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# Coefficient Inequalities for Some Subclasses of Analytic Functions Associated with Conic Domains Involving $q$-calculus

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## Abstract

Main purpose of this paper is to define and study some new classes of analytic functions associated with conic type regions. By using Salagean $q$-differential operator we investigate several interesting properties of these newly defined classes. Comparison of new results with those that were obtained in earlier investigation are given as Corollaries.

*Keywords:* $q$-differential operator, Salagean $q$-differential operator, Janowski functions, $k$-uniformly convex functions, $k$-starlike functions, close-to-convex functions, conic domain.
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## 1. Introduction

Let $\mathcal{A}$ denote the class of functions $f$ analytic in the open unit disc $E = \{z : z \in \mathbb{C}, |z| < 1\}$ and satisfying the normalization condition $f(0) = f'(0) - 1 = 0$. Thus, the functions in $\mathcal{A}$ are represented by the Taylor-Maclaurin series expansion given by

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \quad z \in E. \tag{1.1}$$

Let $\mathcal{S}$ be the subset of $\mathcal{A}$ consisting of the functions that are univalent in $E$. The convolution (Hadamard product) of functions $f, g \in \mathcal{A}$ is defined by

$$(f * g)(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n, \quad z \in E,$$

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where $f(z)$ is given by (1.1) and

$$g(z) = z + \sum_{n=2}^{\infty} b_n z^n, \qquad z \in E.$$

For two functions $f, g \in \mathcal{A}$, we say that $f$ is subordinate to $g$ in $E$, denoted by

$$f(z) \prec g(z) \qquad (z \in E),$$

if there exists a function $w$ where

$$w(0) = 0, \ |w(z)| < 1, \qquad (z \in E),$$

such that

$$f(z) = g(w(z)), \qquad (z \in E).$$

If $g$ is univalent in $E$, then it follows that

$$f(z) \prec g(z) \qquad (z \in E), \ \Rightarrow \ f(0) = 0 \text{ and } f(E) \subset g(E).$$

For more detail see (Miller & Mocanu, 2000). A function $p$ analytic in $E$ and of the form

$$p(z) = 1 + \sum_{n=1}^{\infty} p_n z^n \in \mathcal{P}[A, B] \Leftrightarrow p(z) \prec \frac{1 + Az}{1 + Bz}$$

where $-1 \leq B < A \leq 1$. This class was introduced and investigated by Janowski (Janowski, 1973). In particular, if $A = 1$ and $B = -1$, we obtain the class $\mathcal{P}$ of functions with a positive real part (see (Goodman, 1983)). The classes $\mathcal{P}$ and $\mathcal{P}[A, B]$ are connected by the relation

$$p(z) \in \mathcal{P} \Leftrightarrow \frac{(A+1)\,p(z) - (A-1)}{(B+1)\,p(z) - (B-1)} \in \mathcal{P}[A, B].$$

Now consider, for $k \geq 0$, the classes $k - CV$ and $k - ST$ of $k$-uniformly convex functions and corresponding $k$-starlike functions, respectively, introduced by Kanas and Wisniowska. For some details, see (Kanas, 2003), (Kanas & Wisniowska, 2000), (Kanas & Wisniowska, 1999).
Kanas and Wisniowska (Kanas & Wisniowska, 2000), (Kanas & Wisniowska, 1999) introduced the conic domain $\Omega_k, k \geq 0$ as

$$\Omega_k = \left\{ u + iv : u > k\sqrt{(u-1)^2 + v^2} \right\}.$$

We note that $\Omega_k$ represents the conic region bounded successively by the imaginary axis ($k = 0$), the right branch of hyperbola ($0 < k < 1$), a parabola for $k = 1$, and ellipse for $k > 1$. The extremal functions for these conic regions are

$$p_k(z) = \begin{cases} \frac{1+z}{1-z} & k = 0, \\ 1 + \frac{2}{\pi^2}\left(\log \frac{1+\sqrt{z}}{1-\sqrt{z}}\right)^2 & k = 1, \\ 1 + \frac{2}{1-k^2}\sinh^2\left\{\frac{2}{\pi}(\arccos k)\arctan h\sqrt{z}\right\} & 0 < k < 1, \\ 1 + \frac{1}{k^2-1}\sin\left(\frac{\pi}{2R(t)}\int_0^{\frac{u(z)}{\sqrt{t}}}\frac{\mathrm{d}x}{\sqrt{1-x^2}\sqrt{1-t^2x^2}}\right) + \frac{1^2}{k^2-1} & k > 1, \end{cases} \tag{1.2}$$

where

$$u(z) = \frac{z - \sqrt{t}}{1 - \sqrt{t}z}, \quad z \in E,$$

and $t \in (0, 1)$ is chosen such that $k = \cosh(\pi R'(t)/(4R(t)))$. Here $R(t)$ is Legender's complete elliptic integral of first kind and $R'(t) = R(\sqrt{1-t^2})$ and $R'(t)$ is the complementary integral of $R(t)$ for details see (Ahiezer, 1970), (Hussain *et al.*, 2017), (Kanas & Wisniowska, 2000), (Kanas & Wisniowska, 1999). If $p_k(z) = 1 + Q_1(k)z + Q_2(k)z^2 + ..., z \in E$. Then it was shown in (Kanas & Wisniowska, 2000) that for (1.2) one can have

$$Q_1 := Q_1(k) = \begin{cases} \frac{2A^2}{1-k^2} & 0 \le k < 1, \\ \frac{8}{\pi^2} & k = 1, \\ \frac{\pi^2}{4(k^2-1)\sqrt{t}(1+t)R^2(t)} & k > 1, \end{cases} \tag{1.3}$$

with $A = \frac{2}{\pi} \arccos t$.
The classes $k - UCV$ and $k - ST$ are defined as follows.
A function $f(z) \in \mathcal{A}$ is said to be in the class $k - UCV$, if and only if,

$$\frac{\left(zf'(z)\right)'}{f'(z)} \prec p_k(z), \quad z \in E, \ k \ge 0.$$

A function $f(z) \in \mathcal{A}$ is said to be in the class $k - ST$ , if and only if,

$$\frac{zf'(z)}{f(z)} \prec p_k(z), \quad z \in E, \ k \ge 0.$$

For more study (see (Srivastava *et al.*, 2012), (Srivastava *et al.*, 2009), (Srivastava *et al.*, 2007)). These classes were then generalized to $KD(k, \alpha)$ and $SD(k, \alpha)$ respectively by Shams et al. (Shams *et al.*, 2004) subject to the conic domain $G(k, \alpha), k \ge 0, 0 \le \alpha < 1$, which is

$$G(k, \alpha) = \{w : \Re(w) > k|w - 1| + \alpha\}.$$

Now using the concepts of Janowski functions and the conic domain, Noor and Malik (Noor & Malik, 2011) define the following

