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Expected Value of a Picture Fuzzy Number

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Abstract

This paper proposes a mathematical framework for the definition and computation of the expected interval and expected value of Picture Fuzzy Numbers (PFNs), providing a robust and interpretable tool for ranking and decision-making analysis in contexts characterized by imprecise or uncertain information.

Keywords: Picture fuzzy number, expected interval, expected value.

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Introduction

In recent decades, fuzzy theories have become fundamental tools for modeling uncertainty and imprecision in decision-making and optimization problems. In particular, fuzzy numbers have been employed to represent incomplete or uncertain information, allowing decision-makers to express preferences and evaluations in a more flexible manner than traditional methods.

Building on the classical concept of a fuzzy number introduced by Zadeh (1975*a*), Zadeh (1975*b*), several extensions have been developed, such as Intuitionistic Fuzzy Numbers (IFNs) introduced by Atanassov (1986) and Picture Fuzzy Numbers (PFNs) introduced by Cuong (2013). PFN-s enable the simultaneous modeling of multiple types of information: membership degree, non-membership degree, and hesitation degree. PFNs, in particular, provide a richer framework, including the ability to represent neutral or indeterminate opinions, making them well-suited for applications in multi-criteria decision-making, risk analysis, quality assessment, and other complex domains (Wei & Gao (2018), Qiyas *et al.* (2019), Xian *et al.* (2021), Shit *et al.* (2022), Jaikumar *et al.* (2023), Jana *et al.* (2024), Akdemir & Aydin (2025), Garg *et al.* (2025)). The comparison and ranking of PFNs remain a major challenge, with existing methods often relying on scoring functions or similarity measures.

However, the concept of expected value for PFNs remains insufficiently explored. Transforming a PFN into a scalar indicator through its expected value can facilitate ranking, defuzzification, and integration of

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PFNs into decision-making models, while simultaneously preserving information about uncertainty and hesitation.

This gap motivates the current study: there is a clear need for a rigorous expected value concept for PFNs that can transform the triple of membership, non-membership, and neutral degrees into a single, interpretable scalar for ranking, defuzzification, and decision-making purposes. Developing such a measure would not only extend the theoretical framework of PFNs but also enhance their practical usability in real-world decision-making contexts.

Building on the research on expected interval and expected value for a fuzzy number by Dubois & Prade (1987) and Heilpern (1992), as well as the research on expected interval and expected value for an intuitionistic fuzzy number by Grzegrorzewski (2003) and Nehi & Maleki (2005), this paper introduces the concept of expected interval and expected value for a picture fuzzy number in a general setting, and in particular for the trapezoidal picture fuzzy number.

1. Preliminaries

Definition 1.1. Cuong (2013) Let Ω an universal set. A subset

$$A = \{(x, \mu_A(x), \eta_A(x), \nu_A(x))\,; x \in \Omega\},$$

where $\mu_A : \Omega \to [0,1]$ is the degree of positive membership of x in A, $\eta_A : \Omega \to [0,1]$ represents the degree of neutral membership of x in A and $\nu_A : \Omega \to [0,1]$ is the degree of negative membership of x in A, respectively and μ_A, η_A and ν_A satisfy the condition:

$$0 \leqslant \mu_A(x) + \eta_A(x) + \nu_A(x) \leqslant 1,\ (\forall) x \in \Omega,$$

is a picture fuzzy set (PFS) on Ω.

$\pi_A : \Omega \to [0,1],\ \pi_A(x) = 1 - \mu_A(x) - \eta_A(x) - \nu_A(x)$ is called degree of refusal membership of x in A.

Definition 1.2. Cuong & Kreinovich (2013) Let $A = \{(x, \mu_A(x), \eta_A(x), \nu_A(x))\,; x \in \Omega\}$ be o picture fuzzy set on Ω and $\alpha, \gamma, \beta \in [0,1], \alpha + \gamma + \beta \leq 1$ then the upper (α, γ, β)-cut of A is given by

$$A^{(\alpha,\gamma,\beta)} = \{x \in \Omega : \mu_A(x) \geq \alpha, \eta_A(x) \geq \gamma, \nu_A(x) \leq \beta\}$$

That is, $A^{\alpha} = \{x : \mu_A \geq \alpha\}, A^{\gamma} = \{x : \eta_A \geq \gamma\}, A^{\beta} = \{x : \nu_A \leq \beta\}$ are upper α, γ and β-cut of positive membership, neutral membership and negative membership of a picture fuzzy set A respectively.

