	$-\left(\frac{2}{\sqrt{3}}\right)\frac{1-5\sin^2\theta_W}{\sqrt{1-4\sin^2\theta_W}}$	$\text{-}\frac{\cos\theta_W}{\sqrt{6}}$
J_{1R}, J_{2R}	$\frac{8}{3}\sin^2\theta_W$	$-\left(\frac{8}{\sqrt{3}}\right)\frac{\sin^2\theta_W}{\sqrt{1-4\sin^2\theta_W}}$	0
J_{3L}	$-\frac{10}{3}\sin^2\theta_W$	$\left(\frac{2}{\sqrt{3}}\right)\frac{1-6\sin^2\theta_W}{\sqrt{1-4\sin^2\theta_W}}$	$\frac{\cos\theta_W}{\sqrt{6}}$
J_{3R}	$-\frac{10}{3}\sin^2\theta_W$	$\left(\frac{10}{\sqrt{3}}\right)\frac{\sin^2\theta_W}{\sqrt{1-4\sin^2\theta_W}}$	0

For the second choice, the lepton electric charge operator is represented as

$$Q^{(4,-\frac{1}{4})} = e\left[-T_{15}^{(4)}\frac{\sin\theta}{\sin\theta_W} - \frac{3}{4}\left(\frac{g'}{g}\right)\frac{\cos\theta}{\sin\theta_W}\right], \tag{2.16}$$

allowing for lepton families such as $\begin{pmatrix} \nu_\alpha, & e_\alpha^-, & E_\alpha^-, & E_\alpha'^- \end{pmatrix}_L^T \sim (\mathbf{4}, -\frac{3}{4})$. A phenomenological analysis in this sub-case must assume some new kind of charged lepton $(E_\alpha^-, E_\alpha'^-)$. possibly very heavy.

After a little algebra the coupling matching for both sub-cases arises:

$$\frac{g'}{g} = \frac{\sin\theta_W}{\sqrt{1 - \frac{3}{2}\sin^2\theta_W}}. \tag{2.17}$$

For the quark sector the electric charge operator takes the following representations

$$Q^{(4^*,\frac{5}{12})} = e\left[-T_{15}^{(4^*)}\frac{\sin\theta}{\sin\theta_W} + \frac{5}{12}\left(\frac{g'}{g}\right)\frac{\cos\theta}{\sin\theta_W}\right] \tag{2.18}$$

$$Q^{(4,-\frac{1}{12})} = e\left[-T_{15}^{(4)}\frac{\sin\theta}{\sin\theta_W} - \frac{1}{12}\left(\frac{g'}{g}\right)\frac{\cos\theta}{\sin\theta_W}\right] \tag{2.19}$$

2.2.4. Fermion content of Model C1

A natural fermion outcome occurs in this first choice, namely:

Lepton families

$$f_{\alpha L} = \begin{pmatrix} e_\alpha \\ \nu_\alpha \\ N_\alpha \\ N'_\alpha \end{pmatrix}_L \sim (\mathbf{1}, \mathbf{4}^*, -\tfrac{1}{4}) \qquad (e_{\alpha L})^c \sim (\mathbf{1}, \mathbf{1}, 1) \tag{2.20}$$

Quark families

$$Q_{iL} = \begin{pmatrix} u_i \\ d_i \\ D_i \\ D'_i \end{pmatrix}_L \sim (\mathbf{3}, \mathbf{4}, -1/12) \qquad Q_{3L} = \begin{pmatrix} -d_3 \\ u_3 \\ U \\ U' \end{pmatrix}_L \sim (\mathbf{3}, \mathbf{4}^*, 5/12) \tag{2.21}$$

$$(d_{3L})^c, (d_{iL})^c, (D_{iL})^c, (D'_{iL})^c \sim (\mathbf{3}, \mathbf{1}, +1/3) \tag{2.22}$$

$$(u_{3L})^c, (u_{iL})^c, (U_L)^c, (U'_L)^c \sim (\mathbf{3}, \mathbf{1}, -2/3) \tag{2.23}$$

with $\alpha = 1, 2, 3$ and $i = 1, 2$.

Based on Eq.(2.5) one can compute the neutral charges for model C1. They are presented in Table 2. The outcome is suitable too: the SM fermions exhibit the same neutral charges as they do in the SM framework with respect to the Z boson.