**Definition 1.1.** (See (Noor & Malik, 2011)) A function $p(z)$ is said to be in the class $k - \mathcal{P}[A, B]$, if and only if,

$$p(z) \prec \frac{(A+1)p_k(z) - (A-1)}{(B+1)p_k(z) - (B-1)}, \quad k \ge 0,$$

where $p_k(z)$ is defined in ( 1.2) and $-1 \le B < A \le 1$. Geometrically, the function $p \in k - \mathcal{P}[A, B]$ takes all values from the domain $\Omega_k[A, B]$, $1 \le B < A \le 1$, $k \ge 0$ which is defined as:

$$\Omega_k[A, B] = \left\{w : \Re\left(\frac{(B-1)w - (A-1)}{(B+1)w - (A+1)}\right) > k\left|\frac{(B-1)w - (A-1)}{(B+1)w - (A+1)} - 1\right|\right\},$$

or equivalently $\Omega_k[A,B]$ is a set of numbers $w = u + iv$ such that

$$\begin{aligned}&\left[\left(B^2-1\right)\left(u^2+v^2\right)-2\left(AB-1\right)u+\left(A^2-1\right)\right]^2\\ > \ & k^2\left[\left(-2\left(B+1\right)\left(u^2+v^2\right)+2\left(A+B+2\right)u-2\left(A+1\right)\right)^2+4\left(A-B\right)^2v^2\right].\end{aligned}$$

This domain represents the conic type regains for detail see (Noor & Malik, 2011), (Noor *et al.*, 2017). It can be easily seen that $0-\mathcal{P}[A,B]=\mathcal{P}[A,B]$ introduced in (Janowski, 1973) and $k-\mathcal{P}[1,-1]=\mathcal{P}(p_k)$ introduced in (Kanas & Wisniowska, 1999).
For any non-negative integer $n$, the $q-$integer number $n$, $[n]_q$ is defined by:

$$[n]_q=\frac{1-q^n}{1-q}=1+q+...+q^{n-1}, \qquad [0]_q=0.$$

The $q-$number shifted factorial is defined by $[0]!=1$ and $[n]_q!=[1]_q[2]_q...[n]_q$. Clearly, $\underset{q\to1^-}{Lim}[n]_q=n$ and $\underset{q\to1^-}{Lim}[n]_q!=n!$. In general we will denote $[t]_q=\frac{1-q^t}{1-q}$ also for a non-integer number.

**Definition 1.2.** Let $f\in\mathcal{A}$, and let the $q-$derivative operator or $q-$difference operator be defined by

$$\partial_q f(z)=\frac{f(qz)-f(z)}{(q-1)z} \qquad (z\in E).$$

It is easy to check that for $n\in\mathbb{N}:=\{1,2,...\}$ and $z\in E$

$$\partial_q z^n=[n]_q z^{n-1}.$$

In the field of Geometric Function Theory, various subclasses of the normalized analytic function class $\mathcal{A}$ have been studied from different viewpoints. The $q$-calculus as well as the fractional $q$-calculus provide important tools that have been used in order to investigate various subclasses of $\mathcal{A}$. Moreover, in recent years, such $q$-calculus operators as the fractional $q$-integral and fractional $q$-derivative operators were used to construct several subclasses of analytic functions (see, for example, (Altınkaya & Yalçın, 2017), (Magesh *et al.*, 2018), (Purohit & Raina, 2013), (Srivastava, 1989)).

Throughout this paper we assume $q$ to be a fixed number between 0 and 1.

**Definition 1.3.** (See (Govindaraj & Sivasubramanian, 2018)) For $f\in\mathcal{A}$, let Salagean $q$-differential operator be defined as follows:

$$S_q^0 f(z)=f(z),\ S_q^1 f(z)=z\partial_q f(z),...,S_q^m f(z)=z\partial_q\left(S_q^{m-1}f(z)\right). \tag{1.4}$$

A simple calculation implies

$$S_q^m f(z)=f(z)*F_{m,q}(z),\ z\in E,\ m\in\mathbb{N}U\{0\}=\mathbb{N}_0.$$

where

$$F_{m,q}(z) = z + \sum_{n=2}^{\infty} [n]_q^m z^n. \tag{1.5}$$

Making use of (1.4) and (1.5), the power series of $S_q^m f(z)$ for $f$ of the form (1.1) is given by

$$S_q^m f(z) = z + \sum_{n=2}^{\infty} [n]_q^m a_n z^n.$$

Note that

$$\lim_{q \to 1-} F_{m,q}(z) = z + \sum_{n=2}^{\infty} n^m z^n$$

and

$$\lim_{q \to 1-} S_q^m f(z) = z + \sum_{n=2}^{\infty} n^m a_n z^n$$

which is the familiar Salagean derivative (Salagean, 1983).
Motivated by the recent work presented by Noor and Malik (Noor & Malik, 2011) and (Mahmood *et al.*, 2017), we define some classes of analytic functions associated with conic domains and by using Salagean $q$-differential operator.

**Definition 1.4.** A function $f(z) \in \mathcal{A}$ is said to be in the class $k - \mathcal{ST}_q(m, C, D)$, $k \geq 0$, $-1 \leq D < C \leq 1$, if and only if

$$\Re\left(\frac{(D-1)G_{m,q}(z) - (C-1)}{(D+1)G_{m,q}(z) - (C+1)}\right) > k\left|\frac{(D-1)G_{m,q}(z) - (C-1)}{(D+1)G_{m,q}(z) - (C+1)} - 1\right|,$$

where

$$G_{m,q}(z) = \frac{S_q^{m+1} f(z)}{S_q^m f(z)},$$

or equivalently

$$G_{m,q}(z) \in k - P[C, D].$$

**Definition 1.5.** A function $f(z) \in \mathcal{A}$ is said to be in the class $k - \mathcal{CV}_q(m, C, D)$, $k \geq 0$, $-1 \leq D < C \leq 1$, if and only if

$$\Re\left(\frac{(D-1)H_{m,q}(z) - (C-1)}{(D+1)H_{m,q}(z) - (C+1)}\right) > k\left|\frac{(D-1)H_{m,q}(z) - (C-1)}{(D+1)H_{m,q}(z) - (C+1)} - 1\right|,$$

where

$$H_{m,q}(z) = \frac{z\partial_q S_q^{m+1} f(z)}{S_q^{m+1} f(z)},$$

or equivalently,

$$H_{m,q}(z) \in k - P[C, D].$$

It can be easily seen that

$$f(z) \in k-\mathcal{CV}_q(m,C,D) \iff z\partial_q f(z) \in k-\mathcal{ST}_q(m,C,D). \tag{1.6}$$

**Special cases:**

(i) For $q \to 1^-$, and $m = 0$, then the classes $k-\mathcal{ST}_q(m,C,D)$ and $k-\mathcal{CV}_q(m,C,D)$ reduce into the classes $k-\mathcal{ST}(C,D)$ and $k-\mathcal{CV}(C,D)$ introduced by Noor and Malik in (Noor & Malik, 2011).