Definition 1.3. Qiyas *et al.* (2019) A picture fuzzy number (PFN) $A \in \mathbb{R}$ is denoted as $A = \langle(\mu_A, \eta_A, \nu_A); w_1, w_2, w_3\rangle$ whose positive, neutral and negative membership functions are defined as follows:

$$\mu_A(x) = \begin{cases} f_A^L(x) & if \quad a \leqslant x < b \\ w_1 & if \quad b \leqslant x \leqslant c \\ f_A^R(x) & if \quad c < x \leqslant d \\ 0 & otherwise, \end{cases} \qquad \eta_A(x) = \begin{cases} g_A^L(x) & if \quad a' \leqslant x < b \\ w_2 & if \quad b \leqslant x \leqslant c \\ g_A^R(x) & if \quad c < x \leqslant d' \\ 0 & otherwise, \end{cases} \tag{1.1}$$

$$\nu_A(x) = \begin{cases} h_A^L(x) & if \quad a'' \leqslant x < b \\ w_3 & if \quad b \leqslant x \leqslant c \\ h_A^R(x) & if \quad c < x \leqslant d'' \\ 1 & otherwise, \end{cases}$$

where f_A^L, g_A^L, h_A^R are increasing functions and f_A^R, g_A^R, h_A^L are nonincreasing functions. The values w_1, w_2, w_3 represent the maximum degrees of the positive, neutral and negative membership, $w_1, w_2, w_3 \in [0, 1]$ and $0 \leqslant w_1 + w_2 + w_3 \leqslant 1$.

Definition 1.4. Akram *et al.* (2022) The picture fuzzy number A for which the positive, neutral and negative membership functions of form:

$$\mu_A(x) = \begin{cases} \frac{w_1(x-a)}{b-a} & if \quad a \leqslant x < b \\ w_1 & if \quad b \leqslant x \leqslant c \\ \frac{w_1(d-x)}{d-c} & if \quad c < x \leqslant d \\ 0 & otherwise, \end{cases} \qquad \eta_A(x) = \begin{cases} \frac{w_2(x-a')}{b-a'} & if \quad a' \leqslant x < b \\ w_2 & if \quad b \leqslant x \leqslant c \\ \frac{w_2(d'-x)}{d'-c} & if \quad c < x \leqslant d' \\ 0 & otherwise, \end{cases}$$

$$\nu_A(x) = \begin{cases} \frac{(b-x)+w_3(x-a'')}{b-a''} & if \quad a'' \leqslant x < b \\ w_3 & if \quad b \leqslant x \leqslant c \\ \frac{(x-c)+w_3(d''-x)}{d''-c} & if \quad c < x \leqslant d'' \\ 1 & otherwise, \end{cases}$$

where $a, a', a'', b, c, d, d', d'' \in \mathbb{R}$ with $a'' \leqslant a' \leqslant a \leqslant b \leqslant c \leqslant d \leqslant d' \leqslant d''$ and $w_1, w_2, w_3 \in [0, 1]$, $0 \leqslant w_1 + w_2 + w_3 \leqslant 1$, will be called trapezoidal picture fuzzy number (TrPFN), denoted by $A = \langle (a, b, c, d), (a', b, c, d'), (a'', b, c, d''); w_1, w_2, w_3 \rangle$.

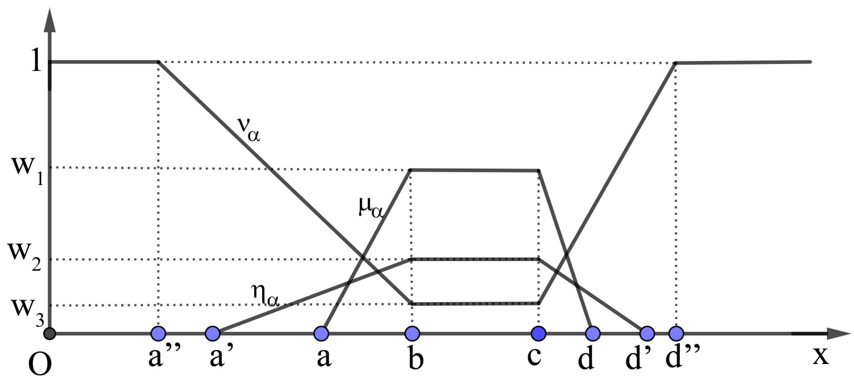

Figure 1. Trapezoidal picture fuzzy number

Remark. For the particular case of $(a, b, c, d) = (a', b, c, d') = (a'', b, c, d'')$, TrPFNs can be characterized as $A = \langle (a, b, c, d); w_1, w_2, w_3 \rangle$ and henceforth called special trapezoidal picture fuzzy numbers (STrPFNs).