Table 2. Coupling coefficients of the neutral currents in 3-4-1 in Model C1

Particle\Coupling$(e/\sin 2\theta_W)$	$Z\bar{f}f$	$Z'\bar{f}f$	$Z''\bar{f}f$
$\nu_{eL}, \nu_{\mu L}, \nu_{\tau L}$	1	$\frac{1-3\sin^2\theta_W}{2\sqrt{2-3\sin^2\theta_W}}$	0
e_L, μ_L, τ_L	$2\sin^2\theta_W - 1$	$\frac{1-3\sin^2\theta_W}{2\sqrt{2-3\sin^2\theta_W}}$	0
$N_{eL}, N_{\mu L}, N_{\tau L}$	0	$-\frac{3\cos^2\theta_W}{2\sqrt{2-3\sin^2\theta_W}}$	$-\cos\theta_W$
$N'_{eL}, N'_{\mu L}, N'_{\tau L}$	0	$-\frac{3\cos^2\theta_W}{2\sqrt{2-3\sin^2\theta_W}}$	$\cos\theta_W$
u_L, c_L	$1 - \frac{4}{3}\sin^2\theta_W$	$\frac{2-9\cos^2\theta_W}{2\sqrt{2-3\sin^2\theta_W}}$	0
d_L, s_L	$-1 + \frac{2}{3}\sin^2\theta_W$	$\frac{2-9\cos^2\theta_W}{2\sqrt{2-3\sin^2\theta_W}}$	0
t_L	$1 - \frac{4}{3}\sin^2\theta_W$	$\frac{2+9\cos^2\theta_W}{2\sqrt{2-3\sin^2\theta_W}}$	0
b_L	$-1 + \frac{2}{3}\sin^2\theta_W$	$\frac{2+9\cos^2\theta_W}{2\sqrt{2-3\sin^2\theta_W}}$	0
$u_R, c_R, t_R, U_{1R}, U'_{iR}$	$-\frac{4}{3}\sin^2\theta_W$	$\frac{4\sin^2\theta_W}{3\sqrt{2-3\sin^2\theta_W}}$	0
$d_R, s_R, b_R, D_{iR}, D'_{iR}$	$+\frac{2}{3}\sin^2\theta_W$	$-\frac{2\sin^2\theta_W}{3\sqrt{2-3\sin^2\theta_W}}$	0
D_{1L}, D_{2L}	$\frac{2}{3}\sin^2\theta_W$	$\frac{-4+9\cos^2\theta_W}{6\sqrt{2-3\sin^2\theta_W}}$	$\cos\theta_W$
D'_{1L}, D'_{2L}	$\frac{2}{3}\sin^2\theta_W$	$\frac{-4+9\cos^2\theta_W}{6\sqrt{2-3\sin^2\theta_W}}$	$-\cos\theta_W$
U_{3L}	$-\frac{4}{3}\sin^2\theta_W$	$\frac{8-9\cos^2\theta_W}{6\sqrt{2-3\sin^2\theta_W}}$	$-\cos\theta_W$
U'_{3L}	$-\frac{4}{3}\sin^2\theta_W$	$\frac{8-9\cos^2\theta_W}{6\sqrt{2-3\sin^2\theta_W}}$	$\cos\theta_W$

2.2.5. Fermion content of Model C2

Some strange charged fermions occur in the second choice, but it is still plausible since these could come very massive.

Lepton families

$$f_{\alpha L} = \begin{pmatrix} \nu_\alpha \\ e^-_\alpha \\ E^-_\alpha \\ E'^-_\alpha \end{pmatrix}_L \sim (\mathbf{1}, \mathbf{4}, -3/4) \qquad (e_{\alpha L})^c, (E_{\alpha L})^c, (E'_{\alpha L})^c \sim (\mathbf{1}, \mathbf{1}, 1) \tag{2.24}$$

Quark families

$$Q_{iL} = \begin{pmatrix} d_i \\ -u_i \\ U_i \\ U'_i \end{pmatrix}_L \sim (\mathbf{3}, \mathbf{4}^*, 5/12) \qquad Q_{3L} = \begin{pmatrix} u_3 \\ d_3 \\ D \\ D' \end{pmatrix}_L \sim (\mathbf{3}, \mathbf{4}, -1/12) \tag{2.25}$$

$$(d_{3L})^c, (d_{iL})^c, (D_L)^c, (D'_L)^c \sim (\mathbf{3}, \mathbf{1}, +1/3) \tag{2.26}$$

$$(u_{3L})^c, (u_{iL})^c, (U_{iL})^c, (U'_{iL})^c \sim (\mathbf{3}, \mathbf{1}, -2/3) \tag{2.27}$$

with $\alpha = 1, 2, 3$ and $i = 1, 2$.