(ii) For $q \to 1^-$, $C = 1$, $D = -1$, and $m = 0$, then the classes $k-\mathcal{ST}_q(m,C,D)$ and $k-\mathcal{CV}_q(m,C,D)$ reduce into the classes $k-\mathcal{ST}$ and $k-\mathcal{UCV}$ introduced by Kanas and Wisniowska in (Kanas & Wisniowska, 2000), (Kanas & Wisniowska, 1999).

(iii) For $q \to 1^-$, $C = 1-2\alpha$, $D = -1$, and $m = 0$, then the classes $k-\mathcal{ST}_q(m,C,D)$ and $k-\mathcal{CV}_q(m,C,D)$ reduce into the classes $SD(k,\alpha)$ and $KD(k,\alpha)$ introduced by Shams et al. in (Shams *et al.*, 2004).

(iv) For $q \to 1^-$, $k = 0$, and $m = 0$, then the classes $k-\mathcal{ST}_q(m,C,D)$ and $k-\mathcal{CV}_q(m,C,D)$ reduce into the classes $\mathcal{S}^*(C,D)$ and $C(C,D)$ introduced by Janowski (Janowski, 1973).

**Definition 1.6.** A function $f(z) \in \mathcal{A}$ is said to be in the class $k-\mathcal{UK}_q(m,A,B,C,D)$, if and only if, for $k \geq 0$, $-1 \leq D < C \leq 1$, $-1 \leq B < A \leq 1$, there exists $g(z) \in k-\mathcal{ST}_q(m,C,D)$, such that

$$\Re\left(\frac{(B-1)L_{m,q}(z)-(A-1)}{(B+1)L_{m,q}(z)-(A+1)}\right) > k\left|\frac{(B-1)L_{m,q}(z)-(A-1)}{(B+1)L_{m,q}(z)-(A+1)}-1\right|,$$

where

$$L_{m,q}(z) = \frac{S_q^{m+1}f(z)}{S_q^m g(z)},$$

or equivalently

$$L_{m,q}(z) \in k-\mathcal{P}[A,B].$$

**Definition 1.7.** A function $f(z) \in \mathcal{A}$ is said to be in the class $k-\mathcal{UQ}_q(m,A,B,C,D)$, if and only if, for $k \geq 0$, $-1 \leq D < C \leq 1$, $-1 \leq B < A \leq 1$, there exists $g(z) \in k-\mathcal{CV}_q(m,C,D)$, such that

$$\Re\left(\frac{(B-1)K_{m,q}(z)-(A-1)}{(B+1)K_{m,q}(z)-(A+1)}\right) > k\left|\frac{(B-1)K_{m,q}(z)-(A-1)}{(B+1)K_{m,q}(z)-(A+1)}-1\right|,$$

where

$$K_{m,q}(z) = \frac{z\partial_q S_q^{m+1}f(z)}{S_q^{m+1}g(z)},$$

or equivalently,

$$K_{m,q}(z) \in k-\mathcal{P}[A,B].$$

It can be easily seen that

$$f(z) \in k-\mathcal{UQ}_q(m,A,B,C,D) \iff z\partial_q f(z) \in k-\mathcal{UK}_q(m,A,B,C,D). \tag{1.7}$$

**Special cases:**

(i) For $q \to 1^-$, and $m = 0$, then the classes $k - \mathcal{UK}_q(m, A, B, C, D)$ and $k - \mathcal{UQ}_q(m, A, B, C, D)$ reduce into the classes $k - \mathcal{UK}(A, B, C, D)$ and $k - \mathcal{UQ}(A, B, C, D)$ introduced by Mahmood at al. in (Mahmood *et al.*, 2017).

(ii) For $q \to 1^-$, $A = 1 - 2\beta$, $B = -1, C = 1 - 2\gamma, D = -1$ and $m = 0$, then the classes $k - \mathcal{UK}_q(m, A, B, C, D)$ and $k - \mathcal{UQ}_q(m, A, B, C, D)$ reduce into the classes $k - \mathcal{UK}(\beta, \gamma)$ and $k - \mathcal{UQ}(\beta, \gamma)$ introduced by AghalaryAghalary and Azadi in (Aghalary & Azadi, 2015).

(iii) For $q \to 1^-$, $A = 1 - 2\beta$, $B = -1, C = 1 - 2\gamma, D = -1$, $k = 0$ and $m = 0$, then the classes $k - \mathcal{UK}_q(m, A, B, C, D)$ and $k - \mathcal{UQ}_q(m, A, B, C, D)$ reduce into the classes $\mathcal{K}(\beta, \gamma)$ and $Q(\beta, \gamma)$ introduced by Libera and Noor in (Libera, 1964), (Noor, 1987).

(iv) For $q \to 1^-$, $k = 0$, and $m = 0$, then the class $k - \mathcal{UK}_q(m, A, B, C, D)$ reduce into the class $\mathcal{K}(A, B, C, D)$ introduced by Silvia in (Silvia, 1983).

(v) For $q \to 1^-$, $k = 0$, $C = 1$, $D = -1$, and $m = 0$, then the class $k - \mathcal{UQ}_q(m, A, B, C, D)$ reduce into the class $Q(A, B)$ introduced by Noor in (Noor, 1989).

(vi) For $q \to 1^-$, $A = 1, B = -1, C = 1, D = -1$, and $m = 0$, then the classes $k - \mathcal{UK}_q(m, A, B, C, D)$ and $k - \mathcal{UQ}_q(m, A, B, C, D)$ reduce into the classes $k - \mathcal{UK}$ and $k - \mathcal{UQ}$ introduced by Acu in (Acu, 2006).

(vii) For $q \to 1^-$, $k = 0$, $A = 1$, $B = -1$, $C = 1$, $D = -1$, and $m = 0$, then the classes $k - \mathcal{UK}_q(m, A, B, C, D)$ and $k - \mathcal{UQ}_q(m, A, B, C, D)$ reduced into the classes $\mathcal{K}$ and $Q$ introduced by Kaplan and Noor et al. in (Kaplan, 1952), (Noor *et al.*, 2009).