Definition 1.5. Akram *et al.* (2022) A STrPFN $A = \langle (a, b, c, d); w_1, w_2, w_3 \rangle$ is non-negative (respectively non-positive), denoted as $A \geq 0$ (respectively $A \leq 0$), if $a \geq 0$ (respectively $d \leq 0$).

Definition 1.6. Akram *et al.* (2022) Two STrPFNs $A = \langle (a_1, b_1, c_1, d_1); w_{1A}, w_{2A}, w_{3A} \rangle$ and $B = \langle (a_2, b_2, c_2, d_2); w_{1B}, w_{2B}, w_{3B} \rangle$ are said to be equal if $a_1 = a_2$, $b_1 = b_2$, $c_1 = c_2$, $d_1 = d_2$, $w_{1A} = w_{1B}$, $w_{2A} = w_{2B}$ and $w_{3A} = w_{3B}$.

2. The expected value for different types of fuzzy numbers

This section refers to some basic concepts related to fuzzy numbers Dubois & Prade (1978), intuitionistic fuzzy numbers Grzegrorzewski (2003) and trapezoidal intuitionistic fuzzy numbers Nehi & Maleki (2005).

2.1. The expected value of a fuzzy number

Let A be an fuzzy number in the set of real numbers $\mathbb{R}$. There exist the numbers $a, b, c, d \in \mathbb{R}, a \leqslant b \leqslant c \leqslant d$, the function increasing continuous $f_A^L : \mathbb{R} \to [0, 1]$ and the function nonincreasing continuous $f_A^R : \mathbb{R} \to [0, 1]$, with which the membership function μ_A is expressed:

$$\mu_A(x) = \begin{cases} f_A^L(x) & if \quad a \leqslant x < b \\ 1 & if \quad b \leqslant x \leqslant c \\ f_A^R(x) & if \quad c < x \leqslant d \\ 0 & otherwise \end{cases}$$

with $0 \leqslant \mu_A(x) \leqslant 1$.

The functions f_A^L and f_A^R are referred to, respectively, as the left-hand side and the right-hand side of the fuzzy number A.

The set α-cut of a fuzzy number A, defined as

$$A^\alpha = \{x \in \mathbb{R} : \mu_A(x) \geq \alpha\}$$

is a closed interval $A^\alpha = [A_1(\alpha), A_2(\alpha)]$, where

$$A_1(\alpha) = \inf\{x \in \mathbb{R}\ \mu_A(x) \geq \alpha\}; \quad A_2(\alpha) = \sup\{x \in \mathbb{R}\ \mu_A(x) \geq \alpha\}.$$

If the sides of the fuzzy number are strictly monotone then, the convention is used that:

$$(f_A^L)^{-1}(\alpha) = A_1(\alpha);\ (f_A^R)^{-1}(\alpha) = A_2(\alpha).$$

Two important notions related to fuzzy numbers are the expected interval $EI(A)$ and the expected value $EV(A)$ of a fuzzy number A, introduced independently in Dubois & Prade (1987) and Heilpern (1992).

The expected interval of a fuzzy number $A = (a, b, c, d)$ is a crisp interval

$$EI(A) = \left[\int_0^1 A_1(\alpha)d\alpha, \int_0^1 A_2(\alpha)d\alpha\right]$$

or, equivalently,

$$EI(A) = [E_1(A), E_2(A)]\,, \tag{2.1}$$

where

$$E_1(A) = b - \int_a^b f_A^L(x)dx; \quad E_2(A) = c + \int_c^d f_A^R(x)dx. \tag{2.2}$$

The expected value of a fuzzy number A is the center of the expected interval $EI(A)$, i.e.

$$EV(A) = \frac{E_1(A) + E_2(A)}{2}. \tag{2.3}$$

For a generalized trapezoidal fuzzy number $A = \langle (a,b,c,d), w\rangle$, $0 \le w \le 1$ for which the membership function is defined as follows

$$\mu_A(x) = \begin{cases} \frac{w(x-a)}{b-a} & if \quad a \leqslant x < b \\ w & if \quad b \leqslant x \leqslant c \\ \frac{(d-x)}{d-c} & if \quad c < x \leqslant d \\ 0 & otherwise, \end{cases}$$

the expected interval is

$$EI(A) = [E_1(A), E_2(A)] = \left[\frac{a+b}{2}\cdot w, \frac{c+d}{2}\cdot w\right]$$

and the expected value is

$$EV(A) = \frac{(a+b+c+d)w}{4}. \tag{2.4}$$

In particular, if $w = 1$ i.e. $A = (a,b,c,d)$ is a trapezoidal fuzzy number, then the expected interval is $EI(A) = \left[\frac{a+b}{2}, \frac{c+d}{2}\right]$ and the expected value is $EV(A) = \frac{a+b+c+d}{4}$.