Based on Eq.(2.5) one can compute the neutral charges for model C1. They are presented in Table 3. As in the previous cases the SM fermions come out with the same neutral charges as they do in the SM framework with respect to the Z boson.

3. Concluding remarks

In this letter we obtained all the possible 3-4-1 models allowed by the general method of solving gauge models with high symmetries that undergo a spontaneous symmetry breaking. All in all, they are three different 3-4-1 models: one belonging to the Class A and two to the Class C. For the three classes of 3-4-1 gauge models the neutral charges (couplings to neutral bosons of the model) are obtained along with the electric corresponding charges. If we restrict ourself to non-exotic electric charges,then only Class C survives. Even more, if heavy charged leptons are still unobserved experimentally, then only subclass A1 remains to be further analyzed from phenomenological point of view. However, all fermion contents are anomaly-free and hence the theoretical models they account for are renormalizable. After some algebraic computations for all the representations involved therein this statement is proved immediately. Therefore, the phenomenological predictions in this promising framework can be valuable indeed. Furthermore, the phenomenology (to be confirmed at present facilities) can be analyzed in detail once each particular model is taken into consideration.

Table 3. Coupling coefficients of the neutral currents in 3-4-1 in Model C2

Particle\Coupling($e/\sin 2\theta_W$)	$Z\bar{f}f$	$Z'\bar{f}f$	$Z''\bar{f}f$
$\nu_{eL}, \nu_{\mu L}, \nu_{\tau L}$	1	$\frac{1-3\sin^2\theta_W}{2\sqrt{2-3\sin^2\theta_W}}$	0
e_L, μ_L, τ_L	$2\sin^2\theta_W - 1$	$\frac{1-3\sin^2\theta_W}{2\sqrt{2-3\sin^2\theta_W}}$	0
$E_{eL}, E_{\mu L}, E_{\tau L}$	0	$-\frac{3\cos^2\theta_W}{2\sqrt{2-3\sin^2\theta_W}}$	$\cos\theta_W$
$E'_{eL}, E'_{\mu L}, E'_{\tau L}$	0	$-\frac{3\cos^2\theta_W}{2\sqrt{2-3\sin^2\theta_W}}$	$-\cos\theta_W$
u_L, c_L	$1-\frac{4}{3}\sin^2\theta_W$	$\frac{2-9\cos^2\theta_W}{2\sqrt{2-3\sin^2\theta_W}}$	0
d_L, s_L	$-1+\frac{2}{3}\sin^2\theta_W$	$\frac{2-9\cos^2\theta_W}{2\sqrt{2-3\sin^2\theta_W}}$	0
t_L	$1-\frac{4}{3}\sin^2\theta_W$	$\frac{2+9\cos^2\theta_W}{2\sqrt{2-3\sin^2\theta_W}}$	0
b_L	$-1+\frac{2}{3}\sin^2\theta_W$	$\frac{2+9\cos^2\theta_W}{2\sqrt{2-3\sin^2\theta_W}}$	0
$u_R, c_R, t_R, U_{1R}, U'_{iR}$	$-\frac{4}{3}\sin^2\theta_W$	$\frac{4\sin^2\theta_W}{3\sqrt{2-3\sin^2\theta_W}}$	0
$d_R, s_R, b_R, D_{iR}, D'_{iR}$	$+\frac{2}{3}\sin^2\theta_W$	$-\frac{2\sin^2\theta_W}{3\sqrt{2-3\sin^2\theta_W}}$	0
D_{3L}	$\frac{2}{3}\sin^2\theta_W$	$\frac{-4+9\cos^2\theta_W}{6\sqrt{2-3\sin^2\theta_W}}$	$\cos\theta_W$
D'_{3L}	$\frac{2}{3}\sin^2\theta_W$	$\frac{-4+9\cos^2\theta_W}{6\sqrt{2-3\sin^2\theta_W}}$	$-\cos\theta_W$
U_{1L}, U_{2L}	$-\frac{4}{3}\sin^2\theta_W$	$\frac{8-9\cos^2\theta_W}{6\sqrt{2-3\sin^2\theta_W}}$	$-\cos\theta_W$
U'_{1L}, U'_{2L}	$-\frac{4}{3}\sin^2\theta_W$	$\frac{8-9\cos^2\theta_W}{6\sqrt{2-3\sin^2\theta_W}}$	$\cos\theta_W$

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