**Lemma 1.1.** *(See (Rogosinski, 1943)) Let $h(z) = 1 + \sum_{n=1}^{\infty} c_n z^n$ be subordinate to $H(z) = 1 + \sum_{n=1}^{\infty} C_n z^n$. If $H(z)$ is univalent in $E$ and $H(E)$ is convex, then*

$$|c_n| \leq |C_1|, \quad n \geq 1.$$

**Lemma 1.2.** *(See (Noor & Malik, 2011)) Let $h(z) = 1 + \sum_{n=1}^{\infty} c_n z^n \in k - \mathcal{P}[A, B]$, then*

$$|c_n| \leq |Q_1(k, A, B)|, |Q_1(k, A, B)| = \frac{A - B}{2} |Q_1(k)|,$$

*where $|Q_1(k)|$ is given by (1.3).*

## 2. Main Results

**Theorem 2.1.** *A function $f \in \mathcal{A}$ and of the form (1.1) is in the class $k - \mathcal{ST}_q(m, C, D)$, if it satisfies the condition*

$$\sum_{n=2}^{\infty} \left\{2(k+1)(q[n-1]_q) + \left|[n]_q (D+1) - (C+1)\right|\right\} [n]_q^m |a_n| \leq C - D, \tag{2.1}$$

*where $-1 \leq D < C \leq 1$, $k \geq 0$.*

*Proof.* Assuming that (2.1) holds, then it suffices to show that

$$k\left|\frac{(D-1)G_{m,q}(z)-(C-1)}{(D+1)G_{m,q}(z)-(C+1)}-1\right|-\Re\left\{\frac{(D-1)G_{m,q}(z)-(C-1)}{(D+1)G_{m,q}(z)-(C+1)}-1\right\}<1.$$

We have

$$\begin{aligned}
& k\left|\frac{(D-1)G_{m,q}(z)-(C-1)}{(D+1)G_{m,q}(z)-(C+1)}-1\right|-\Re\left\{\frac{(D-1)G_{m,q}(z)-(C-1)}{(D+1)G_{m,q}(z)-(C+1)}-1\right\}\\
\leq\ & (k+1)\left|\frac{(D-1)G_{m,q}(z)-(C-1)}{(D+1)G_{m,q}(z)-(C+1)}-1\right|\\
=\ & (k+1)\left|\frac{(D-1)S_q^{m+1}f(z)-(C-1)S_q^m f(z)}{(D+1)S_q^{m+1}f(z)-(C+1)S_q^m f(z)}-1\right|\\
=\ & 2(k+1)\left|\frac{S_q^m f(z)-S_q^{m+1}f(z)}{(D+1)S_q^{m+1}f(z)-(C+1)S_q^m f(z)}\right|\\
=\ & 2(k+1)\left|\frac{\sum_{n=2}^{\infty}([n]_q-1)\,[n]_q^m\, a_n z^n}{(D-C)\,z+\sum_{n=2}^{\infty}\left\{(D+1)\,[n]_q-(C+1)\right\}[n]_q^m\, a_n z^n}\right|\\
\leq\ & 2(k+1)\left\{\frac{\sum_{n=2}^{\infty} q\,[n-1]_q\,[n]_q^m\,|a_n|}{C-D-\sum_{n=2}^{\infty}\left|(D+1)\,[n]_q-(C+1)\right|[n]_q^m\,|a_n|}\right\}.
\end{aligned}$$

The last expression is bounded above by 1 if

$$\sum_{n=2}^{\infty}\left\{2(k+1)q\,[n-1]_q+\left|[n]_q\,(D+1)-(C+1)\right|\right\}[n]_q^m\,|a_n|\leq C-D.$$

This completes the proof. □

When $q\to 1^-$, $m=0$, we have the following known result, proved by Noor and Malik in (Noor & Malik, 2011).

**Corollary 2.1.** *A function $f\in\mathcal{A}$ and of the form (1.1) is in the class $k-\mathcal{ST}(C,D)$, if it satisfies the condition*

$$\sum_{n=2}^{\infty}\{2(k+1)(n-1)+|n(D+1)-(C+1)|\}\,|a_n|\leq|D-C|\,.$$

When $q\to 1^-$, $m=0$, $C=1-2\alpha$, $D=-1$ with $0\leq\alpha<1$, then we have the following known result, proved by Shams et al. in (Shams *et al.*, 2004).

**Corollary 2.2.** *A function $f\in\mathcal{A}$ and of the form (1.1) is in the class $SD(k,\alpha)$, if it satisfies the condition*

$$\sum_{n=2}^{\infty}\{n(k+1)-(k+\alpha)\}\,|a_n|\leq 1-\alpha,$$

*where $0\leq\alpha<1$ and $k\geq 0$.*

When $q \to 1^-$, $k = 0$, $m = 0$, $C = 1 - 2\alpha$, $D = -1$ with $0 \leq \alpha < 1$, then we have the following known result, proved by Silverman in (Silverman, 1975).

**Corollary 2.3.** *A function $f \in \mathcal{A}$ and of the form (1.1) is in the class $S^*(\alpha)$, if it satisfies the condition*

$$\sum_{n=2}^{\infty} \{n - \alpha\} |a_n| \leq 1 - \alpha, \qquad 0 \leq \alpha < 1.$$

**Theorem 2.2.** *A function $f \in \mathcal{A}$ and of the form (1.1) is in the class $k - \mathcal{CV}_q(m, C, D)$, if it satisfies the condition*

$$\sum_{n=2}^{\infty} \left\{2(k+1)(q\,[n-1]_q) + \left|[n]_q\,(D+1) - (C+1)\right|\right\} [n]_q^{m+1} |a_n| \leq C - D,$$

*where $-1 \leq D < C \leq 1$, $k \geq 0$.*

The proof follows immediately by using Theorem 2.1 and (1.6).

When $q \to 1^-$, $m = 0$, then, we have the following known result, proved by Noor and Malik in (Noor & Malik, 2011).

**Corollary 2.4.** *A function $f \in \mathcal{A}$ and of the form (1.1) is in the class $k - \mathcal{UCV}(C, D)$, if it satisfies the condition*

$$\sum_{n=2}^{\infty} n\,\{2(k+1)(n-1) + |n(D+1) - (C+1)|\}\,|a_n| \leq C - D.$$

**Theorem 2.3.** *If $f(z) \in k - \mathcal{ST}_q(m, C, D)$ and is of the form (1.1). Then*

$$|a_n| \leq \prod_{j=0}^{n-2} \left( \frac{\left|Q_1(k)(C-D) - 2q[j]_q\,[j+1]_q^m\,D\right|}{2q\,[j+1]_q\,[j+2]_q^m} \right), \qquad n \geq 2, \tag{2.2}$$