These results have been employed in various methods for ranking fuzzy numbers. For example Jimnez (1996) a direct comparison of the expected intervals is proposed, while in Asady (2013) the approximation of the fuzzy number includes not only the expected interval but also the core of the fuzzy number.

The expected value (2.4), together with the variance, constitutes fundamental characteristics of a generalized trapezoidal fuzzy number, on the basis of which a novel and efficient similarity measure was developed in Dutta & Borah (2023) and subsequently applied to decision-making problems.

2.2. *The expected value of a intuitionistic fuzzy number*

Let A be an intuitionistic fuzzy number (IFN) in the set of real numbers $\mathbb{R}$. There exist the numbers $a, b, c, d, a', b', c', d' \in \mathbb{R}$, $a' \leqslant a \leqslant b' \leqslant b \leqslant c \leqslant c' \leqslant d \leqslant d'$, the increasing functions $f_A^L, g_A^R : \mathbb{R} \to [0,1]$ and the nonincreasing functions $f_A^R, g_A^L : \mathbb{R} \to [0,1]$, so that the membership function μ_A and the non-membership function ν_A are defined as:

$$\mu_A(x) = \begin{cases} f_A^L(x) & if \quad a \leqslant x < b \\ 1 & if \quad b \leqslant x \leqslant c \\ f_A^R(x) & if \quad c < x \leqslant d \\ 0 & otherwise, \end{cases} \qquad \nu_A(x) = \begin{cases} g_A^L(x) & if \quad a' \leqslant x < b' \\ 0 & if \quad b' \leqslant x \leqslant c' \\ g_A^R(x) & if \quad c' < x \leqslant d' \\ 1 & otherwise, \end{cases}$$

with $0 \leqslant \mu_A(x) + \nu_A(x) \leqslant 1$.

The expected interval and the expected value of an intuitionistic fuzzy number $A = \langle (a,b,c,d)(a',b',c',d')\rangle$ have been defined in Grzegrorzewski (2003).

The expected interval of a intuitionistic fuzzy number A is a crisp interval $EI(A)$ given by

$$EI(A) = [E_1(A), E_2(A)],$$

where

$$\begin{aligned} E_1(A) &= \frac{a'+b}{2} + \frac{1}{2}\int_{a'}^{b'} g_A^L(x)dx - \frac{1}{2}\int_a^b f_A^L(x)dx; \\ E_2(A) &= \frac{c+d'}{2} + \frac{1}{2}\int_c^d f_A^R(x)dx - \frac{1}{2}\int_{c'}^{d'} g_A^R(x)dx. \end{aligned} \tag{2.5}$$

Remark. For a generalized trapezoidal intuitionistic fuzzy number (GTrIFN) $A = \langle(a,b,c,d)(a',b',c',d'); w_1, w_2\rangle$ for which the membership function and the nonmembership function are defined as follows

$$\mu_A(x) = \begin{cases} \frac{w_1(x-a)}{b-a} & if \quad a \leqslant x < b \\ w_1 & if \quad b \leqslant x \leqslant c \\ \frac{w_1(d-x)}{d-c} & if \quad c < x \leqslant d \\ 0 & otherwise, \end{cases} \qquad \nu_A(x) = \begin{cases} \frac{(b'-x)+w_2(x-a')}{b'-a'} & if \quad a' \leqslant x < b' \\ w_2 & if \quad b' \leqslant x \leqslant c' \\ \frac{(x-c')+w_2(d'-x)}{d'-c'} & if \quad c' < x \leqslant d' \\ 1 & otherwise, \end{cases}$$

similarly to (2.5) we obtain:

$$\begin{aligned} E_1(A) &= \tfrac{a'-b'w_2+bw_1}{2} + \tfrac{1}{2}\int_{a'}^{b'} \tfrac{(b'-x)+w_2(x-a')}{b'-a'}dx - \tfrac{1}{2}\int_a^b \tfrac{w_1(x-a)}{b-a}dx = \tfrac{(a'+b')(1-w_2)+(a+b)w_1}{4} \\ E_2(A) &= \tfrac{w_1c+d'-c'w_2}{2} + \tfrac{1}{2}\int_c^d \tfrac{w_1(d-x)}{d-c}dx - \tfrac{1}{2}\int_{c'}^{d'} \tfrac{(x-c')+w_2(d'-x)}{d'-c'}dx = \tfrac{(c'+d')(1-w_2)+(c+d)w_1}{4}. \end{aligned}$$

The expected value for a GTrIFN is the center of the expected interval $EI(A) = [E_1(A), E_2(A)]$, i.e.

$$EV(A) = \frac{E_1(A) + E_2(A)}{2} = \frac{(a+b+c+d)w_1 + (a'+b'+c'+d')(1-w_2)}{8}. \tag{2.6}$$

In particular, for $w_1 = 1$ and $w_2 = 0$ that is, for $A = \langle(a,b,c,d)(a',b',c',d'); 1, 0\rangle$, the expected value of a trapezoidal intuitionistic fuzzy number as given in Ye (2011) is recovered:

$$EV(A) = \frac{a+b+c+d+a'+b'+c'+d'}{8}. \tag{2.7}$$

Another noteworthy particular case is the expected value for a generalized trapezoidal intuitionistic fuzzy number in which $b = b'$ and $c = c'$ i.e., $A = \langle(a,b,c,d)(a',b,c,d'); w_1, w_2\rangle$:

$$EV(A) = \frac{(a+d)w_1 + (a'+d')(1-w_2) + (b+c)(1+w_1-w_2)}{8}. \tag{2.8}$$

If, in addition, $a = a'$ and $d = d'$, then the expected value of a generalized trapezoidal intuitionistic fuzzy number $A = \langle(a,b,c,d); w_1, w_2\rangle$ is obtained:

$$EV(A) = \frac{(a+b+c+d)(1+w_1-w_2)}{8}. \tag{2.9}$$

Using the concept of the expected value of an intuitionistic fuzzy number, various ranking methods have been developed, which have subsequently been applied to practical problems Ye (2011), Nishad & Singh (2014), Chakraborty *et al.* (2015), Li & Chen (2015), Liu *et al.* (2016).

2.3. *The expected value of a picture fuzzy number*

Let $A = \langle(\mu_A, \eta_A, \nu_A); w_1, w_2, w_3\rangle$ be a picture fuzzy number for which μ_A, η_A and ν_A are defined as in (1.1). The (α, γ, β)-cut section of A, as defined in (1.2) consists of three closed intervals:

$$\begin{aligned} A^\alpha &= [A_1(\alpha), A_2(\alpha)]; \quad \alpha \in [0, w_1] \\ A^\gamma &= [A_1(\gamma), A_2(\gamma)]; \quad \gamma \in [0, w_2] \\ A^\beta &= [A_1(\beta), A_2(\beta)]; \quad \beta \in [w_3, 1], \end{aligned} \tag{2.10}$$

where

$$\begin{aligned} A_1(\alpha) &= \inf\{x \in \mathbb{R}\ \mu_A(x) \geq \alpha\} & A_2(\alpha) &= \sup\{x \in \mathbb{R}\ \mu_A(x) \geq \alpha\} \\ A_1(\gamma) &= \inf\{x \in \mathbb{R}\ \eta_A(x) \geq \gamma\} & A_2(\gamma) &= \sup\{x \in \mathbb{R}\ \eta_A(x) \geq \gamma\} \\ A_1(\beta) &= \inf\{x \in \mathbb{R}\ \nu_A(x) \leq \beta\} & A_2(\beta) &= \sup\{x \in \mathbb{R}\ \nu_A(x) \leq \beta\} \end{aligned}$$

In this case the following relations hold: $(f_A^L)^{-1}(\alpha) = A_1(\alpha)$; $(f_A^R)^{-1}(\alpha) = A_2(\alpha)$, $(g_A^L)^{-1}(\gamma) = A_1(\gamma)$; $(g_A^R)^{-1}(\gamma) = A_2(\gamma)$, $(h_A^L)^{-1}(\beta) = A_1(\beta)$; $(h_A^R)^{-1}(\beta) = A_2(\beta)$.