*where $|Q_1(k)|$ is defined by (1.3).*

*Proof.* By definition, for $f(z) \in k - \mathcal{ST}_q(m, C, D)$, we have

$$\frac{S_q^{m+1} f(z)}{S_q^m f(z)} = p(z), \tag{2.3}$$

where

$$p(z) \in k - P[C, D].$$

Now from (2.3), we have

$$S_q^{m+1} f(z) = S_q^m f(z) p(z),$$

which implies that

$$\begin{aligned}
z+\sum_{n=2}^{\infty}[n]_q^{m+1}a_nz^n &= \left(1+\sum_{n=1}^{\infty}c_nz^n\right)\left(z+\sum_{n=2}^{\infty}[n]_q^{m}a_nz^n\right)\\
z+\sum_{n=2}^{\infty}[n]_q^{m+1}a_nz^n &= \left(1+\sum_{n=1}^{\infty}c_nz^n\right)\left(\sum_{n=1}^{\infty}[n]_q^{m}a_nz^n\right)\\
z+\sum_{n=2}^{\infty}[n]_q^{m+1}a_nz^n &= \sum_{n=1}^{\infty}[n]_q^{m}a_nz^n+\left(\sum_{n=1}^{\infty}[n]_q^{m}a_nz^n\right)\left(\sum_{n=1}^{\infty}c_nz^n\right)\\
\sum_{n=2}^{\infty}\left\{[n]_q-1\right\}[n]_q^{m}a_nz^n &= \left(\sum_{n=1}^{\infty}[n]_q^{m}a_nz^n\right)\left(\sum_{n=1}^{\infty}c_nz^n\right)\\
\sum_{n=2}^{\infty}q\,[n-1]_q\,[n]_q^{m}a_nz^n &= \left(\sum_{n=1}^{\infty}[n]_q^{m}a_nz^n\right)\left(\sum_{n=1}^{\infty}c_nz^n\right).
\end{aligned}\tag{2.4}$$

By using Cauchy product formula on R.H.S of (2.4), we have

$$\sum_{n=2}^{\infty}q\,[n-1]_q\,[n]_q^{m}a_nz^n=\sum_{n=1}^{\infty}\left[\sum_{j=1}^{n-1}[j]_q^{m}a_jc_{n-j}\right]z^n.\tag{2.5}$$

Equating coefficients of $z^n$ on both sides of (2.5), we have

$$q\,[n-1]_q\,[n]_q^{m}a_n=\sum_{j=1}^{n-1}[j]_q^{m}a_jc_{n-j},\quad [1]_q^m=1,\ a_1=1.$$

This implies that

$$|a_n|\le\frac{1}{q\,[n-1]_q\,[n]_q^{m}}\sum_{j=1}^{n-1}[j]_q^{m}\left|a_j\right|\left|c_{n-j}\right|,\ \ [1]_q^m=1,\ a_1=1.$$

Using lemma (1.2), we have

$$|a_n|\le\frac{|Q_1(k)|\,(C-D)}{2q\,[n-1]_q\,[n]_q^{m}}\sum_{j=1}^{n-1}[j]_q^{m}\left|a_j\right|,\qquad [1]_q^m=1,a_1=1.\tag{2.6}$$

Now we prove that

$$\begin{aligned}
&\frac{|Q_1(k)|\,(C-D)}{2q\,[n-1]_q\,[n]_q^{m}}\sum_{j=1}^{n-1}[j]_q^{m}\left|a_j\right|\\
\le\ &\prod_{j=1}^{n-1}\left(\frac{\left|Q_1(k)(C-D)-2q[j-1]_q\,[j]_q^{m}D\right|}{2q\,[j]_q\,[j+1]_q^{m}}\right),\\
&\frac{|Q_1(k)|\,(C-D)}{2q\,[n-1]_q\,[n]_q^{m}}\sum_{j=1}^{n-1}[j]_q^{m}\left|a_j\right|\\
\le\ &\prod_{j=0}^{n-2}\left(\frac{\left|Q_1(k)(C-D)-2q[j]_q\,[j+1]_q^{m}C\right|}{2q\,[j+1]_q\,[j+2]_q^{m}}\right).
\end{aligned}$$

For this, we use the induction method.
For $n=2$ from (2.6), we have

$$|a_2|\le\frac{|Q_1(k)|\,(C-D)}{2q\,[2]_q^{m}}.$$

From (2.2), we have

$$|a_2| \leq \frac{|Q_1(k)|\,(C-D)}{2q\,[2]_q^m}.$$

For $n = 3$ from (2.6), we have

$$\begin{aligned} |a_3| &\leq \frac{|Q_1(k)|\,(C-D)}{2q[2]_q\,[3]_q^m}\left\{1+[2]_q^m\,|a_2|\right\} \\ &\leq \frac{|Q_1(k)|\,(C-D)}{2q[2]_q\,[3]_q^m}\left\{1+\frac{|Q_1(k)|\,(C-D)}{2q}\right\}. \end{aligned}$$

From (2.2), we have

$$\begin{aligned} |a_3| &\leq \frac{(C-D)\,|Q_1(k)|}{2q\,[2]_q^m}\left\{\frac{\left|Q_1(k)(C-D)-2q\,[2]_q^m\,D\right|}{2q[2]_q\,[3]_q^m}\right\}, \quad [1]_q = 1, \\ &\leq \frac{(C-D)\,|Q_1(k)|}{2q\,[2]_q^m}\left\{\frac{|Q_1(k)|\,(C-D)+2q\,[2]_q^m\,|D|}{2q[2]_q\,[3]_q^m}\right\} \\ &\leq \frac{(C-D)\,|Q_1(k)|}{2q[2]_q\,[3]_q^m}\left\{1+\frac{|Q_1(k)|\,(C-D)}{2q\,[2]_q^m}\right\}. \end{aligned}$$

Let the hypothesis be true for $n = t$.
From (2.6), we have

$$|a_t| \leq \frac{|Q_1(k)|\,(C-D)}{2q\,[t-1]_q\,[t]_q^m}\sum_{j=1}^{t-1}[j]_q^m\,|a_j|, \quad a_1 = 1, \quad [1]_q^m.$$

From (2.2), we have

$$\begin{aligned} |a_t| &\leq \prod_{j=0}^{t-2}\left(\frac{\left|Q_1(k)(C-D)-2q[j]_q\,[j+1]_q^m\,D\right|}{2q\,[j+1]_q\,[j+2]_q^m}\right) \\ &\leq \prod_{j=0}^{t-2}\left(\frac{\left|Q_1(k)(C-D)+2q[j]_q\,[j+1]_q^m\right|}{2q\,[j+1]_q\,[j+2]_q^m}\right). \end{aligned}$$

By the induction hypothesis, we have

$$\begin{aligned} &\frac{|Q_1(k)|\,(C-D)}{2q\,[t-1]_q\,[t]_q^m}\sum_{j=1}^{t-1}[j]_q^m\,|a_j| \\ &\leq \prod_{j=0}^{t-2}\left(\frac{\left|Q_1(k)(C-D)+2q[j]_q\,[j+1]_q^m\right|}{2q\,[j+1]_q\,[j+2]_q^m}\right). \end{aligned} \tag{2.7}$$

Multiplying both sides by

$$\frac{\left|Q_1(k)(C-D)+2q[t-1]_q\,[t]_q^m\right|}{2q\,[t+1]_q\,[t+2]_q^m},$$

we have

$$\frac{\left|Q_1(k)(C-D)+2q[t-1]_q\,[t]_q^m\right|}{2q\,[t+1]_q\,[t+2]_q^m}\times\prod_{j=0}^{t-2}\left(\frac{\left|Q_1(k)(C-D)+2q[j]_q\,[j+1]_q^m\right|}{2q\,[j+1]_q\,[j+2]_q^m}\right)$$

$$\geq \left\{\frac{\left|Q_1(k)(C-D)+2q[t-1]_q\,[t]_q^m\right|}{2q\,[t+1]_q\,[t+2]_q^m}\right\}\times\frac{|Q_1(k)|\,(C-D)}{2q\,[t-1]_q\,[t]_q^m}\sum_{j=1}^{t-1}[j]_q^m\left|a_j\right|,$$

$$\prod_{j=0}^{t-2}\left(\frac{\left|Q_1(k)(C-D)+2q[j]_q\,[j+1]_q^m\right|}{2q\,[j+1]_q\,[j+2]_q^m}\right)$$

$$\geq \begin{cases} \frac{|Q_1(k)|(C-D)}{2q[t+1]_q[t+2]_q^m}\left\{\frac{|Q_1(k)|(C-D)}{2q[t-1]_q[t]_q^m}\sum_{j=1}^{t-1}[j]_q^m\left|a_j\right|\right\} \\ +\frac{2q[t-1]_q[t]_q^m}{2q[t+1]_q[t+2]_q^m}\left\{\frac{|Q_1(k)|(C-D)}{2q[t-1]_q[t]_q^m}\sum_{j=1}^{t-1}[j]_q^m\left|a_j\right|\right\}, \end{cases}$$

$$\geq \frac{|Q_1(k)|\,(C-D)}{2q\,[t+1]_q\,[t+2]_q^m}\left\{\frac{|Q_1(k)|\,(C-D)}{2q\,[t-1]_q\,[t]_q^m}\sum_{j=1}^{t-1}[j]_q^m\left|a_j\right|+\sum_{j=1}^{t-1}[j]_q^m\left|a_j\right|\right\},$$

$$\prod_{j=0}^{t-2}\left(\frac{\left|Q_1(k)(C-D)+2q[j]_q\,[j+1]_q^m\right|}{2q\,[j+1]_q\,[j+2]_q^m}\right)$$

$$\geq \frac{|Q_1(k)|\,(C-D)}{2q\,[t+1]_q\,[t+2]_q^m}\left\{|a_t|+\sum_{j=1}^{t-1}[j]_q^m\left|a_j\right|\right\},$$

$$= \frac{|Q_1(k)|\,(C-D)}{2q\,[t+1]_q\,[t+2]_q^m}\sum_{j=1}^{t}[j]_q^m\left|a_j\right|.$$

That is,

$$\frac{|Q_1(k)|\,(A-B)}{2q\,[t+1]_q\,[t+2]_q^m}\sum_{j=1}^{t}[j]_q^m\left|a_j\right|$$

$$\leq \prod_{j=0}^{t-2}\left(\frac{\left|Q_1(k)(C-D)+2q[j]_q\,[j+1]_q^m\right|}{2q\,[j+1]_q\,[j+2]_q^m}\right).$$

which shows that inequality (2.7) is true for $n = t + 1$. Hence the required result. □

When $m = 0$, $q \to 1^-$, we have the following known result, proved by Noor and Malik in (Noor & Malik, 2011).

**Corollary 2.5.** *A function $f \in \mathcal{A}$ and of the form (1.1) is in the class $k - \mathcal{ST}[C, D]$, if it satisfies the condition*

$$|a_n| \leq \prod_{j=0}^{n-2}\left(\frac{|Q_1(k)(C-D)-2jD|}{2\,(j+1)}\right).$$

When $m = 0$, $q \to 1^-$, $C = 1$, $D = -1$, then we have the following known result, proved by Kanas and Wisniowska in (Kanas & Wisniowska, 2000).

**Corollary 2.6.** *A function $f \in \mathcal{A}$ and of the form (1.1) is in the class $k - \mathcal{ST}$, if it satisfies the condition*

$$|a_n| \leq \prod_{j=0}^{n-2} \left( \frac{|Q_1(k) + j|}{(j+1)} \right).$$

When $m = 0$, $q \to 1^-$, $k = 0$, then $Q_1(k) = 2$ and we get the following known result, proved in (Janowski, 1973).

**Corollary 2.7.** *A function $f \in \mathcal{A}$ and of the form (1.1) is in the class $\mathcal{ST}[C, D]$, if it satisfies the condition*

$$|a_n| \leq \prod_{j=0}^{n-2} \left( \frac{|(C-D) - jD|}{(j+1)} \right), \qquad -1 \leq D < C \leq 1.$$

*When $m = 0$, $q \to 1^-$, $C = 1 - 2\alpha$, $D = -1$, with $0 \leq \alpha < 1$, then we have the following known result, proved by Shams et al. in (Shams et al., 2004).*

**Corollary 2.8.** *A function $f \in \mathcal{A}$ and of the form (1.1) is in the class $SD(k, \alpha)$, if it satisfies the condition*

$$|a_n| \leq \prod_{j=0}^{n-2} \left( \frac{|Q_1(k)(1-\alpha) + j|}{(j+1)} \right), \qquad -1 \leq D < C \leq 1.$$

**Theorem 2.4.** *If $f(z) \in k - \mathcal{CV}_q(m, C, D)$ and is of the form (1.1). Then*

$$|a_n| \leq \frac{1}{[n]_q} \prod_{j=0}^{n-2} \left( \frac{\left|Q_1(k)(C-D) - 2q[j]_q\,[j+1]_q^m\, D\right|}{2q\,[j+1]_q\,[j+2]_q^m} \right), \qquad (n \geq 2).$$

The proof follows immediately by using Theorem (2.3) and the relation (1.6).

When $m = 0$, $q \to 1^-$, we have the following known result, proved by Noor and Sarfaraz in (Noor & Malik, 2011).