Proposition 1. *The expected interval of a picture fuzzy number A is a crisp interval $EI(A)$ given by*

$$EI(A) = [E_1(A), E_2(A)], \tag{2.11}$$

where

$$\begin{aligned} E_1(A) &= \frac{a'' + b(w_1 + w_2 - w_3)}{2} + \frac{1}{2}\int_{a''}^{b} h_A^L(x)dx - \frac{1}{2}\int_{a}^{b} f_A^L(x)dx - \frac{1}{2}\int_{a'}^{b} g_A^L(x)dx; \\ E_2(A) &= \frac{d'' + c(w_1 + w_2 - w_3)}{2} + \frac{1}{2}\int_{c}^{d'} f_A^R(x)dx + \frac{1}{2}\int_{c}^{d} g_A^R(x)dx - \frac{1}{2}\int_{c}^{d''} h_A^R(x)dx. \end{aligned} \tag{2.12}$$

Proof. Since the picture fuzzy number A can be decomposed into three fuzzy numbers corresponding to the membership function μ_A, the neutrality function η_A and the non-membership function ν_A, with continuous and strictly monotonic sides f_A^L, f_R^L, g_A^L, g_A^R, h_A^L, h_A^R,we have that

$$\begin{aligned} E_1(A) &= \frac{a'' - bw_3}{2} + \frac{1}{2}\int_{a''}^{b} h_A^L(x)dx + \frac{bw_1}{2} - \frac{1}{2}\int_{a}^{b} f_A^L(x)dx + \frac{bw_2}{2} - \frac{1}{2}\int_{a'}^{b} g_A^L(x)dx; \\ E_2(A) &= \frac{cw_1}{2} + \frac{1}{2}\int_{c}^{d'} f_A^R(x)dx + \frac{cw_2}{2} + \frac{1}{2}\int_{c}^{d} g_A^R(x)dx + \frac{d'' - cw_3}{2} - \frac{1}{2}\int_{c}^{d''} h_A^R(x)dx. \end{aligned}$$

□

Definition 2.1. The expected value of a picture fuzzy number A is the center of the expected interval $EI(A)$, i.e.

$$EV(A) = \frac{E_1(A) + E_2(A)}{2}. \tag{2.13}$$

Theorem 2.1. *For the trapezoidal picture fuzzy number $A = \langle (a, b, c, d), (a', b, c, d'), (a'', b, c, d''); w_1, w_2, w_3\rangle$ defined as in (1.4), the expected value is:*

$$EV(A) = \frac{(a + d)w_1 + (a' + d')w_2 + (a'' + d'')(1 - w_3) + (b + c)(1 + w_1 + w_2 - w_3)}{8}. \tag{2.14}$$

Proof. Formulas (2.12) are applied to calculate the endpoints of the expected interval, $E_1(A)$ and $E_2(A)$.

$$\begin{aligned} E_1(A) &= \tfrac{a''+b(w_1+w_2-w_3)}{2} + \tfrac{1}{2}\int_{a''}^{b} \tfrac{(b-x)+w_3(x-a'')}{b-a''}dx - \tfrac{1}{2}\int_{a}^{b} \tfrac{w_1(x-a)}{b-a}dx - \tfrac{1}{2}\int_{a'}^{b} \tfrac{w_2(x-a')}{b-a'}dx = \\ &= \tfrac{aw_1+a'w_2+a''(1-w_3)+b(1+w_1+w_2-w_3)}{4}, \\ E_2(A) &= \tfrac{d''+c(w_1+w_2-w_3)}{2} + \tfrac{1}{2}\int_{c}^{d} \tfrac{w_1(d-x)}{d-c}dx + \tfrac{1}{2}\int_{c}^{d'} \tfrac{w_2(d'-x)}{d'-c}dx - \tfrac{1}{2}\int_{c}^{d''} \tfrac{(x-c)+w_3(d''-x)}{d''-c}dx = \\ &= \tfrac{dw_1+d'w_2+d''(1-w_3)+c(1+w_1+w_2-w_3)}{4}, \end{aligned}$$

and according to (2.13) we obtain (2.14). □

Remark. In particular, if $a = a' = a''$ si $d = d' = d''$, for the trapezoidal picture fuzzy number $A = \langle(a, b, c, d); w_1, w_2\rangle$, the expected value becomes:

$$EV(A) = \frac{(a + b + c + d)(1 + w_1 + w_2 - w_3)}{8}. \tag{2.15}$$

A similar result was obtained in Akram *et al.* (2021).

3. Conclusion

The paper addresses the concepts of expected interval and expected value for picture fuzzy numbers, which provide the foundation for developing a ranking method for picture fuzzy numbers, similar to that in Grzegrorzewski (2003) for the case of intuitionistic fuzzy numbers.

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