**Corollary 2.9.** *A function $f \in \mathcal{A}$ and of the form (1.1) is in the class $k - \mathcal{UCV}[C, D]$, if it satisfies the condition*

$$|a_n| \leq \frac{1}{n} \prod_{j=0}^{n-2} \left( \frac{|Q_1(k)(C-D) - 2jD|}{2\,(j+1)} \right).$$

**Theorem 2.5.** *A function $f \in \mathcal{A}$ and of the form (1.1) is in the class $k - \mathcal{UK}_q(m, A, B, C, D)$, if it satisfies the condition*

$$\sum_{n=2}^{\infty} \left\{ 2(k+1) \left| b_n - [n]_q\, a_n \right| + \left| (B+1)\,[n]_q\, a_n - (A+1) b_n \right| \right\} [n]_q^m \leq A - B, \tag{2.8}$$

*where $-1 \leq D < C \leq 1$, $-1 \leq B < A \leq 1$, $k \geq 0$.*

*Proof.* Assuming that (2.8) holds, then it suffices to show that

$$k \left| \frac{(B-1)L_{m,q}(z) - (A-1)}{(B+1)L_{m,q}(z) - (A+1)} - 1 \right| - \Re \left\{ \frac{(B-1)L_{m,q}(z) - (A-1)}{(B+1)L_{m,q}(z) - (A+1)} - 1 \right\} < 1.$$

We have

$$\begin{aligned}
& k\left|\frac{(B-1)L_{m,q}(z)-(A-1)}{(B+1)L_{m,q}(z)-(A+1)}-1\right|-\Re\left\{\frac{(B-1)L_{m,q}(z)-(A-1)}{(B+1)L_{m,q}(z)-(A+1)}-1\right\} \\
\leq\ & (k+1)\left|\frac{(B-1)L_{m,q}(z)-(A-1)}{(B+1)L_{m,q}(z)-(A+1)}-1\right| \\
=\ & (k+1)\left|\frac{(B-1)S_q^{m+1}f(z)-(A-1)S_q^m g(z)}{(B+1)S_q^{m+1}f(z)-(A+1)S_q^m g(z)}-1\right| \\
=\ & 2(k+1)\left|\frac{S_q^m g(z)-S_q^{m+1}f(z)}{(B+1)S_q^{m+1}f(z)-(A+1)S_q^m g(z)}\right| \\
=\ & 2(k+1)\left|\frac{\sum_{n=2}^{\infty}\left\{b_n-[n]_q a_n\right\}[n]_q^m z^n}{(B-A)z+\sum_{n=2}^{\infty}\left\{(B+1)[n]_q a_n-(A+1)b_n\right\}[n]_q^m z^n}\right| \\
\leq\ & 2(k+1)\left\{\frac{\sum_{n=2}^{\infty}\left|b_n-[n]_q a_n\right|[n]_q^m}{A-B-\sum_{n=2}^{\infty}\left|(B+1)[n]_q a_n-(A+1)b_n\right|[n]_q^m}\right\}.
\end{aligned}$$

The last expression is bounded above by 1 if

$$\sum_{n=2}^{\infty}\left\{2(k+1)\left|b_n-[n]_q a_n\right|+\left|(B+1)[n]_q a_n-(A+1)b_n\right|\right\}[n]_q^m \leq A-B.$$

This completes the proof. □

When $q \to 1^-$, $m = 0$, we have the following known result, proved by Mahmood at al. (Mahmood *et al.*, 2017).

**Corollary 2.10.** *A function $f \in \mathcal{A}$ and of the form (1.1) is in the class $k-\mathcal{UK}(A,B,C,D)$, if it satisfies the condition*

$$\sum_{n=2}^{\infty}\{2(k+1)|b_n-na_n|+|(B+1)na_n-(A+1)b_n|\}\leq A-B.$$

**Theorem 2.6.** *A function $f \in \mathcal{A}$ and of the form (1.1) is in the class $k-\mathcal{UQ}_q(m,A,B,C,D)$, if it satisfies the condition*

$$\sum_{n=2}^{\infty}[n]_q^{m+1}\left\{2(k+1)\left|b_n-[n]_q a_n\right|+\left|(B+1)[n]_q a_n-(A+1)b_n\right|\right\}\leq A-B,$$

*where $-1 \leq D < C \leq 1$, $-1 \leq B < A \leq 1, k \geq 0$.*

The proof follows immediately by using Theorem 2.1 and (1.7).

When $q \to 1^-$, $m = 0$, we have the following known result, proved by Mahmood at al. (Mahmood *et al.*, 2017)

**Corollary 2.11.** *A function $f \in \mathcal{A}$ and of the form (1.1) is in the class $k - \mathcal{UQ}(A, B, C, D)$, if it satisfies the condition*

$$\sum_{n=2}^{\infty} n\left\{2(k+1)\left|b_n - na_n\right| + \left|(B+1)na_n - (A+1)b_n\right|\right\} \leq A - B.$$

When $q \to 1^-$, $m = 0$, $A = 1 - 2\beta$, $B = -1$, $C = 1$, $D = -1$ with $0 \leq \beta < 1$, then we have the following known result, proved by Subramanian et al. in (Subramanian *et al.*, 2003).

**Corollary 2.12.** *A function $f \in \mathcal{A}$ and of the form (1.1) is in the class $\mathcal{UQ}(\beta)$, $g(z) = z$, if it satisfies the condition*

$$\sum_{n=2}^{\infty} n^2 \left|a_n\right| \leq 1 - \beta.$$

**Theorem 2.7.** *If $f(z) \in k - \mathcal{UK}_q(m, A, B, C, D)$ and is of the form (1.1). Then*

$$\left|a_n\right| \leq \begin{cases} \frac{1}{[n]_q} \prod_{i=0}^{n-2} \left( \frac{\left|Q_1(k)(C-D) - 2q[i]_q[i+1]_q^m D\right|}{2q[i+1]_q[i+2]_q^m} \right) \\ + \frac{\left|Q_1(k)\right|(A-B)}{2[n]_q[n]_q^m} \sum_{j=1}^{n-1} [j]_q^m \prod_{i=0}^{j-2} \left( \frac{\left|Q_1(k)(C-D) - 2q[i]_q[i+1]_q^m D\right|}{2q[i+1]_q[i+2]_q^m} \right), \quad n \geq 2. \end{cases}$$

*where $|Q_1(k)|$ is defined by (1.3).*

*Proof.* Let us take

$$\frac{S_q^{m+1} f(z)}{S_q^m g(z)} = p(z), \tag{2.9}$$

where

$$p(z) \in k - \mathcal{P}[A, B] \text{ and } g(z) \in k - \mathcal{ST}_q(m, C, D).$$

Now from (2.9), we have

$$S_q^{m+1} f(z) = S_q^m g(z) p(z),$$

which implies that

$$\begin{aligned} z + \sum_{n=2}^{\infty} [n]_q^{m+1} a_n z^n &= \left(1 + \sum_{n=1}^{\infty} c_n z^n\right)\left(z + \sum_{n=2}^{\infty} [n]_q^m b_n z^n\right), \\ z + \sum_{n=2}^{\infty} [n]_q^{m+1} a_n z^n &= \left(1 + \sum_{n=1}^{\infty} c_n z^n\right)\left(\sum_{n=1}^{\infty} [n]_q^m b_n z^n\right), \\ z + \sum_{n=2}^{\infty} [n]_q^{m+1} a_n z^n &= \sum_{n=1}^{\infty} [n]_q^m b_n z^n + \left(\sum_{n=1}^{\infty} [n]_q^m b_n z^n\right)\left(\sum_{n=1}^{\infty} c_n z^n\right), \\ \sum_{n=2}^{\infty} \left\{[n]_q a_n - b_n\right\} [n]_q^m z^n &= \left(\sum_{n=1}^{\infty} [n]_q^m b_n z^n\right)\left(\sum_{n=1}^{\infty} c_n z^n\right), \end{aligned} \tag{2.10}$$

By using Cauchy product formula on R.H.S of (2.10), we have

$$\sum_{n=2}^{\infty} \left\{[n]_q a_n - b_n\right\} [n]_q^m z^n = \sum_{n=1}^{\infty} \left[\sum_{j=1}^{n-1} [j]_q^m b_j c_{n-j}\right] z^n. \tag{2.11}$$

Equating coefficients of $z^n$ on both sides of (2.11), we have

$$\left\{[n]_q a_n - b_n\right\}[n]_q^m = \sum_{j=1}^{n-1}[j]_q^m b_j c_{n-j}, \quad ,a_0 = 1,$$

$$[n]_q^{m+1} a_n = [n]_q^m b_n + \sum_{j=1}^{n-1}[j]_q^m b_j c_{n-j}.$$

This implies that

$$[n]_q^{m+1}|a_n| \le [n]_q^m |b_n| + \sum_{j=1}^{n-1}[j]_q^m \left|b_j\right|\left|c_{n-j}\right|, \quad a_1 = 1. \tag{2.12}$$

Since $p(z) \in k - \mathcal{P}[A,B]$, therefore by using lemma 1.2 on (2.12), we have

$$[n]_q^{m+1}|a_n| \le [n]_q^m |b_n| + \sum_{j=1}^{n-1}\frac{|Q_1(k)|(A-B)}{2}[j]_q^m \left|b_j\right|. \tag{2.13}$$

Again $g(z) \in k - \mathcal{ST}_q(m,C,D)$, therefore by using Theorem 2.3 on (2.13), we have

$$[n]_q^{m+1}|a_n| \le \begin{cases} [n]_q^m \prod_{i=0}^{n-2}\left(\frac{\left|Q_1(k)(C-D)-2q[i]_q[i+1]_q^m D\right|}{2q[i+1]_q[i+2]_q^m}\right) \\ +\frac{|Q_1(k)|(A-B)}{2}\sum_{j=1}^{n-1}[j]_q^m \prod_{i=0}^{j-2}\left(\frac{\left|Q_1(k)(C-D)-2q[i]_q[i+1]_q^m D\right|}{2q[i+1]_q[i+2]_q^m}\right), \end{cases}$$

which implies that

$$|a_n| \le \begin{cases} \frac{1}{[n]_q}\prod_{i=0}^{n-2}\left(\frac{\left|Q_1(k)(C-D)-2q[i]_q[i+1]_q^m D\right|}{2q[i+1]_q[i+2]_q^m}\right) \\ +\frac{|Q_1(k)|(A-B)}{2[n]_q[n]_q^m}\sum_{j=1}^{n-1}[j]_q^m \prod_{i=0}^{j-2}\left(\frac{\left|Q_1(k)(C-D)-2q[i]_q[i+1]_q^m D\right|}{2q[i+1]_q[i+2]_q^m}\right). \end{cases}$$

□

When $q \to 1^-$, $m = 0$, we have the following known result, proved by Mahmood et al. (Mahmood *et al.*, 2017).

**Corollary 2.13.** *If $f(z) \in k - \mathcal{UK}(m,A,B,C,D)$ and is of the form (1.1). Then*

$$|a_n| \le \begin{cases} \frac{1}{n}\prod_{i=0}^{n-2}\left(\frac{|Q_1(k)(C-D)-2iD|}{2(i+1)}\right) \\ +\frac{|Q_1(k)|(A-B)}{2n}\sum_{j=1}^{n-1}\prod_{i=0}^{j-2}\left(\frac{|Q_1(k)(C-D)-2iD|}{2(i+1)}\right), \quad n \ge 2, \end{cases}$$

*where $Q_1(k)$ is defined by (1.3).*

When $q \to 1^-$, $m = 0$, $A = 1$, $B = -1$, $C = 1$, $D = -1$, we have the following known result, proved by Noor et al. (Noor *et al.*, 2009).

**Corollary 2.14.** *If $f(z) \in k - \mathcal{UK}(0,1,-1,1,-1)$ and is of the form (1.1). Then*

$$|a_n| \le \frac{(|Q_1(k)|)_{n-1}}{n!} + \frac{|Q_1(k)|}{n}\sum_{j=0}^{n-1}\frac{(|Q_1(k)|)_{j-1}}{(j-1)!}, \quad n \ge 2.$$

When $q \to 1^-$, $m = 0$, $k = 0$, $A = 1$, $B = -1$, $C = 1$, $D = -1$, we have the following known result, proved by Kaplan (Kaplan, 1952).

**Corollary 2.15.** *If $f(z) \in 0 - \mathcal{UK}(0, 1, -1, 1, -1) = \mathcal{K}$ and is of the form (1.1). Then*

$$|a_n| \leq n, \quad n \geq 2.$$

**Theorem 2.8.** *If $f(z) \in k - \mathcal{UQ}_q(m, A, B, C, D)$ and is of the form (1.1). Then*

$$|a_n| \leq \begin{cases} \frac{1}{[n]_q^2} \prod_{i=0}^{n-2} \left( \frac{\left|Q_1(k)(C-D)-2q[i]_q[i+1]_q^m D\right|}{2q[i+1]_q[i+2]_q^m} \right) \\ + \frac{|Q_1(k)|(A-B)}{2[n]_q^2[n]_q^m} \sum_{j=1}^{n-1} [j]_q^m \prod_{i=0}^{j-2} \left( \frac{\left|Q_1(k)(C-D)-2q[i]_q[i+1]_q^m D\right|}{2q[i+1]_q[i+2]_q^m} \right), \end{cases}$$

*where $|Q_1(k)|$ is defined by (1.3).*

*Proof.* The proof follows immediately by using Theorem 2.7 and (1.7). □

When $q \to 1^-$, $m = 0$, we have the following known result, proved by Mahmood at al. (Mahmood *et al.*, 2017).

**Corollary 2.16.** *If $f(z) \in k - \mathcal{UK}(m, A, B, C, D)$ and is of the form (1.1). Then*

$$|a_n| \leq \begin{cases} \frac{1}{n^2} \prod_{i=0}^{n-2} \left( \frac{|Q_1(k)(C-D)-2iD|}{2(i+1)} \right) \\ + \frac{|Q_1(k)|(A-B)}{2n^2} \sum_{j=1}^{n-1} \prod_{i=0}^{j-2} \left( \frac{|Q_1(k)(C-D)-2iD|}{2(i+1)} \right), \quad n \geq 2. \end{cases}$$

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